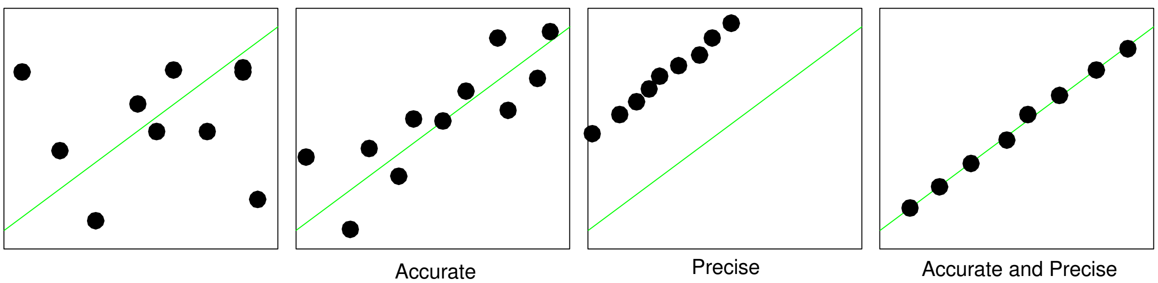
Calibration & Noise

Calibration

Accuracy: (mean) The difference between the expected value of a sensor's output and the actual value of the quantity being measured. The mean of a set of data is related to the accuracy of the data.

Precision: (standard deviation) The variations in a measured signal due to noise or other phenomena. The standard deviation of a set of data is a measure of the precision.

For example, if the temperature of a room from 0C to 10C is measured, the following data shows measurements which are neither accurate nor precise, accurate, precise, and both.



Measured T vs Actual T

Theoretical Calibration: Define a mapping based upon how an ideal system should behave:

$$y = f(x)$$

End-Point Calibration: Define a linear mapping such that the data passes through the endpoints:

$$y = ax + b$$

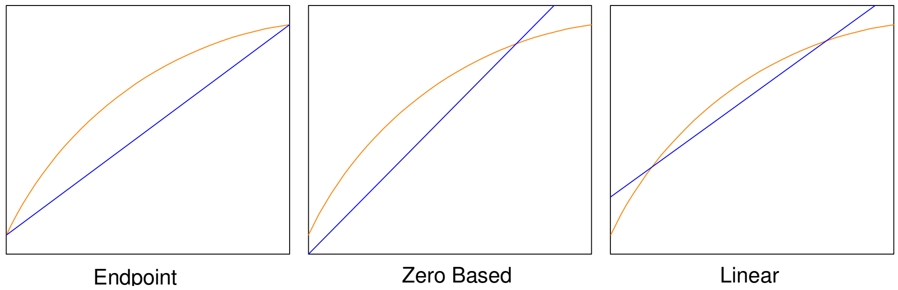
such that $y(x_1)=y_1$, $y(x_n)=y_n$.

Zero-Based Calibration: Define a linear mapping which passes through the origin and minimizes the mean squared error: This is useful in that it forces the output to zero when the measurement is zero.

$$y = ax$$

Linear Calibration: Define a linear mapping which minimizes the mean squared error.

$$y = ax + b \qquad \text{a, b free}$$



Linear Calibration Schemes

Calibration:

Calibration improves the accuracy and precision of a sensor by defining a mathematical relationship between what you measured and what the reading should be. For example, the third plot above could be improved by subtracting a constant from the measured temperature. This would be one type of calibration. In general, calibration attempts to map the measurement to its ideal value by some function:

$$y = f(x)$$

where 'x' is the measured quantity and 'y' is an improved estimate of what you're trying to measure.

Different functions are as follows:

Linear Calibration: Try to fit a straight line to the data:

$$y = ax + b$$

Polynomial Calibration: Try to fit a polynomial to the data:

$$y = ax^2 + bx + c$$

Nonlinear Calibration: Try to fit some other function to the data.

