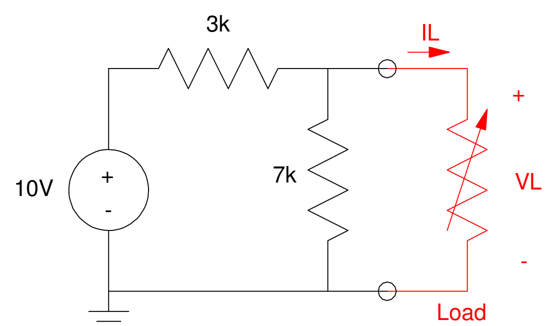
2-Port Models

Thevenin Equivalent:

Thevenin equivalents are a tool where you simplify a circuit to a voltage source and a resistance. The idea is that for any linear circuit, the output voltage/current relationship follows a straight line. For example, consider the following circuit:

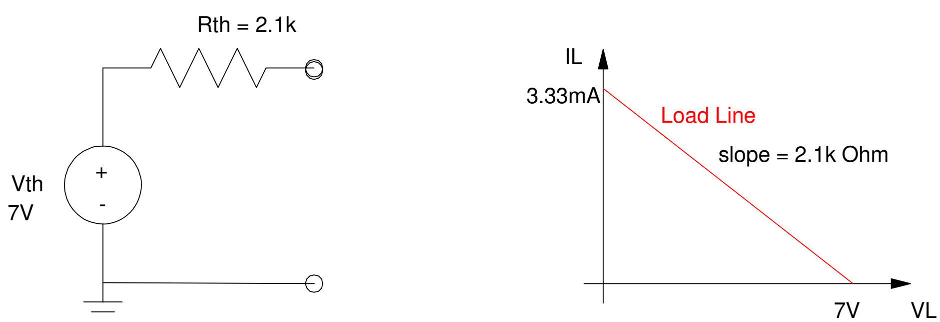


Sample Circuit

Place a resistor across the load and vary it from zero to infinity. The voltage and current at the load will change in a linear fashion - with the result termed the load line.

Thevenin equivalents replace the circuit with a simple model which follows the same load line. The way you find the Thevenin terms are:

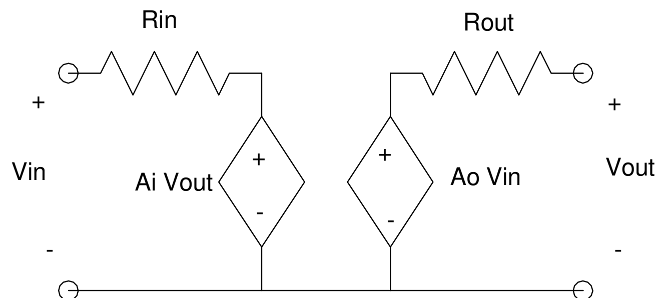
- V_{th}: The voltage at the load with R_L = infinity (7V)
- R_{th}: The resistance looking in with sources turned off: 3k||7k = 2.1k
- I_{short}: The current I_L when RL = 0. $I_{short} = V_{th} / R_{th}$: I_{short} = 10V/3V = 3.33mA



Thevenin Equivalent for Above Circuit along with its Load Line

2-Port Models

A 2-port model is a Thevenin equivalent for a circuit with an input and an output - such as an amplifier. Since the input can affect the output, the Thevenin voltage source at the output is replaced with a voltage controlled voltage source. Sometimes, the output can also affect the input. Likewise, the input has a like Thevenin equivalent: a Thevenin resistance along with a voltage-controlled voltage-source:



2-Port Model

Like a Thevenin equivalent, 2-Port models are tools which help with circuit analysis:

- Thevenin equivalents can make circuit analysis much simpler.
- 2-Port models can make multi-stage amplifier analysis much simpler

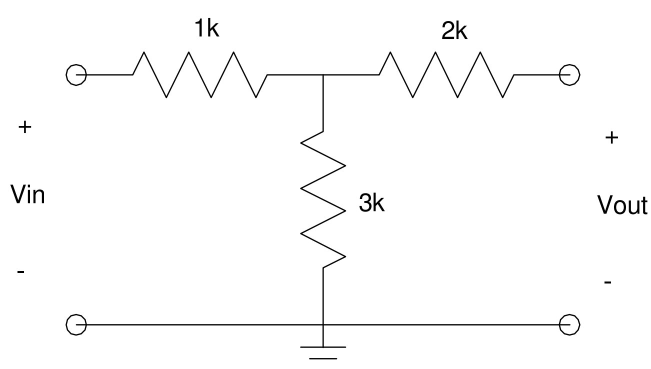
2-Port Parameters:

