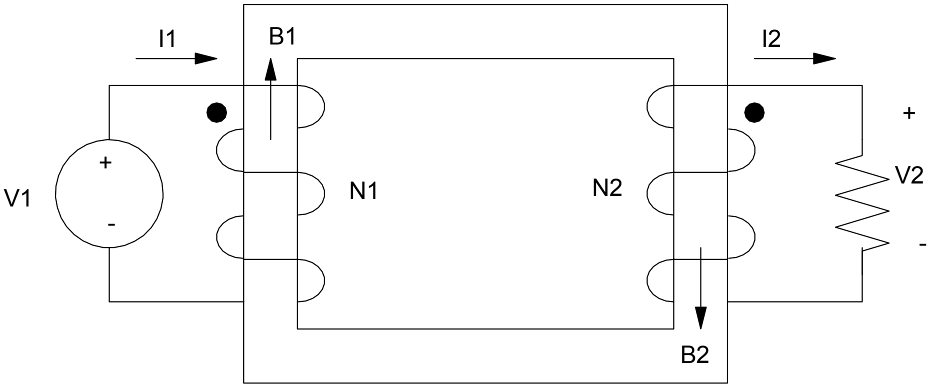
Transformers

Ideal Iron Core Transformer:

A transformer is an inductor with two or more sets of windings around a common core.



Transformers are used in power systems since they allow you to convert from one voltage to another. Assuming all the magnetic flux is confined to the core, then

$$B1 = B2$$

This results in

$$N_1 I_1 = N_2 I_2$$

or

$$I_s = \left(\frac{N_1}{N_2}\right) I_1$$

Since power out equals power in:

$$V_1 I_1 = V_2 I_2$$

or

$$V_2 = \left(\frac{N_2}{N_1}\right) V_1$$

Transformers change the AC voltage by the ratio of the turns ratio

Transformers change the AC current by the inverse ratio of the turns ratio

Transformers also change the impedance as seen through the transformer. If a load is connected to the right

$$V_2 = I_2 Z_2$$

This impedance as seen from the left side of the transformer looks like

$$Z_1 = \frac{V_1}{I_1}$$

$$Z_1 = \left(\frac{\left(\frac{N_1}{N_2}\right) V_2}{\left(\frac{N_2}{N_1}\right) I_2} \right)$$

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 \left(\frac{V_2}{I_2}\right)$$

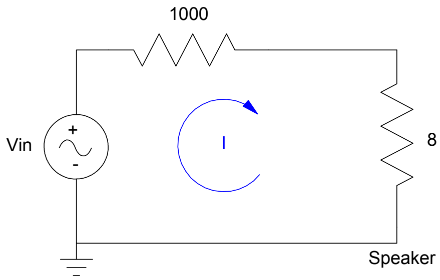
$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

Impedances change as the square of the turns ratio when going through a transformer.

This allows multiple uses for transformers.

Example 1: An amplifier has an output impedance of 1k Ohm. Design a circuit which allows this amplifier to drive an 8 Ohm speaker at 90% efficience.

Solution: You can't connect the amplifier to the 8 Ohm speaker directly - the impedance mismatch makes the efficiency close to zero:



The efficiency is

$$\eta = \frac{\text{Power Out}}{\text{Power In}}$$

$$\eta = \frac{I^2 \cdot 8\Omega}{I^2 \cdot 1008\Omega} = 0.0079$$

In order for the efficiency to by 90%, you need to

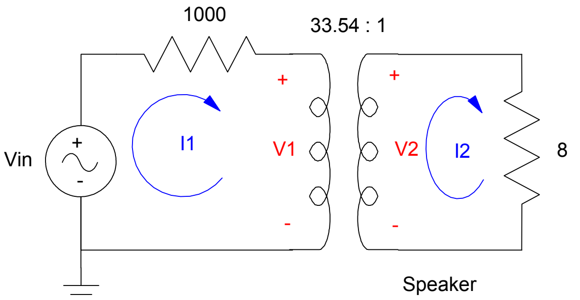
- Increase the load resistance, as seen by the amplifier, to 9000 Ohms, or
- Decrease the source resistance, as seen by the load, to 0.888 Ohms.