Least Squares Curve Fitting:

Given a function,

$$y = f(x)$$

determine (a, b, c) to approximate y() as

$$\hat{y} \approx ax^2 + bx + c$$

such that the sum-squared difference is minimized

$$J = \sum (y_i - \hat{y}_i)^2$$

Solution: Collect n data points and write this in matrix form

$$y_i = \begin{bmatrix} x_i^2 & x_i & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

For all n data points:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

or

$$Y_{nx1} = B_{nx3} A_{3x1}$$

You can't invert an nx3 matrix, so multiply by B^T:

$$B^T B = B^T B A$$

$B^T B$ is a square matrix, so multiply by its inverse:

$$A = (B^T B)^{-1} B^T Y$$

Weighted Least Squares:

Repeat, but weigh each equation with weighting qi:

$$J = \sum q_i (y_i - \hat{y}_i)^2$$

Solution: Degine a diagonal weighting matrix, Q:

$$Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & \ddots \end{bmatrix}$$

Take the original equation:

$$Y = BA$$

Multiply by the weighting matrix

$$QY = QBA$$

Multiply by XT:

$$B^T QY = B^T QBA$$

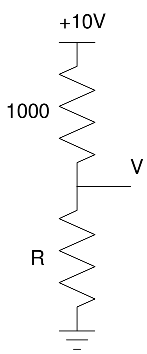
Solve for A

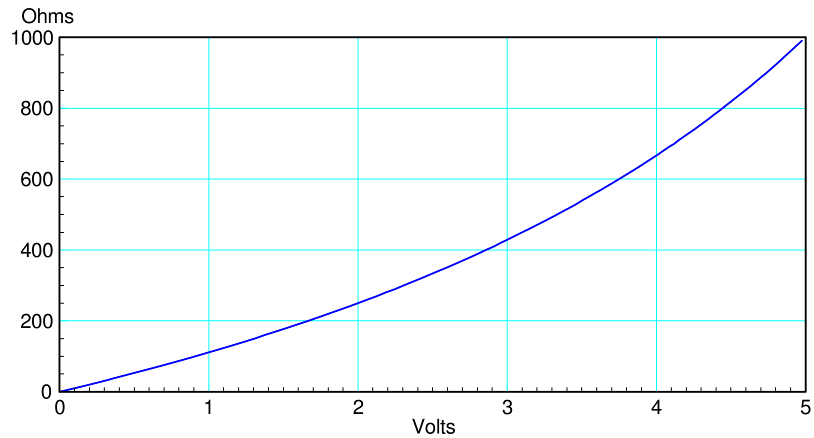
$$\boxed{A = (B^T QB)^{-1} B^T QY}$$

(weighted least squares solution)

Examples: Regression using MATLAB:

Problem: Determine the resistance from the voltage over the range of 0 < R < 1000 Ohms





Solution (Least Squares): Express the problem as

$$Y = BA$$

where Y is the output, X is a known function matrix, and A is a constant but unknown matrix. The least squares solution for A will then be

$$A = (B^T B)^{-1} B^T Y$$

The estimated output is then

$$Y_e = BA$$

and the error in this estimate is

$$E = Y - Y_e$$

Zero-Based Calibration: First, set this up as

$$R \approx aV = AB$$

Let the basis function be R:

$$B = [V]$$

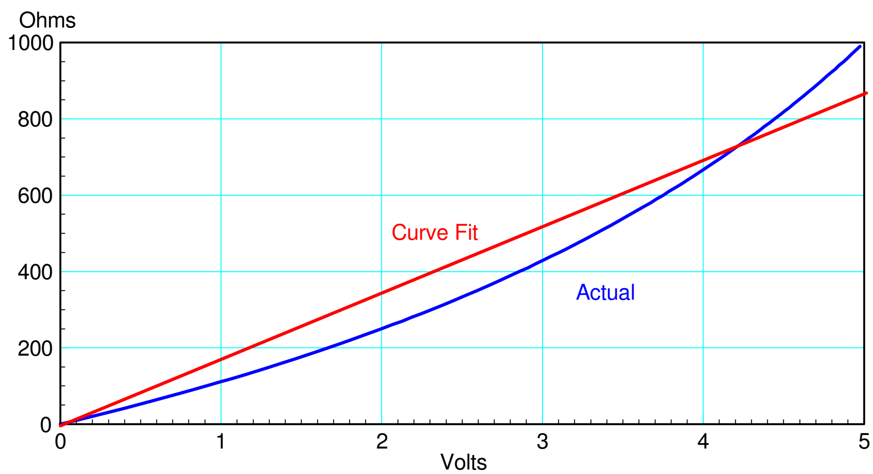
Solve for A as

In Matlab:

```
R = [0:1:1000]';
V = R ./ (1000+R) * 10;
B = V;

A = inv(B'*B)*B'*R

 A  = 169.53

plot(V,R,'b',V,B*A,'r');
xlabel('Volts');
ylabel('Ohms');
title('Zero Based Calibration');
```

