## ECE 761 - Homework \#11

Control of a 2-Link Robotic Arm


Use the dynamics you derived in homework \#10 for a 2-link arm. The dynamics should look something like this (equations for $\mathrm{m} 1=\mathrm{m} 2=1 \mathrm{~kg}$. Your numbers will be slightly different due to using different masses).

$$
\left[\begin{array}{cc}
\left(4+2 c_{2}\right) & \left(1+c_{2}\right) \\
\left(1+c_{2}\right) & 1
\end{array}\right]\left[\begin{array}{c}
\ddot{\theta}_{r 1} \\
\ddot{\theta}_{r 2}
\end{array}\right]=T-g\left[\begin{array}{c}
3 c_{1}+c_{12} \\
c_{12}
\end{array}\right]+\left[\begin{array}{c}
2 s_{2} \dot{\theta}_{1} \dot{\theta}_{2}+s_{2} \dot{\theta}_{2}^{2} \\
-s_{2} \dot{\theta}_{1}^{2}
\end{array}\right]
$$

1) Simulate the motion of a $R R$ robot under freefall $(T=0)$ from the initial condition of $[0,0]$. Check to see if the motion is reasonable (gravity is down).

Problem 2-6: Write a program (modify PD_Control.txt) to trace out a star (or another shape you like) with corners at

|  | P 1 | P 2 | P 3 | P 4 | P 5 | $\mathrm{P} 6=\mathrm{P} 1$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| X | 1.0000 | 1.2939 | 0.5245 | 1.4755 | 0.7061 | 1.0000 |
| Y | -1.5000 | 0.3090 | -0.8090 | -0.8090 | 0.3090 | -1.5000 |

and a 2 -second motion from point to point

Simulate and plot the tracking error for
2) PD control

$$
T=100\left(\theta_{r}-\theta\right)+14\left(-\frac{d \theta}{d t}\right)
$$

3) PD + Feedforward Control (gravity)

- The gravity matrix here is for $\mathrm{m} 1=\mathrm{m} 2=1 \mathrm{~kg}$ (your numbers will be different)
- If done correctly, the feedforward term (gravity) should cancel the gravity term in your dynamics

$$
T=100\left(\theta_{r}-\theta\right)+14\left(-\frac{d \theta}{d t}\right)+g\left[\begin{array}{c}
3 c_{1}+c_{12} \\
c_{12}
\end{array}\right]
$$

4) PD + Gravity + Velocity feedforward terms

$$
T=100\left(\theta_{r}-\theta\right)+14\left(\frac{d \theta_{r}}{d t}-\frac{d \theta}{d t}\right)+g\left[\begin{array}{c}
3 c_{1}+c_{12} \\
c_{12}
\end{array}\right]
$$

5) PD + Gravity + Velocity + Coriolis feedforward terms

- The he gravity and coroilis matrices are for $\mathrm{m} 1=\mathrm{m} 2=1 \mathrm{~kg}$ (your numbers will differ slightly)
- If done correctly, the feedforward terms should cancel corresponding terms in your dynamics

$$
T=100\left(\theta_{r}-\theta\right)+14\left(\frac{d \theta_{r}}{d t}-\frac{d \theta}{d t}\right)+g\left[\begin{array}{c}
3 c_{1}+c_{12} \\
c_{12}
\end{array}\right]-\left[\begin{array}{c}
2 s_{2} \dot{\theta}_{1} \dot{\theta}_{2}+s_{2} \dot{\theta}_{2}^{2} \\
-s_{2} \dot{\theta}_{1}^{2}
\end{array}\right]
$$

6) PD + Gravity + Velocity + Coriolis + Acceleration

- ditto

$$
\begin{aligned}
& T=100\left(\theta_{r}-\theta\right)+14\left(\frac{d \theta_{r}}{d t}-\frac{d \theta}{d t}\right)+g\left[\begin{array}{c}
3 c_{1}+c_{12} \\
c_{12}
\end{array}\right]-\left[\begin{array}{c}
2 s_{2} \dot{\theta}_{1} \dot{\theta}_{2}+s_{2} \dot{\theta}_{2}^{2} \\
-s_{2} \dot{\theta}_{1}^{2}
\end{array}\right] \\
& +\left[\begin{array}{cc}
\left(4+2 c_{2}\right) & \left(1+c_{2}\right) \\
\left(1+c_{2}\right) & 1
\end{array}\right]\left[\begin{array}{c}
\ddot{\theta}_{r 1} \\
\ddot{\theta}_{r 2}
\end{array}\right]
\end{aligned}
$$

