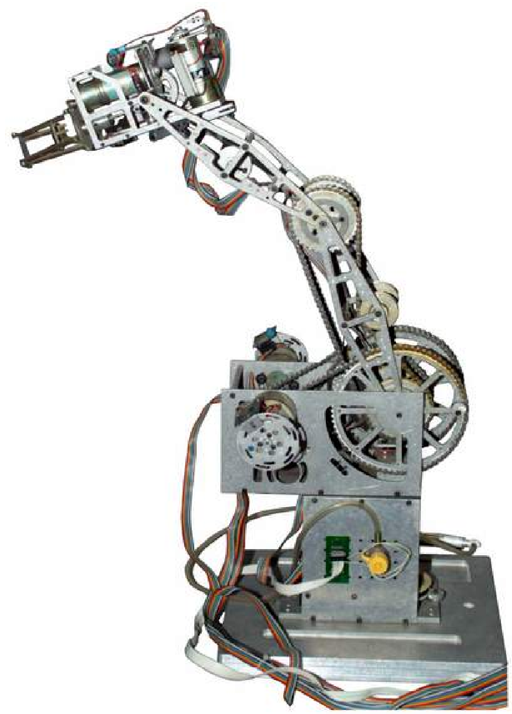
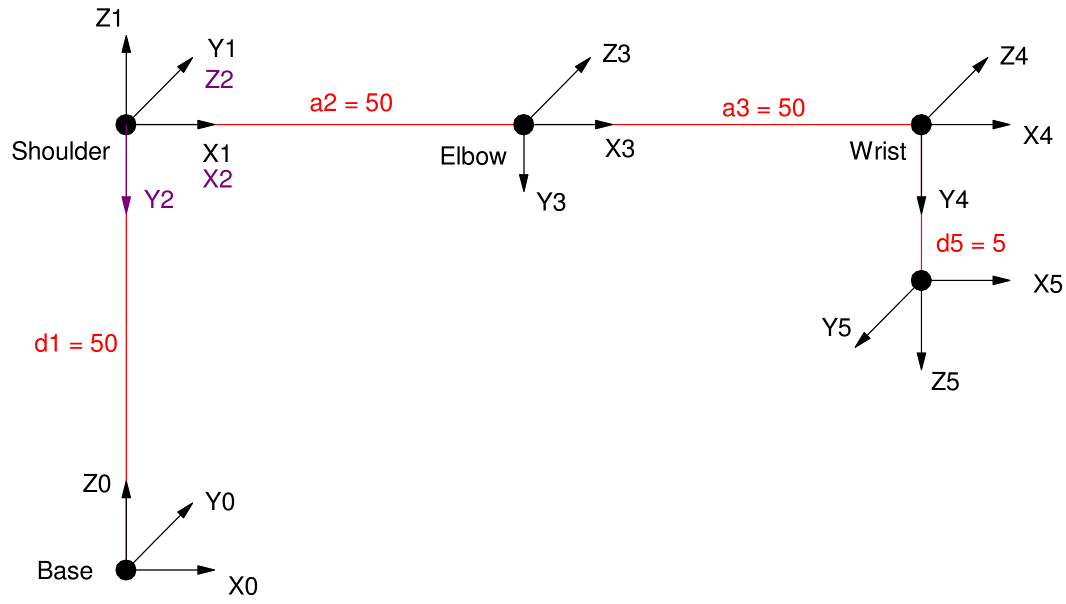
Inverse Kinematics for a Rhino Robot

Rhino Robot: Forward Kinamatics



Rhino Robot: http://www.theoldrobots.com/images40/rinoarm4.JPG

The Rhino Robot is a 4 degree-of-freedom robot used to illustrate programming and control of robot manipulators for classroom settings. To determine the tip position given the joint angles (termed forward kinematics), define the reference frames. One valid definition for the reference frames for a Rhino robot are as follows:



Reference Frames for a Rhino Robot

This results in the following specifications for a Rhino Robot:

Link i	α_{i-1} The angle between the Zi-1 and Zi axis (twist)	a_{i-1} The distance from Zi-1 to Zi measured along the Xi-1 axis	d_i The distance from Xi-1 to Xi measured along the Zi axis (cm)	Q_i The angle between Xi-1 and Xi measured about the Zi axis
1	0 deg	0	d1 = 50	Q1
2	-90 deg	0	0	Q2
3	0 deg	a2 = 50	0	Q3
4	0 deg	a3 = 50	0	Q4
5	-90 deg	0	d5 = 5	Q5
6				

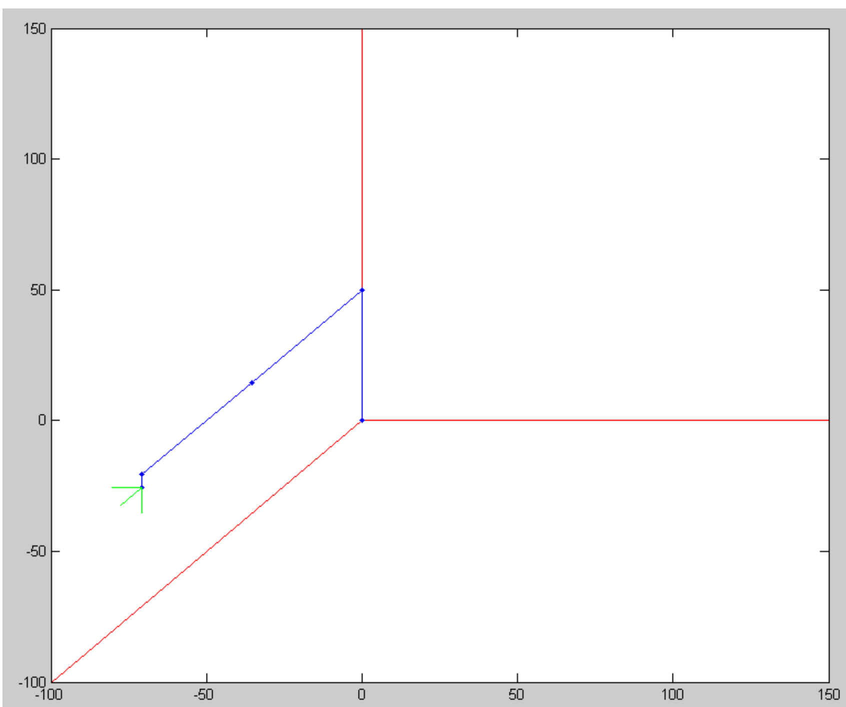
With all the axis aligned, the forward kinematics simplify somewhat. The previous Matlab routine RRR works for a Rhino robot as well if you change the robot definition:

```
function [Tip] = Rhino(W, TIP)

Q     = [W(1), W(2), W(3), W(4), W(5)];
alpha = [0, -pi/2, 0, 0, -pi/2];
a = [0, 0, 50, 50, 0];
d = [50, 0, 0, 0, 5];


T01a = Transform(alpha(1), a(1), 0,    Q(1));
T01  = Transform(alpha(1), a(1), d(1), Q(1));
T12a = Transform(alpha(2), a(2), 0,    Q(2));
T12  = Transform(alpha(2), a(2), d(2), Q(2));
T23a = Transform(alpha(3), a(3), 0,    Q(3));
T23  = Transform(alpha(3), a(3), d(3), Q(3));
T34a = Transform(alpha(4), a(4), 0,    Q(4));
T34  = Transform(alpha(4), a(4), d(4), Q(4));
T45a = Transform(alpha(5), a(5), 0,    Q(5));
T45  = Transform(alpha(5), a(5), d(5), Q(5));

(the rest of the code is almost identical to the RRR.m file)
```



Screen Shot of the Rhino Robot at home position: Q = {0, 0, 0, 0, 0}

Forward Kinematics: Given the joint angles, the tip position is:

$$P_0 = T_{01} T_{12} T_{23} T_{34} T_{45} P_5$$

where P5 is the origin of reference frame #5 (the tip). For example, at zero position (shown above), the tip position is

```
>> Rhino([0,0,0,0,0], zeros(4,1))

x  100.0000
y    0.0000
z   45.0000
     1.0000
```

Inverse Kinematics solves the inverse problem: given the tip position, determine the joint angles.

Inverse Kinematics: Algebraic Solution:

Given the joint angles, the tip position is known via transformation matricies. From before, the transform matrix to go from reference frame 1 to reference frame 0 is

$$T_{01} = R_x(\alpha_0) D_x(a_0) R_z(\theta_1) D_z(d_1)$$

or

$$T_{01} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_0 \\ s\theta_1 c\alpha_0 & c\theta_1 c\alpha_0 & -s\alpha_0 & -s\alpha_0 d_1 \\ s\theta_1 s\alpha_0 & c\theta_1 s\alpha_0 & c\alpha_0 & c\alpha_0 d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For a Rhino robot

$$T_{01} = R_x(0) D_x(0) R_z(\theta_1) D_z(50)$$

meaning

$$T_{01} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transform for going from reference frame 1 to 2 is

$$T_{12} = R_x(\alpha_1) D_x(a_1) R_z(\theta_2) D_z(d_2)$$

$$T_{12} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transform for going from reference frame 2 to 3 is

$$T_{23} = R_x(\alpha_2)D_x(a_2)R_z(\theta_3)D_z(d_3)$$

$$T_{23} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 50 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transform for going from frame 3 to 4 (fame 4 is the tip) is

$$T_{34} = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & 50 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The tip position is at

$$P_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Relative to frame zero, this is

$$P_0 = T_{01}T_{12}T_{23}T_{34}P_4$$

or

$$P_0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & 50 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & -s_4 & 0 & 50 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

There is some redundancy in the Rhino robot: there are four angles to define the tip position (which has three contraints). The wrist position has only 3 degrees of freedom (angles 1 .. 3) and three contraints (x, y, z)

$$P_0 = \begin{vmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} c_3 & -s_3 & 0 & 50 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 50 \\ 0 \\ 0 \\ 1 \end{vmatrix}$$

Multiplying this out...

$$P_0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50c_3 + 50 \\ 50s_3 \\ 0 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (50c_3 + 50)c_2 - 50s_2 s_3 \\ 0 \\ -(50c_3 + 50)s_2 - 50c_2 s_3 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} ((50c_3 + 50)c_2 - 50s_2 s_3)c_1 \\ ((50c_3 + 50)c_2 - 50s_2 s_3)s_1 \\ -(50c_3 + 50)s_2 - 50c_2 s_3 + 50 \\ 1 \end{bmatrix}$$

or, to solve for the three angles, simply solve the following three equations for three unknowns:

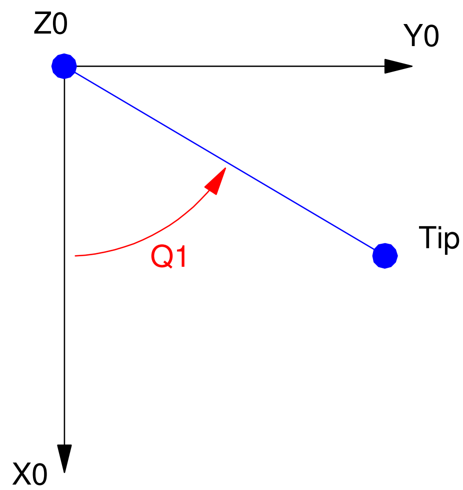
$$x = ((50c_3 + 50)c_2 - 50s_2 s_3)c_1$$

$$y = ((50c_3 + 50)c_2 - 50s_2 s_3)s_1$$

$$z = -(50c_3 + 50)s_2 - 50c_2 s_3 + 50$$

Inverse Kinematics: Geometric Solution

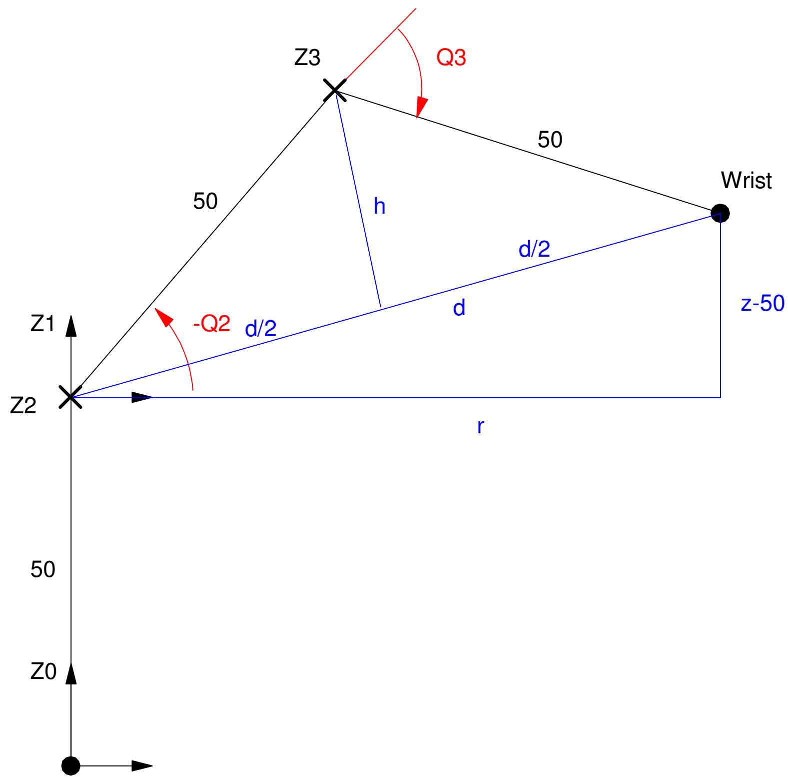
The top view of a Rhino Robot tells you Q1:



Top view of a Rhino robot

$$\theta_1 = \arctan\left(\frac{x_{tip}}{y_{tip}}\right)$$

To find the angles 2 and 3, take a cross section along the plane of the robot (or assume it is at 0 degrees). The side view of a Rhino Robot is



Side view of a Rhino robot

The distances are

$$r = \sqrt{x_{tip}^2 + y_{tip}^2}$$

$$d = \sqrt{r^2 + (z-50)^2}$$

$$h = \sqrt{50^2 - \left(\frac{d}{2}\right)^2}$$

$$\theta_a = \arctan\left(\frac{z-50}{r}\right)$$

$$\theta_b = \arctan\left(\frac{h}{d/2}\right)$$

$$\theta_2 = \theta_a + \theta_b$$

$$-\theta_3 = 180^0 - 2 \cdot \arctan\left(\frac{d/2}{h}\right)$$

Check: Assume the wrist is at (50, 0, 0)

$$\theta_1 = \arctan\left(\frac{0}{50}\right) = 0^0$$

$$r = 50$$

$$d = 70.71$$

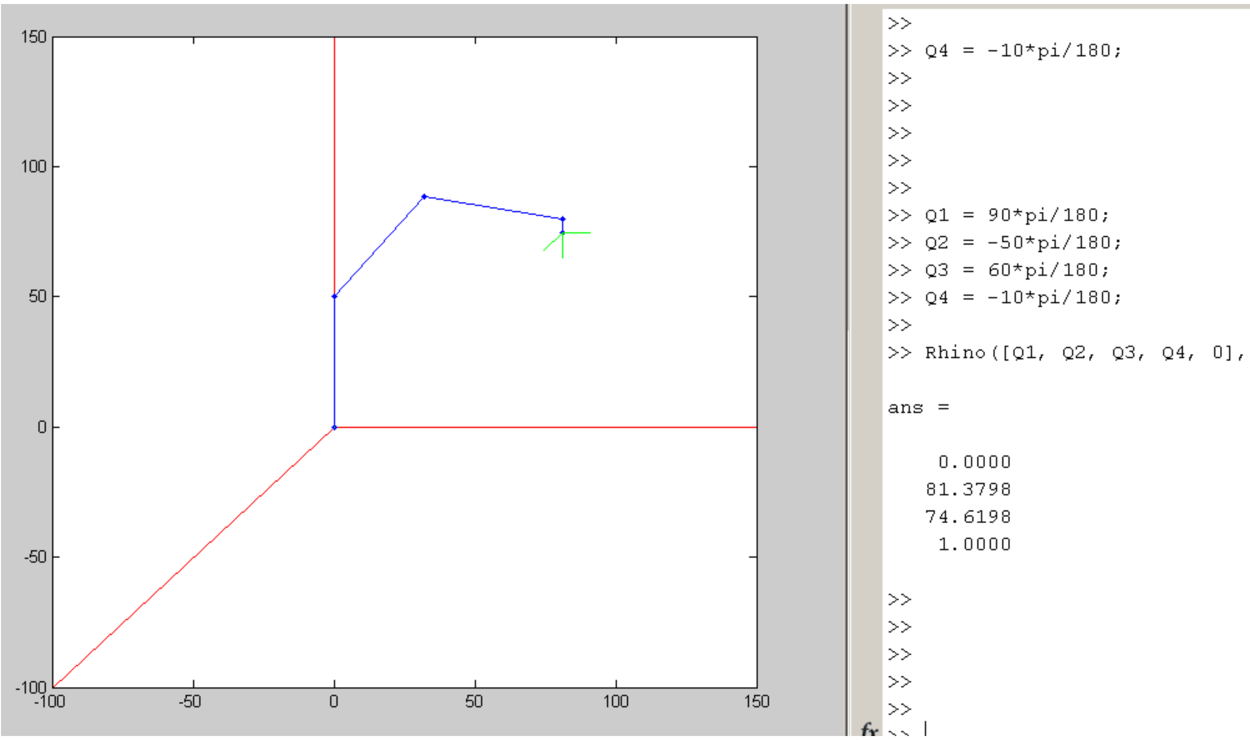
$$h = 35.3553$$

$$\theta_a = \arctan\left(\frac{-50}{50}\right) = -45^0$$

$$\theta_b = \arctan\left(\frac{35.3553}{35.3553}\right) = +45^0$$

$$\theta_2 = \theta_a + \theta_b = 0^0$$

$$\theta_3 = 2\arctan\left(\frac{35.3553}{35.3553}\right) - 180^0 = -90^0$$




```
>> Q1 = 90*pi/180;
>> Q2 = -50*pi/180;
>> Q3 = 60*pi/180;
>> Q4 = -(Q2+Q3);
>> Tip = Rhino([Q1, Q2, Q3, -(Q2+Q3), 0], [0;0;0;1])

    0.0000
   81.3798
   74.6198
    1.0000

>> r = sqrt(Tip(1)^2 + Tip(2) ^ 2)

   81.3798

>> z = Tip(3) + 5

   79.6198

>> d = sqrt(r^2 + (z-50)^2)

   86.6025

>> h = sqrt(50^2 - (d/2)^2)

   25.0000

>> qa = atan2(z-50,r) * 180/pi

    20

>> qb = atan2(h, d/2) * 180/pi
```

```
   30.0000

>> q2 = -(qa + qb)

  -50.0000

>> q3 = pi - 2*atan2(d/2, h)

    1.0472

>> q3*180/pi

   60.0000

>>
```