so the best you can do is

$$R \approx 169.53R$$



Accuracy:

```
x = mean(R - B*A)
      x  = -20.967
```

Precision:

```
s = std(R - B*A)
```

s = 68.82

Endpoint Calibration: Next try

$$R \approx aV + b$$

$$R = \begin{bmatrix} V & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = BA$$

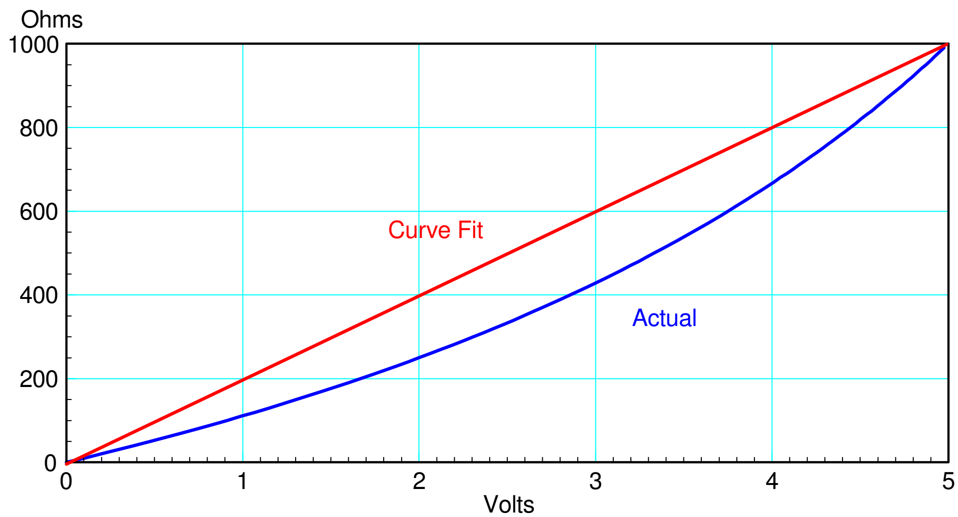
subject to the constrain that it passes through the endpoints.

Option 1) Repeat but just use the endpoints to find a and b.

Option 2) Use weighted least squares.

$$R \approx 200V$$

Plotting the results



Accuracy:

```
x = mean(V - B*A)
     x   = -113.69
```

Precision

```
s = stdev(V - B*A)
```

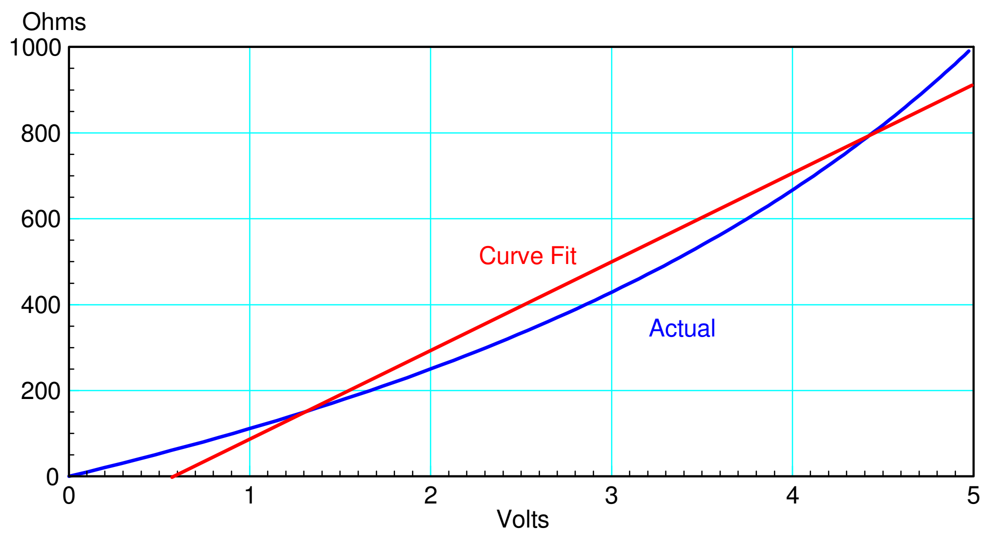
s = 51.65

Linear Interpolation: Repeat, but use all points (uniform weighting)

