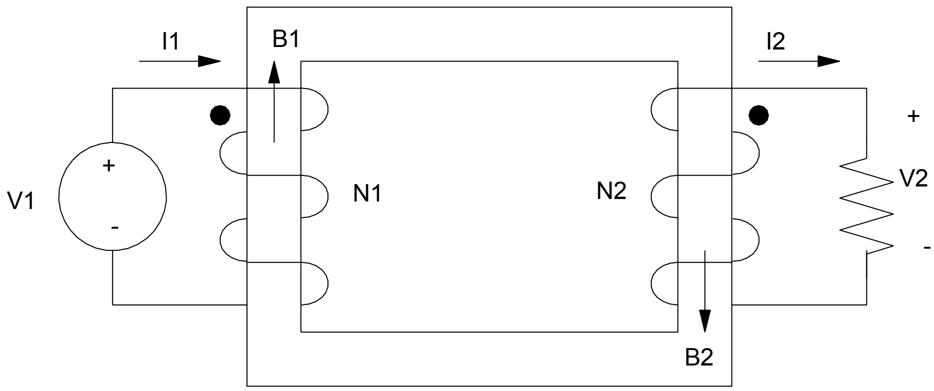
Transformer Testing

Ideal Iron Core Transformer:

A transformer is an inductor with two or more sets of windings around a common core.



Our goal is to develop a model for the transformer.

First, let's assume I2=0 (the coil to the right is open). In this case, you just have an inductor. For an ideal inductor,

$$v_1 = L\frac{di_1}{dt}$$

If V1 is sinusoidal with an amplitude of 1 (arbitrarilly):

$$v_1 = \cos(\omega t) = 1\angle 0^0$$

$$i_1 = \frac{-1}{L\omega}\sin(\omega t) = \frac{1}{L\omega}\angle 90^0$$

Note that the voltage and current are 90 degrees apart. This results in the power delivered to the inductor being:

$$P = v_1 i_1 = (\cos(\omega t))\left(\frac{-1}{L\omega}\sin(\omega t)\right) = \frac{-1}{2L\omega}\sin(2\omega t)$$

The power is on average zero. It bounces from positive to negative - meaning the magnetic field is built up (storing energy) and collapses (releasing energy) twice per cycle. Using phasor notation:

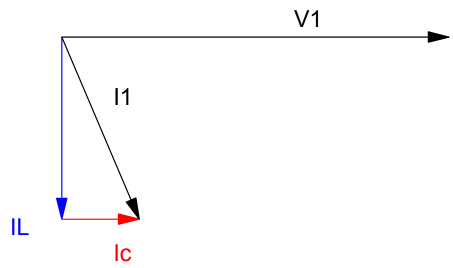
$$P = \frac{1}{2}vi^* = \frac{1}{2}(1\angle 0^0)\left(\frac{1}{L\omega}\angle -90^0\right) = \frac{1}{2L\omega}\angle -90^0 = \frac{j}{2L\omega}$$

The real part of the power (zero) is the heat produced. The complex part of the power is the energy being stored and released in the magnetic field over and over again.

To improve this model, add the losses due to:

- Eddy currents
- Hysteresis
- Armature Resistance:

This results in the current having a component perpindicular to the voltage (IL from the inductance) and a a component parallel to the voltage (Ic: the core losses)

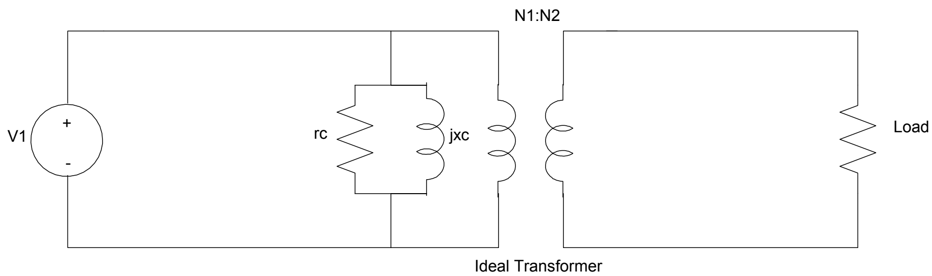


A model for a transformer (or an inductor) with no load is then as follows where

- rc models the energy dissipated in the core losses and the eddy currents

  $\frac{v_1^2}{r_c} = P_e + P_c$:

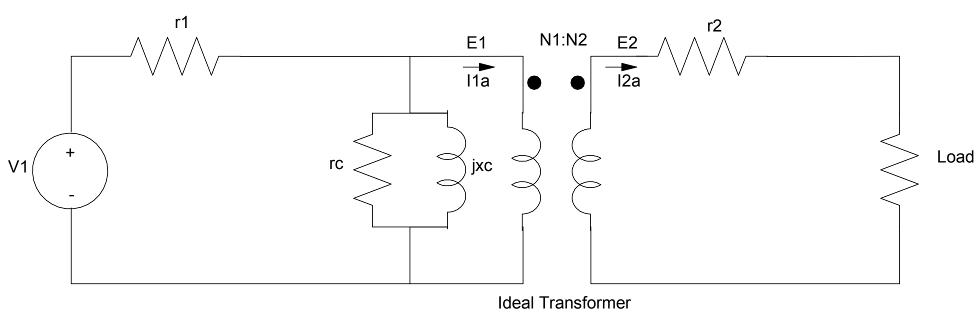
- jxc models the inductance of the transformer



Next, add in the i2r resistance losses. Since the current will vary with the load, add this in series. Similarly, there will be losses on the load side as well:

r1 = the resistance of the high side of the transformer

r2 = the resitance of hte low side of the transformer



Finally, assume the load is present, resulting in I2 being non-zero. For an energy balance:

$$v_{1a}i_{1a} = v_{2a}i_{2a}$$

The flux is the same throughout the transformer. The voltage is related to the number of windings:

$$v_{2a} = \frac{N_2}{N_1}v_{1a}$$

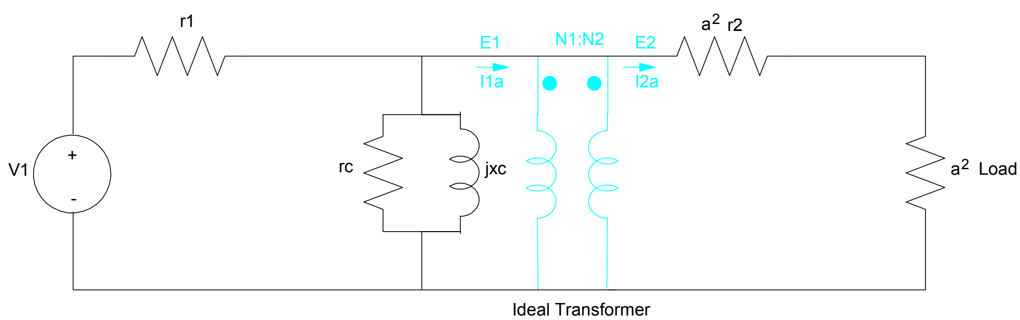
For the power to balance,

$$i_{2a} = \frac{N_1}{N_2}i_{1a}$$

The impedances seen through a transformer change as the square of the turn ratio squared:

$$Z_1 = \frac{v_{1a}}{i_{1a}} = \left(\frac{N_1}{N_2}\right)^2 \frac{v_{2a}}{i_{2a}} = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

The curcuit seen by the source (V1) is then as follows where $a^2 = \left(\frac{N_1}{N_2}\right)^2$



Often times, the core current and losses are small relative to the load. In this case, you can ignore the core resistance and inductance.

Parameters from No-Load Tests

Given a transformer, determine it's parameters.

1. Open Circuit Test.

r1: Disconnect the load. Measure the resistance of the source side of the transformer. (Apply a voltage and measure the resulting current at DC.)

xc: Disconnect the load. Measure the inductance of the source side of the transformer at the operating frequency (usually 60Hz).

rc: Disconnect the load. Apply an AC source. Measure the power lost by energizing the transformer using a Watt meter. Also measure the current and voltage.

$$e_1 = v_1 - r_1 i_1$$

$$P = e_1 i_1 \cos\theta$$

$$P = \frac{e_1^2}{r_c}$$

Short Circuit Test: Short the load (Load = 0 Ohms).