To determine each of the four 2-port model parameters, four tests are run:

- Ai: Set Vout = 1V and measure Vin. Ai = Vin
- Ao: Set Vin = 1V and measure Vout. Ao = Vout
- Rin: Set Vout = 0V and measure the resistance seen at the input
- Rout: Set Vin = 0V and measure the resistance seen at the output

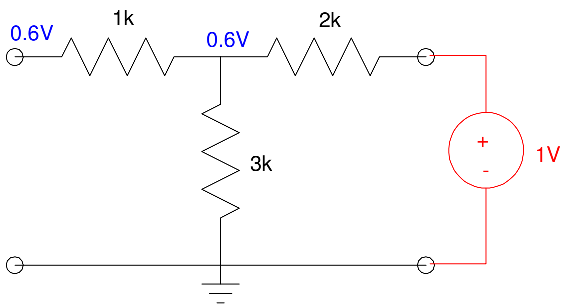
Essentially, devise a test on the 2-port model to find a given parameter. Do the same with the circuit you are analyzing.

Example: Determine the 2-port model for the following circuit:

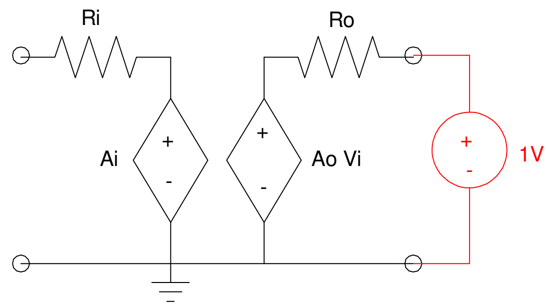


Ai: Set Vout = 1V, measure the voltage at Vin. Ai = Vin

- Ai = 0.6



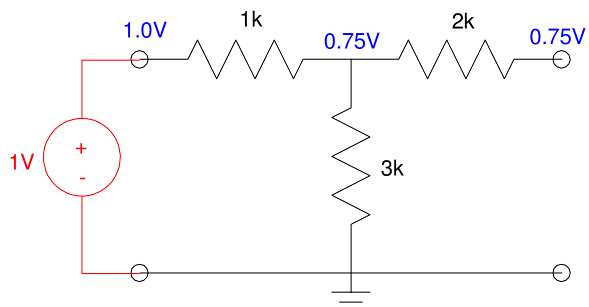
Circuit



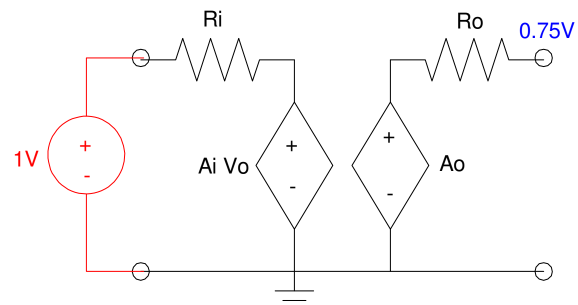
2-Port Model

Ao: Set Vin = 0V, measure the voltage at Vout. Ao = Vout

- Ao = 0.75



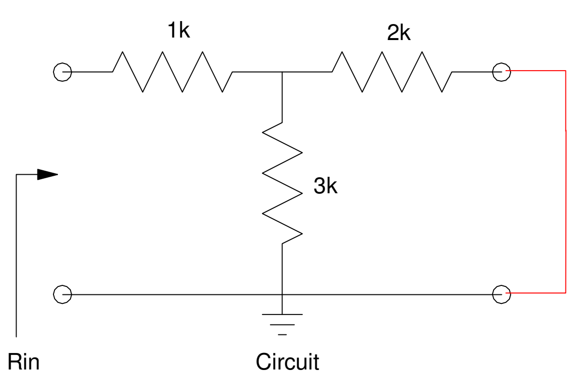
Circuit



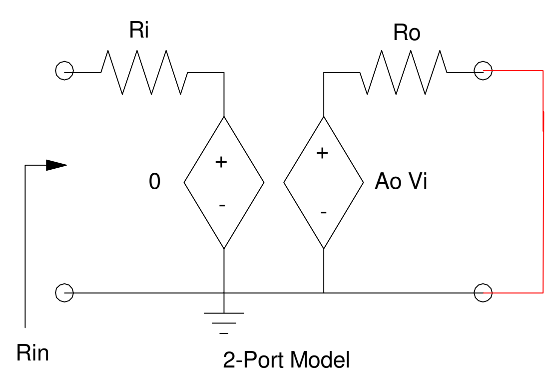
2-Port Model

Rin: Set Vout = 0V, measure the resistance at Vin.