$$V \approx aR + b$$

```
B = [V, V.^0];
A = inv(B'*B)*B'*R

a    201.715
b    -118.913
```

$$V \approx 201.715V - 118.913$$



Note that

- R(V=0) is no longer zero.
- In return, you have a more accurate calibration scheme (on average)

Accuracy:

```
x = mean(R - B*A)
     x  = 0
```

Precision

```
s = stdev(V - B*A)
    s  =  51.59
```

Polynomial Calibration: The data looks like a quadratic function. Hence, a relationship like

$$R \approx aV^2 + bV + c$$

may work better. To do this, change the basis funciton (B) to

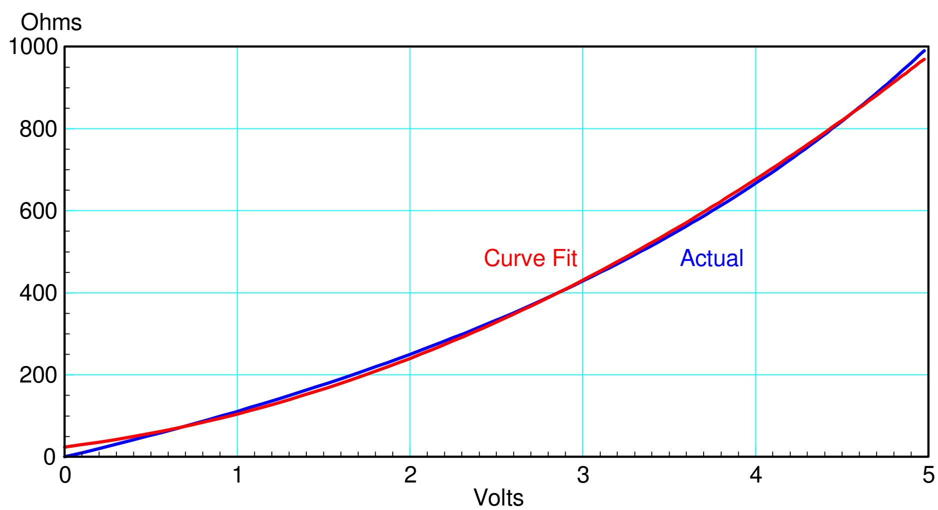
$$R = \begin{bmatrix} V^2 & V & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = BA$$

and solve just like before:

```
B = [V.^2, V, V.^0];
A = inv(B'*B)*B'*R

a    27.54
b    52.98
c    24.01
```

meaning

$$R \approx 27.54V^2 + 52.98V + 24.01$$



Accuracy

```
x = mean(V - B*A)
    x  =   0
```

Precision

```
s = stdev(V - B*A)
    s  =  9.000
```

Summary:

Calibration Scheme	R = f(V)	Accuracy mean(error)	Precision std(error)
Zero-Based	R = 169.53 V	-20.97	68.82
Endpoint	R = 200 V	-113.69	51.65
Linear Interpolation	R = 201.7 V - 118.9	0	51.6
Polynomial Interpolation	$R = 27.54V^2 + 52.98V + 24.01$	0	9.0

Noise

One problem commonly encountered is how to measure a remote location. For example, if you wanted to measure the temperature at the top of a smoke stack, it would be much more convenient to place the monitoring equipment on the ground rather than at the top of the smoke stack. In order to do this, wires are used to transfer the voltage from the sensor to the data recording instruments.

One problem with using wires to transfer data is that wires also act as antennas. By using a long stretch of wire, one might wind up with more noise than signal at the base of the smoke stack. During this week, different connections of wires from a sensor to an amplifier will be investigated in terms of their susceptibility to noise. By the end of the week, the student should be able to identify sources of noise in a given transmission line set up and suggest changes to reduce the noise level in the circuit.

