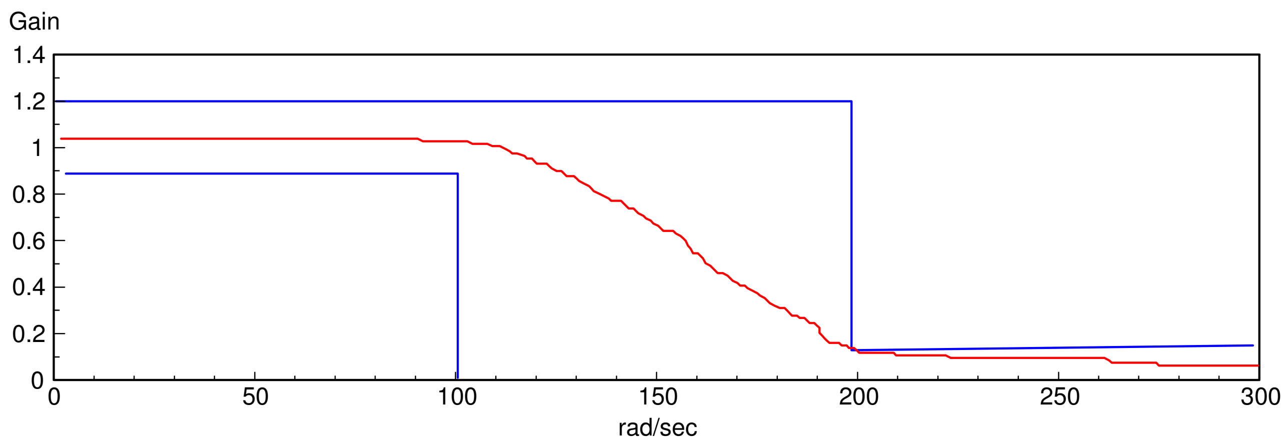
Active Filters: Design Example

Design a filter which

- Has a maximum gain < 1.2
- Has a gain > 0.9 for frequencies below 100 rad/sec and
- Has a gain < 0.1 for frequencies above 200 rad/sec



Filter Requirements

Step 1: Determine the order of the filter

The gain drops off as $\left(\frac{1}{\omega}\right)^n$ for an nth-order filter. Assuming the gain at 100 is one, the order required is

$$\left(\frac{100}{200}\right)^n < 0.1$$

$$n > 3.32$$

So, you need at least a 4th-order filter to meet these requirements. Let n=5 just to be safe.

Step 2: Determine the type of filter

Assume a Butterworth low-pass filter because Butterworth filters are easy to design.

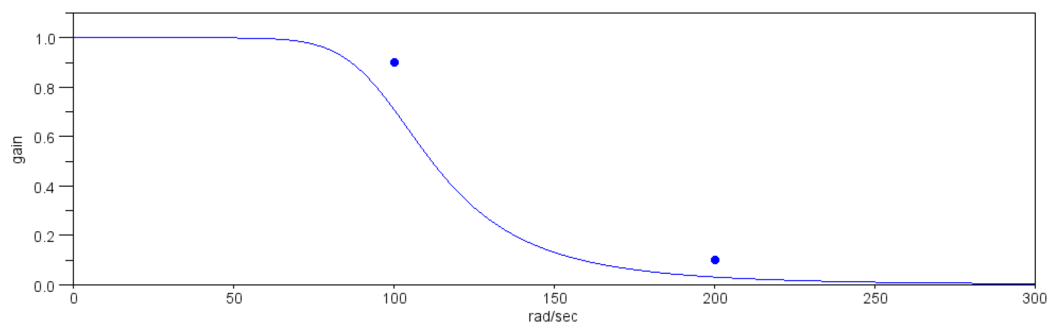
Step 3: Choose the filter's corners.

Assume a corner at 100 rad/sec as a start. This gives

$$G(s) = \left(\frac{100^5}{(s+100)\left(s+100\angle \pm 36^0\right)\left(s+100\angle \pm 72^0\right)} \right)$$

Plotting this in SciLab

```
-->w = [0:300]';
-->s = j*w;
-->s1 = -100;
-->s2 = -100*exp(j*36*%pi/180)
-->s3 = -100*exp(-j*36*%pi/180)
-->s4 = -100*exp(j*72*%pi/180)
-->s5 = -100*exp(-j*72*%pi/180)
-->G = 100^5 ./ ( (s-s1) .* (s-s2) .* (s-s3) .* (s-s4) .* (s-s5) );
-->plot(w,abs(G))
-->plot([100,200],[0.9,0.1],'.');
-->xlabel('rad/sec');
-->ylabel('gain');
```



5th-Order Butterworth Filter with Corner at 100 rad/sec

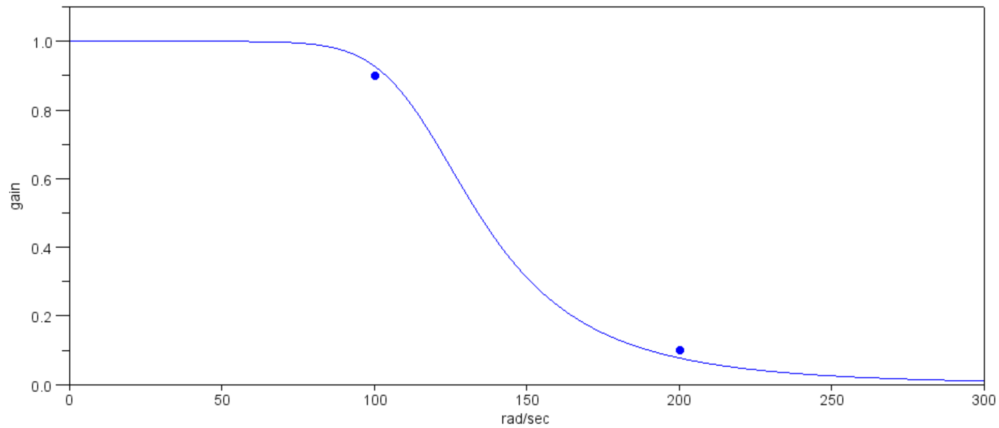
To meet the design requirements

- The maximum gain must be less than 1.2 (it is)
- The gain at 100 must be more than 0.9 (it isn't)
- The gain at 200 must be less than 0.1 (it is)

You can slide the graph to the right - meaning make the corner more than 100 rad/sec. Using trial-and-error, 120 works well, resulting in

```
-->plot(w*1.2,abs(G))
-->plot([100,200],[0.9,0.1],'.');
```

note: rather than recompute G(s) with a corner at 120 rad/sec (which would stretch the above graph by a factor of 1.2), frequency is scaled by 1.2. It has the same effect but is an easier way to adjust the corner frequency while checking what corner meets the design specs.



Frequency Response of a 5th Order Butterworth Low-Pass Filter with Corners at 120 rad/sec

This works, so

$$G(s) = \left(\frac{120^5}{(s+120)\left(s+120\angle\pm 36^0\right)\left(s+120\angle\pm 72^0\right)} \right)$$

Circuit Implementation:

To build this circuit, build it in three sections:

$$G_1 = \left(\frac{120}{(s+120)} \right)$$

$$G_2 = \left(\frac{120^2}{\left(s+120\angle+36^0\right)\left(s+120\angle-36^0\right)} \right)$$

$$G_3 = \left(\frac{120^2}{\left(s+120\angle+72^0\right)\left(s+120\angle-72^0\right)} \right)$$

Section 1:

$$\frac{1}{RC} = 120$$

C = 10uF, R = 830 *note: R is 10x smaller than stage 2 to avoid loading*

Section 2:

$$\frac{1}{RC} = 120$$

C = 1uF, R = 8.3k

$$3 - k = 2\cos(36^0)$$

$$k = 1.3820 = 1 + \frac{R_1}{R_2}$$

R2 = 100k, R1 = 38.2k

Section 3:

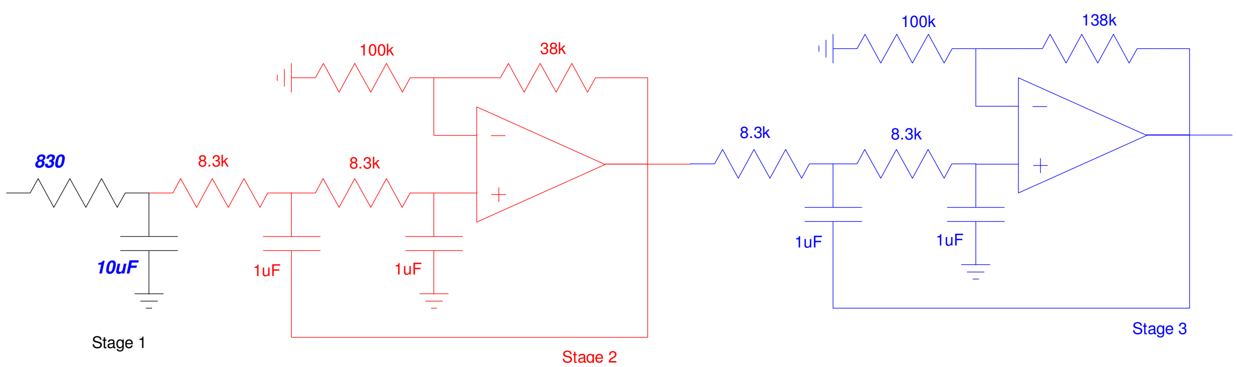
$$\frac{1}{RC} - 120$$

C = 1uF, R = 8.3k

$$3 - k = 2\cos(72^0)$$

$$k = 2.3820 = 1 + \frac{R_1}{R_2}$$

R2 = 100k, R1 = 138.2k



5th Order Butterworth low pass filter with a corner at 120 rad/sec