- $R_{in} = 1k + 3k||2k = 2.2k\Omega$



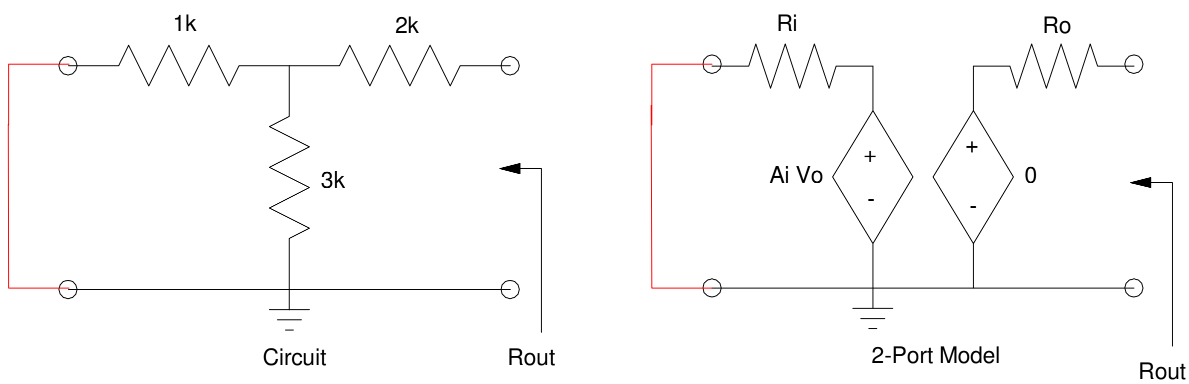
Circuit



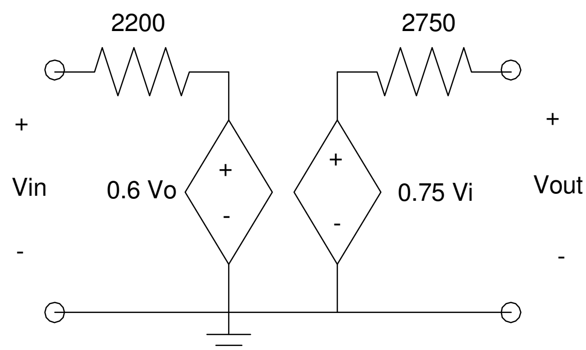
2-Port Model

Rout: Set Vin = 0V, measure the resistance at Vout

- Rout = $2k + 1k||3k = 2750\Omega$

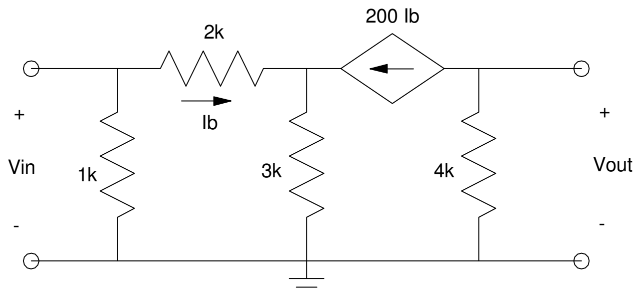


So, the 2-port model is:

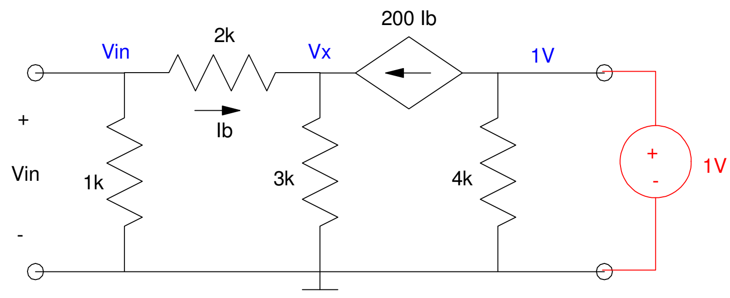


Sometimes the Thevenin resistance isn't so obvious. In this case, you might have to apply a t test voltage, compute the resulting current, and computer resistance as R = V/I