Definitions:

Consider the problem of trying to measure a voltage, Vs, remotely.

Ideally, the output voltage is proportional to Vs.

Common-Mode Gain: The output proportional to the sum (or average) of Va and Vb. Common mode gain is ideally zero so that noise along the length of the line cancels out.

Differential Gain: The output proportional to the difference of Va and Vb.

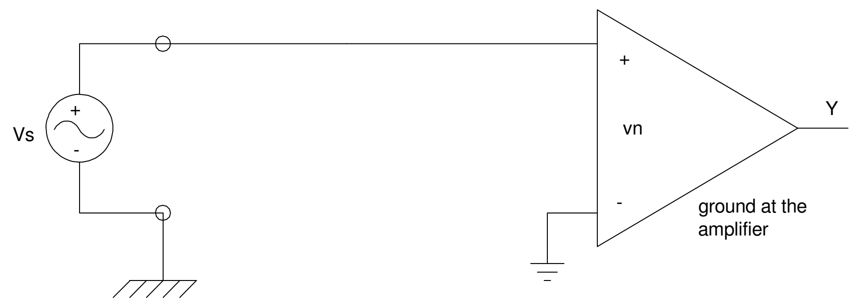
Common Mode Rejection Ratio: The ratio of the differential gain to the common-mode gain. The larger this number is, the better the amplifier is.

Ground Loops: All grounds are not equal. The potential at one point may be different than another point. (For example, there's about a 4V difference between Cincinnati and Columbus. If you take a wire, ground one end in Cincinnati and one end in Columbus, you'll likewise get some current flow. Over smaller distances, noise (from radio signals, transformers, etc.) may affect the potential at one location and not another.

Signal-to-Noise Ratio: The ratio of the energy at Vn due to the signal (Vs) to the energy of Vn due to other sources.

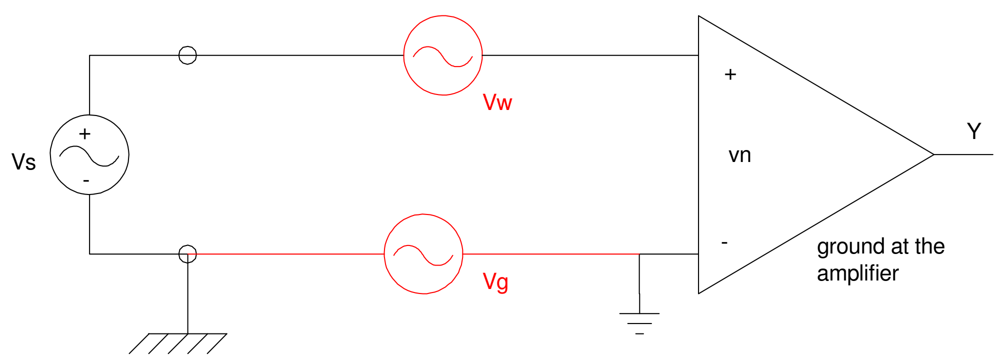
Basic Circuit

Case 1: The simplest circuit - and also the worse - for measuring a voltage remotely is as follows:



In order to save wire, the sensor is grounded locally. A single wire is then used to transfer this data to the data recorder. This voltage is then compared with its local ground.

The problem with this circuit is several. This can be seen by adding two more voltages to this circuit to signify noise sources.



Since the two grounds may be at different potentials, a voltage source, Vg is added between nodes b and d. Second, since the wire acts an antenna, the voltage induced on this wide is signified by Vw.