Apply a voltage to the source side and measure the power, voltage, and current.

$$P = vi\cos\theta = i^2 z^*$$

$$z = r + j\omega X =$$

Example: A 50-kVA 2400/120V (20:1)transformer has the following data. Assume zero resistance for r1:

Open circuit test: Measured on the low side:

- watt meter = 396 Watts
- ammeter: 9.65A rms
- voltmeter: 120V rms

Short-circuit test: (rms)

- watt meter = 810W
- ammeter reading = 20.8A rms
- volt meter: 92V rms

From the open-circuit test:

$$P = \frac{V^2}{R}$$

$$r_c = R = \frac{V^2}{P} = \frac{(120V)^2}{396W} = 36.36\Omega$$

The total impedance is

$$|Z| = \frac{V}{I} = \frac{120V}{9.65A} = 12.43\Omega$$

$$\left|\frac{1}{Z}\right| = \left|\frac{1}{r_c} + \frac{1}{jx_c}\right| = \frac{1}{12.43\Omega}$$

$$\frac{1}{x_c} = \sqrt{\left(\frac{1}{12.43}\right)^2 - \left(\frac{1}{36.36}\right)^2}$$

$$x_c = 13.23\Omega$$

Note that this is on the low side. To translate these through the transformer, scale be the turn ratio squared:

$$r_c = 20^2 \cdot 36.36\Omega = 14,544\Omega$$

$$jx_c = j5292\Omega$$

With the short circuit test

$$Z = (r_c)||(jx_2)||a^2(r_2 + jx_2)$$

$$Z = \left(\frac{V}{I}\right)\angle\theta$$

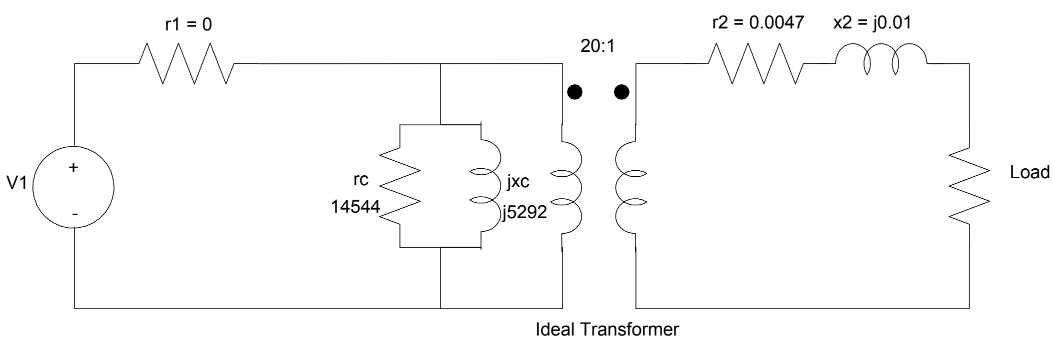
$$P = VI\cos\theta$$

$$\theta = 64.95^0$$

$$Z = 4.4231\angle 64.95^0 = (14544\Omega)||(j5292\Omega)||a^2(r_2 + jx_2)$$

$$a^2(r_2 + jx_2) = 1.8748 + j4.0105\Omega$$

$$r_2 + jx_2 = 0.0047 + j0.01\Omega$$



Effficiency and Voltage Regulation

Efficiency is defined as the energy to the load divided by the total energy delivered. For example, assume the load is 10kW at 120V

Load = 1.44 Ohms

Translating all impedances to the low-side:

$$Z = (36.36\Omega)||(j13.23\Omega)||(0.0047 + j0.01 + 1.44)\Omega$$

$$Z = 1.3724 + j0.1533$$

The total current draw is

$$I = \frac{V}{Z} = \frac{120V}{Z} = 86.897\angle 6.37^0$$

The current to the load is

$$I_L = \frac{120V}{1,4447 + j0.01} = 83.06\angle 0.396^0$$

The total power delivered is

$$P = VI^* = 10363 + j1157$$

The power to the load is

$$P = VI_L^* = 9967 + j69$$

The efficiency is then

$$Eff = \frac{9967}{10363} = 96.2\%$$

The efficiency is zero at no load (of course) and increases as the load increases.

The voltage at the load is from voltage division

$$|V_L| = \left| \left(\frac{1.44\Omega}{1.44\Omega + (0.0047 + j0.01)} \right) 120V \right| = 119.60V$$

The voltage regulation is the change in voltage as you go from no load to full load:

$$V_{reg} = \left(\frac{120V - 119.60V}{120V} \right) = 0.0033 = 0.33\%$$