A transformer does this. As seen by the amplifier:

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 \cdot 8\Omega = 9000\Omega$$

$$\left(\frac{N_1}{N_2}\right) = 33.54$$

A step-down transformer with a 33.541 turns ratio will result in an amplifier which is 90% efficient.

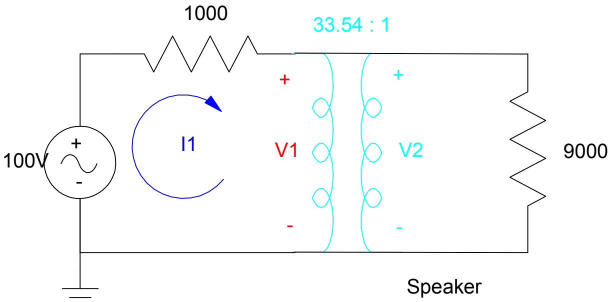


To analyze this circuit, either

- Bring all voltages and impedances left to the source side, or
- Bring them right to the load side.

Either way works. If the source is 100V and you take all voltages and impedance left, then 8 Ohms becomes

$$Z_1 = \left(\frac{33.54}{1}\right)^2 \cdot 8\Omega = 9000\Omega$$



By voltage division:

$$V_1 = \left(\frac{9000}{1000+9000}\right) 100V = 90V$$

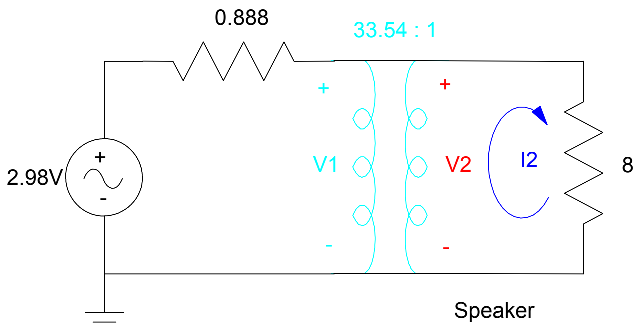
$$I_1 = \frac{100V}{10000\Omega} = 10mA$$

$$P_1 = V_1 I_1 = 0.9W$$

If you take all voltages and impedances right to the load side, then the 1000 Ohmes becomes

$$Z = \left(\frac{1}{33.54}\right)^2 \cdot 1000\Omega = 0.8888\Omega$$

$$V_{in} = \left(\frac{1}{33.54}\right) 100V = 2.98V$$



The voltages and currents are then

$$V_2 = \left(\frac{8}{8+0.8888}\right)2.98V = 2.682V$$

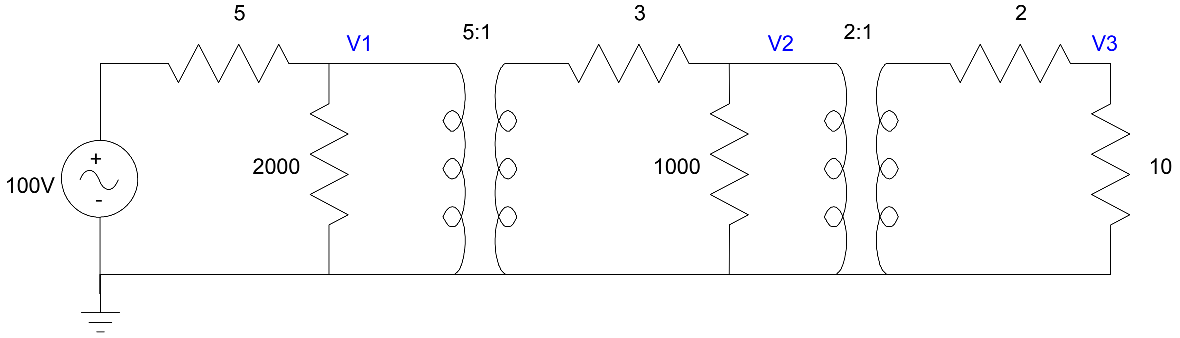
$$I_2 = \frac{2.98V}{8.8888\Omega} = 0.3353A$$

The output power is

$$P_2 = V_2 I_2 = 0.9W$$

Note that energy is conserved: power out = power in.

Example 2: For a more complex circuit with multiple transformers, it is often easiest to transfer all voltages and impedances to one side of a transformer. For example, find the voltages in the following circuit:



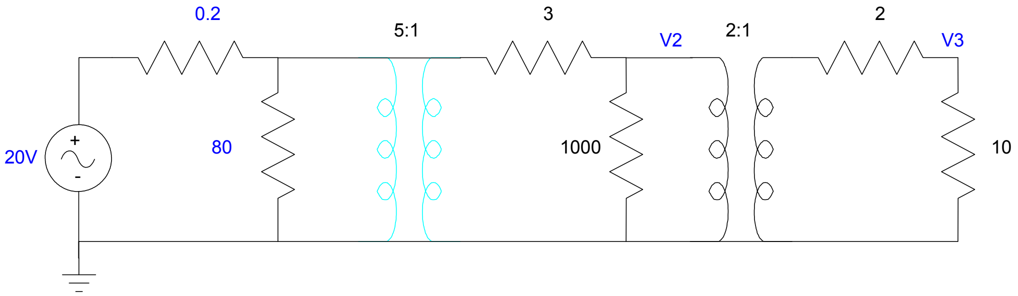
The direction you go (left or right) doesn't really matter. It it a little easier to interprit the results if you consider what you're looking for:

- If you want to know the current the 100V source provides, it makes sense to bring all impedances left.
- If you want to know the voltage and current at the load, it makes sense to bring all voltages and impedances right.

Let's bring everything right, just because. Going through the 5:1 transformer reults in

Impedances being scaled as $\left(\frac{1}{5}\right)^2$

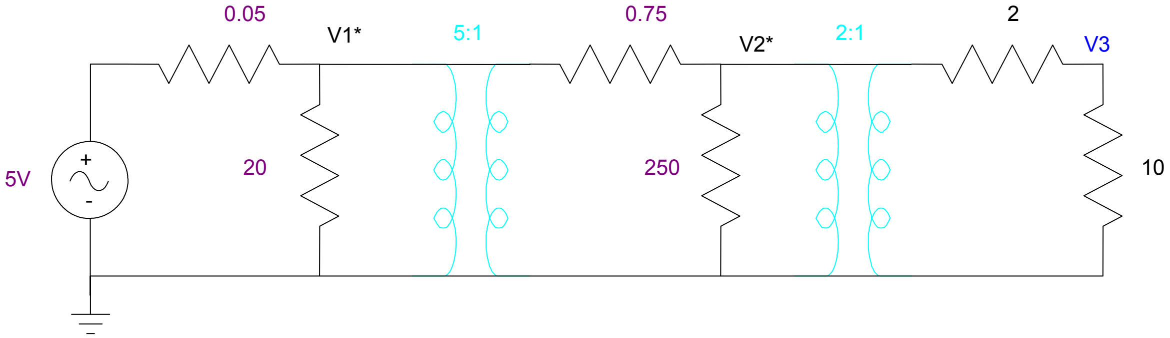
Voltages scaled as $\left(\frac{1}{5}\right)$



Going through the 2:1 transformer results in

Impedances being scaled as $\left(\frac{1}{2}\right)^2$

Voltages being scaled as $\left(\frac{1}{2}\right)$



You can now determine the node voltages. Note, however that

- Voltage V1* is different from V1 by the transformer turns ratio.
- You could transfer voltages through both transformers in one step by using the net turn ratio of 10:1

$$\left(\frac{V_1^*-5}{0.2}\right)+\left(\frac{V_1^*}{20}\right)+\left(\frac{V_1^*-V_2^*}{0.75}\right)=0$$

$$\left(\frac{V_2^*-V_1^*}{0.75}\right)+\left(\frac{V_2^*}{200}\right)+\left(\frac{V_2^*-V_3}{2}\right)=0$$

$$\left(\frac{V_3-V_2^*}{2}\right)+\left(\frac{V_3}{10}\right)=0$$