Ideally, the output should be

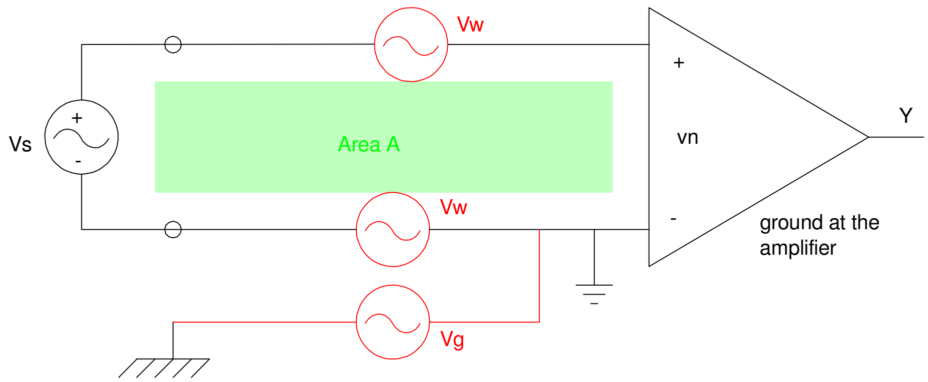
$$Vn = Vs$$

Instead, output voltage is

$$Vn = Vs + Vg + Vw$$

Hence, this circuit is sensitive to common mode noise (Vg+Vw).

Case 2: In order to eliminate this common-mode noise, a pair of wires could be used:



Instead of grounding the sensor, a second wire is run back to the volt meter. In this case, the measured voltage is not sensitive to the common-mode voltage (Vg) or the voltage induced on the two wires (Vw - assuming it is the same).

If there is a finite area enclosed by the wires, however, any changing magnetic fields will induce a voltage around this loop. Calling this induced loop voltage VA, the output voltage is now:

$$Vn = Vw + Vs - Vw + VA$$

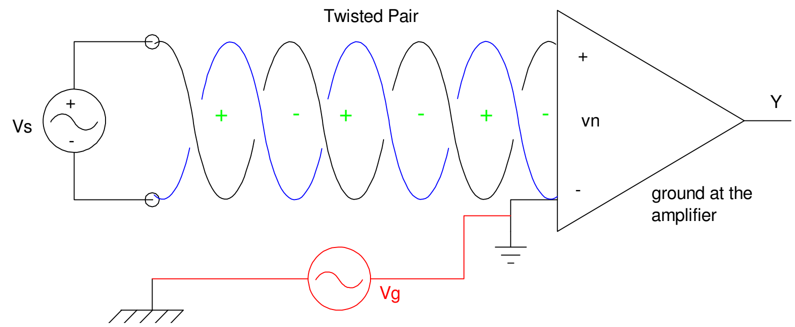
While this circuit is much better than the previous one, it will still tend to be noisy - especially if the area A is large.

Case 3: Twisted Pair: In order to minimize the area between the two wires, the wired could be twisted together. In this way, the induced voltages will tend to cancel as the voltages add and subtract as the loops in the wire wind around and around. In theory, the worst case for induced noise will only occur with an odd number of twists (where the area is still quite small).

In short, twisted pairs of wires are able to reject common mode noise and are fairly immune to induced voltages.

It's likewise a very commonly used method for transferring data from one site to another.

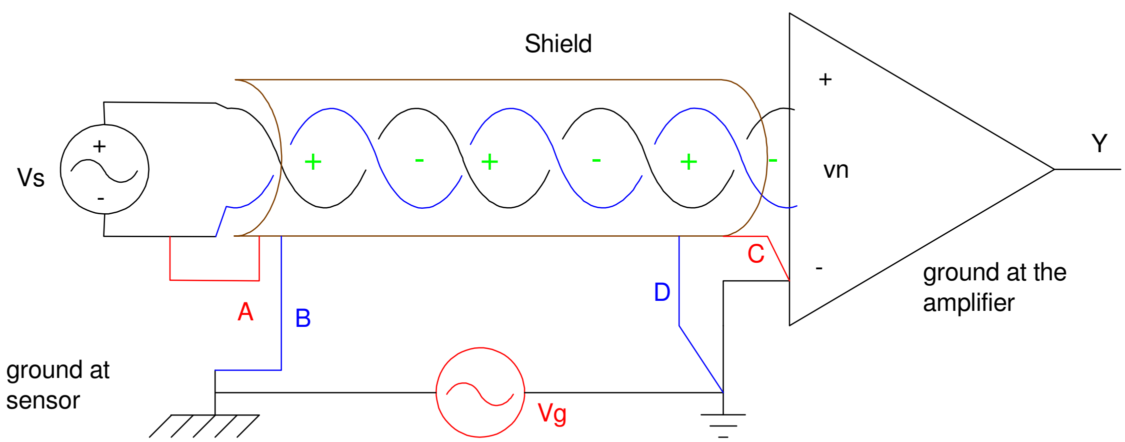
Shielded Twisted Pair: A second type of wire commonly used is a shielded wire. The idea here is to shield the signal in a grounded enclosure. In theory, the electric field on one side of the conductor due to fields on the other should be zero. One should, therefore, be able to transfer a voltage from one point to another through a shielded cable without any noise.



Some variations on how to connect a shielded cable are as follows:

Case 4: Shielded Wires with a Grounded Amplifier: If the amplifier is grounded, the shield could be connected to ground a) at the sensor side to the ground wire, b) at the sensor side to the sensor ground, c) at the amplifier side to the amplifier's ground wire, or d) at the amplifier's side to the amplifier's ground.

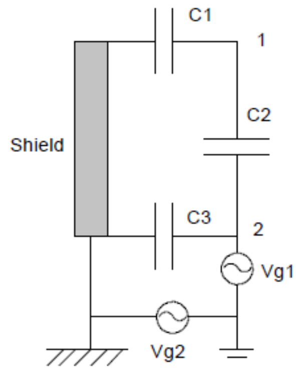
(note that grounding both ends is bad. This will cause currents to flow in the shield which then couple to the lines through the capacitance between the lines.)



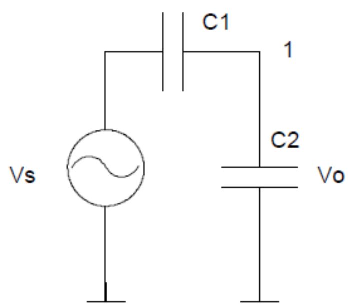
Due to the two grounds being at a slightly different potential, Vg2 is added to signify this difference. To model the slight voltage drop induced across the amplifier's ground wire, Vg1 is added. The goal is to pick which grounding location minimizes the effect of Vg1 and Vg2 on Vs.

Connection A: The worst location to connect the ground is to the ground wire on the sensor side. If no shield were used, the electric fields which induce a voltage on line 2 will also tend to induce the same voltage on line 1. Using a differential amplifier likewise causes these voltages to cancel. By adding a shield tied to point A, however, all of the induced voltages are added to line 2 and no voltages are added to line 1. This connection likewise imbalances the two lines.

Connection B: The circuit for the conduction with ground location B is as follows:



Here, C1, C2, and C3 signify the small capacitance between the wires. The voltage V12, is then found using voltage division for capacitors:

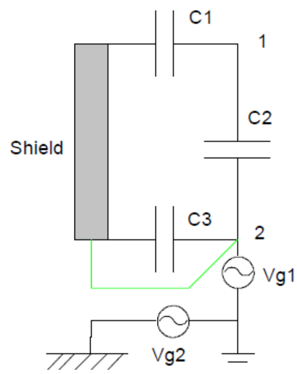


$$V_o = \left(\frac{C_1}{C_1+C_2}\right) V_s$$

and is

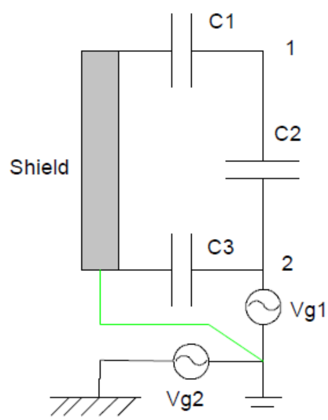
$$V_{12} = \left(\frac{C_1}{C_1+C_2}\right)(V_{g1} + V_{g2})$$

Connection C:



Connecting the ground to the ground wire of the instrumentation amplifier results in the noise sources having no effect on the signal.

Connection D:

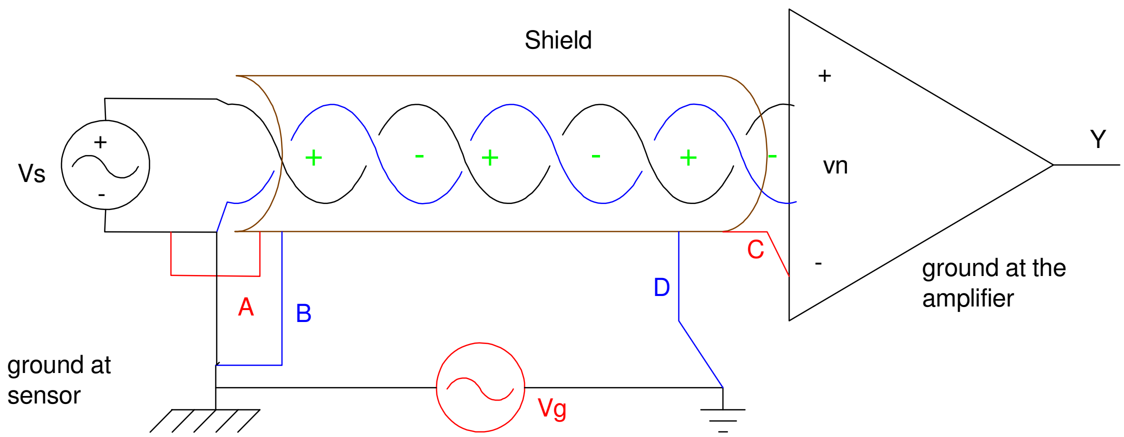


Grounding the shield to the ground at the amplifier's side results in the signal picked up by the amplifier's ground wires appearing at the output with a gain of

$$V_{12} = \left(\frac{C_1}{C_1+C_2}\right) V_{g1}$$

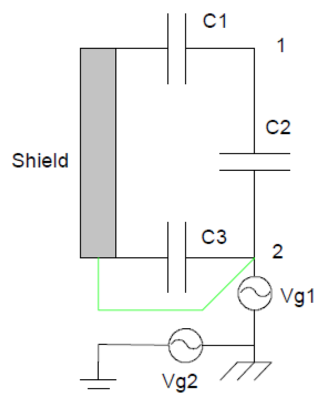
Note that Vg1 is typically very small - especially if short leads are used for the amplifier's ground wires. Hence, case C and D will not differ that much, although connection C is theoretically the better of the two.

Case 5: Shielded Twisted Pair with a Grounded Sensor: If the sensor is designed such that it has to be grounded, the circuit for the sensor - wiring - amplifier becomes as follows:



The goal is to ground the shield (at A, B, C, D) such that the signal at Vn is affected by Vg1 and Vg2 as little as possible.

Connection A: If the ground is connected to the sensor's ground wire on the sensor side, the circuit looks like the following:

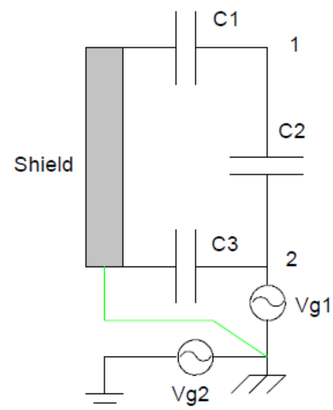


In this case,

$$V12 = 0$$

The shield will prevent electric fields from affecting the signal wires and will not add any voltages to the measured signal.

Connection B:

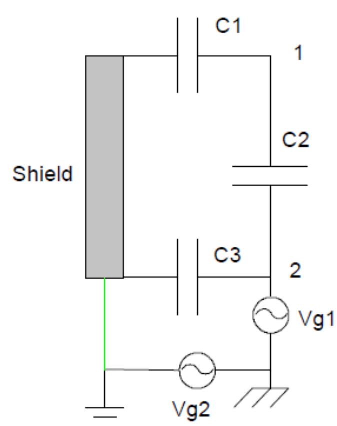


$$V_{12} = \left(\frac{C_1}{C_1+C_2}\right) V_{g1}$$

Connection B is only slightly inferior to connection A. Only the voltages which are induced on the sensor's grounding wire will be added to the amplifier's input.

Connection C: In this case, Connection C is clearly bad. As before, this uses the shield to take all the induced voltages and apply them to one of the lines and not the other - maximizing the noise received.

Connection D:



$$V_{12} = \left(\frac{C_1}{C_1+C_2}\right) (V_{g1} + V_{g2})$$