Example 2: Find the 2-port model for the following circuit:



Ai: Set Vout = 1V, measure Vin



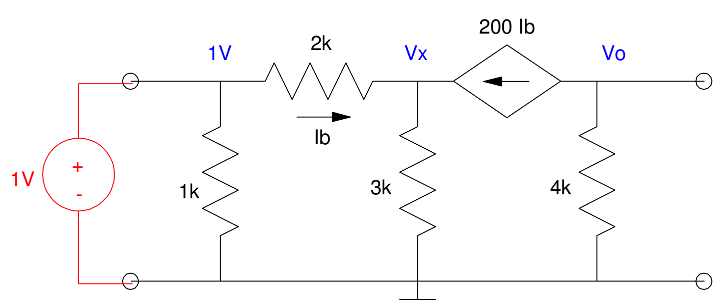
Compute Vx using voltage nodes:

$$\left(\frac{V_x}{1k+2k}\right) + \left(\frac{V_x}{3k}\right) - 200\left(\frac{0-V_x}{3k}\right) = 0$$

$$V_x = 0$$

So, **Ai = 0**

Ao: Set Vin = 1V, measure the voltage at Vo



Find Vx using voltage nodes:

$$\left(\frac{V_x-1}{2k}\right)+\left(\frac{V_x}{3k}\right)-200\left(\frac{1-V_x}{3k}\right)=0$$

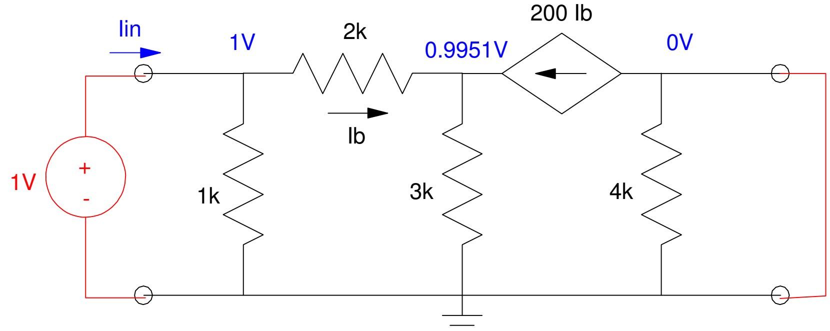
$$V_x = 0.9951V$$

$$I_b=\left(\frac{1-V_x}{2k}\right)=2.469\mu A$$

$$V_o=-(200Ib)4k=1.9753V$$

Ao = 1.9753

Rin: Set Vo = 0V, measure the resistance at the input.



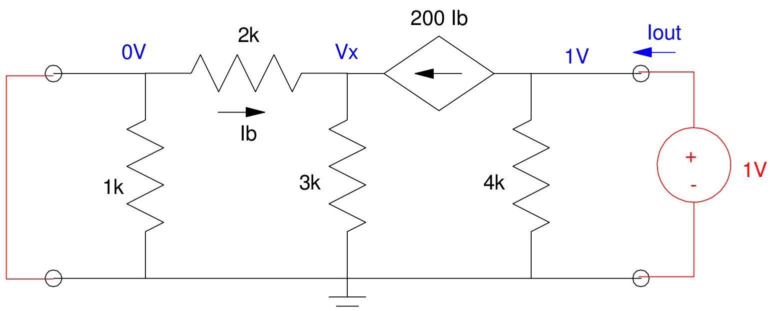
From the previous analysis, Vx = 0.9951V,

$$I_{in}=\left(\frac{1V-0.9951V}{2k}\right)+\left(\frac{1V}{1k}\right)=1.0017mA$$

$$R_{in}=\frac{V_{in}}{I_{in}}=\frac{1V}{1.0017mA}=998\Omega$$

Rout: Set Vin = 0V, measure the resistance at Vout.

This isn't obvious, so add a 1V source at the output and compute the resulting current



Solve for Vx:

$$\left(\frac{V_x-0}{2k}\right)+\left(\frac{V_x}{3k}\right)-200\left(\frac{0-V_x}{2k}\right)=0$$

Vx = 0

$$I_{out} = 0 + \left(\frac{1V}{4k}\right) = 250uA$$

so

$$R_{out} = \frac{V_{out}}{I_{out}} = \frac{1V}{250uA} = 4k\Omega$$

The resulting 2-port model is then: