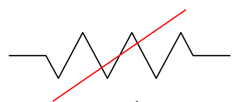
Temperature Sensors

Consider the problem of trying to measure temperature. Measuring the motion of atoms directly - essentially what temperature is - is not an easy task. Instead, it is easier to measure a second phenomena which is correlated with temperature. In this section, two such devices will be investigated:

- the Resistance Temperature Device (RTD), and
- the thermister.

RTD: "Resistance Temperature Device"

Symbol:



Description: In metals, at 0K the resistance is low due to electrons traveling unhindered in the conduction bands. As temperature increases, the atoms vibrate and impede the electron flow. Hence, resistance increases as temperature increases.

Model: A polynomial model is often used:

$$R = R_0(1 + a_1T + a_2T^2 + a_3T^3 + ...)$$

where Ro is the resistance at a given temperature and T is the difference between the actual temperature and the temperature where Ro is defined.

The more terms used in the model, the more accurate it is, of course. Typically, a first-order model is used for simplicity, however:

$$R = R_0(1 + aT)$$

Note that all metals have resistance which varies with temperature. For example

Be	a = 2.5%/C	most sensitive
Ni	a = 0.681%/C	
Fe	a = 0.651%/C	
Cu	a = 0.43%/C	
Al	a = 0.429%/C	
Pt	a = 0.385%/C	
Nd	a = 0.16%/C	least sensitive

Hence, any metal could be used to make an RTD. Typically only Cu, Ni, and Pt are used, however, due to their ease of use in circuit components. Copper and Nickel are relatively inexpensive but vary considerably with time and from element to element. Platinum is much more expensive, but works t o +850C and is more repeatable.

Example: Find the resistance of a Copper RTD at 25C given that R(5C) = 10k.

Solution: Using the model

$$R = R_0(1 + a \cdot T)$$

Since Ro is defined at 5C, at 25C T=+20.

$$R = 10,000 \cdot (1 + 0.0043 \cdot 20C) = 10,860\Omega$$

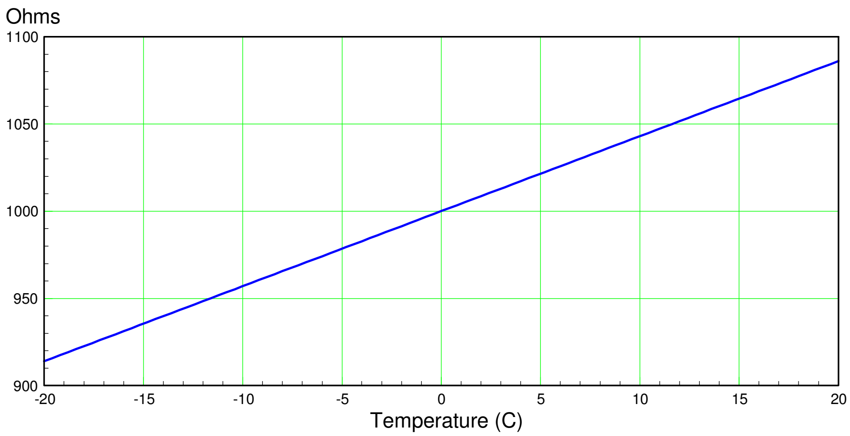
Example 2: Design a circuit to output

- -10V at 20C, and
- +10V at +20C

Assume a copper RTD with

$$R = 1000\ (1 + 0.0043T)\ \Omega$$

where T is the temperature in degrees C.



Solution

- Use a voltage divider to convert resistance to voltage.
- Use an instrumentation amplifier to convert this voltage to -10V to +10V

At -20C,

- R = 914 Ohms
- X = 4.775V
- Y = -10V

At +20C

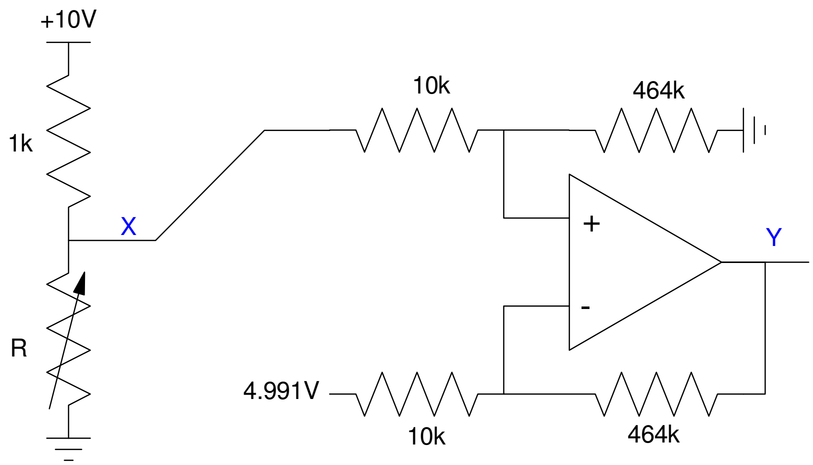
- R = 1086 Ohms
- X = 5.206V

- Y = +10V

$$gain = \left(\frac{10V-(-10V)}{5.206V-4.775V}\right) = 46.42$$

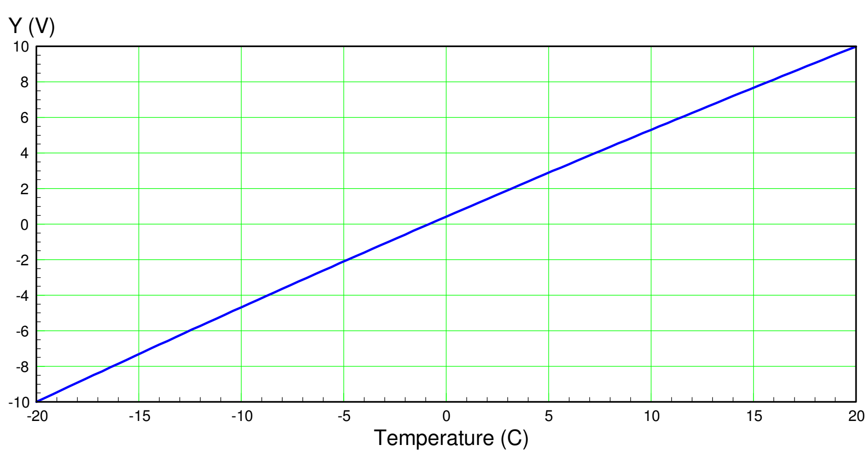
$$offset = \left(\frac{5.206V+4.775V}{2}\right) = 4.991$$

resulting in



Checking in Matlab:

```
T = [-20:0.1:20];
R = 1000 * (1 + 0.0043*T);
X = R ./ (1000 + R) * 10;
Y = 46.42 * (X - 4.991);
plot(T,Y);
xlabel('Temperature (C)');
ylabel('Y (Volts)');
```



Note:

- The voltage goes from -10V to +10V as temperature goes from -20V to +20V as desired
- The result isn't exactly linear: the voltage at 0C isn't 0V. This is due to the nonlinear relationship of the voltage divider.

Example 3: If you can measure this resistor to within 1 Ohm, how precisely can you determine the temperature?

Solution: The sensitivity of this sensor is

$$\frac{dR}{dT} = a \cdot R_0$$

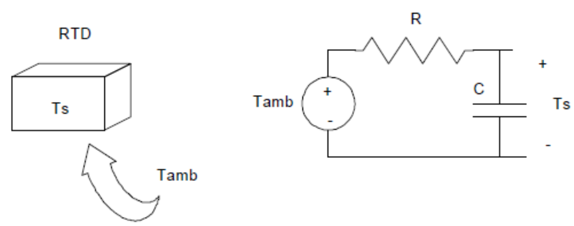
$$dT = \frac{dR}{a \cdot R_0} = \frac{1\Omega}{0.0043 \cdot 10{,}000}$$

$$dT = 0.023C$$

You can measure the temperature to within 0.0233 degrees.

Dynamic Model of an RTD

Since the RTD is a physical device, it will have some thermal inertial. In addition, it will not have perfect contact with the substance being measured - resulting in a finite thermal resistance between the sensor and the object being measured. This can be modeled with the following electrical analog:



- R = the thermal resistance (degrees C per Watt)
- C = the thermal inertia (specific heat - Joules per degree C)

Similarly, the step response of this system will follow the exponential rise of an RC circuit

$$T_s = \left(\frac{1}{RCs+1}\right) T_{amb}$$

or for a step change in the ambient temperature:

$$T_s = T_{amb} + (T_s(0) - T_{amb}) \cdot \exp\left(\frac{-t}{RC}\right)$$

The significance of this is that a time lag exists between the sensor's temperature (and hence reading) and the actual temperature being measured. This lag results in two effects:

- First, the reading will be incorrect until you wait for the sensor to come to equilibrium,
- Second, the sensor acts as a low-pass filter, removing the high-frequency data

Example: An RTD has a nominal resistance of 100 Ohms. When 5V is applied to the RTD, it maintains a temperature 10C above the ambient temperature. If the specific heat of the sensor is 0.5J/C, find how long you must wait before the reading is valid.

Solution: The thermal resistance (units: degrees C per Watt) is

$$P = \frac{V^2}{R} = \frac{5^2}{100\Omega} = 250mW$$

$$R = \left(\frac{10^0 C}{250mW}\right) = 40\frac{^0C}{W} \Rightarrow 40\Omega$$

The thermal capacitance (J/C) is 0.5

The RC time constant is then

$$RC = 40\Omega \cdot 0.5F = 20s$$

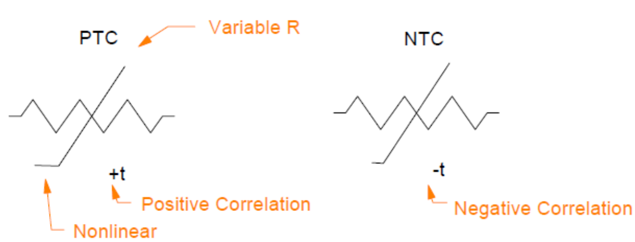
The transient for this sensor then decays as $e^{-t/20}$. If you want the reading to be within 2% of its steady-state value, you need to wait four time constants. Hence, you need to wait 80 seconds before you start taking data.

$$2\% = 0.02 = e^{-t/20}$$

$$t = 80 \text{ seconds}$$

Thermisters - "Thermally Sensitive Resistor"

The symbol for a thermister is shown below. Thermisters have a strong nonlinear relationship between temperature and resistance - which is denoted by a break in the line across the resistor. Unlike RTD's. thermisters can have a resistance which increases or decreases with temperature. These are called PTC and NTC (for positive temperature coefficient and negative temperature coefficient) devices.



Like the RTD, a thermister is a resistor designed so that its value changes with temperature. While RTD's are based upon the temperature-dependent relationship of resistance in a metal, thermisters use semiconductors instead.

For a semiconductor, the number of free electrons and holes is governed by

$$np = n_i^2$$

$$n_i = A_0 T^3 e^{-E_g/kT}$$

where n is the number of electrons, p is the number of holes, and ni is the intrinsic carrier concentration. Note that as temperature increases, the intrinsic carrier concentration also increases. Similarly, the conductivity should increase (resistance decreases) as temperature increases.

Alternatively, if the semiconductor is heavily doped, it acts more like a metal than a semiconductor. If doped heavily enough, the resistance will increase with temperature (as in a metal) rather than decrease. This creates the PTC version of the thermister.

Thermister Model:

Because of this exponential relationship between the number of carriers and temperature, a "good" model to fit data for a thermister is typically of the form

$$R = R_0 \cdot \exp\left(\frac{B}{T} - \frac{B}{T_0}\right)$$

where Ro is the resistance at reference temperature To, T is the temperature in degrees Kelvin, and B is the "characteristic temperature" of the thermister.

Since the intrinsic carrier concentration also has a T^3 term, this model is not exact. Higher-order models will be more exact (of course) - typically as follows:

2-Parameter Model: $R = \exp\left(A + \frac{B}{T}\right)$ ±0.3C over 50C

3-parameter Model: $R = \exp\left(A + \frac{B}{T} + \frac{C}{T^3}\right)$ ±0.01C over 100C

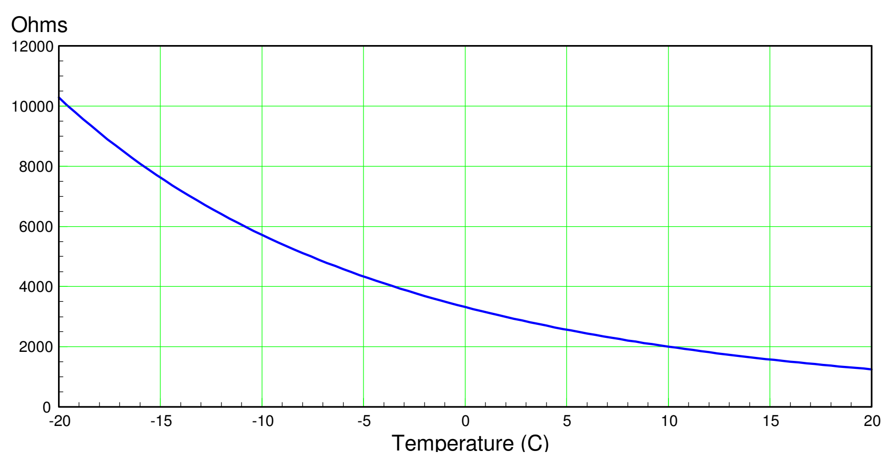
4-Parameter Model: $R = \exp\left(A + \frac{B}{T} + \frac{C}{T^2} + \frac{D}{T^3}\right)$ ±0.00015C over 100C

Example: Design a circuit to output

- -10V at -20C
- +10V at +20C

Assume a thermistor with (T in Kelvin)

$$R = 1000 \cdot \exp\left(\frac{3905}{T} - \frac{3905}{298}\right)\ \Omega$$



Note:

- Thermistors have a *much* sensitivity (change in resistance) than RTD's (good), but
- They are highly nonlinear.

Solution: Again, use an instrumentation amplifier and a voltage divider. It tends to work best if the voltage divider uses a resistor equal to the geometric mean of the endpoints

$$R = \sqrt{R_a R_b}$$

Analysis:

At -20C (253K)

- R = 10,285 Ohms
- X = 7.742V
- Y = -10V

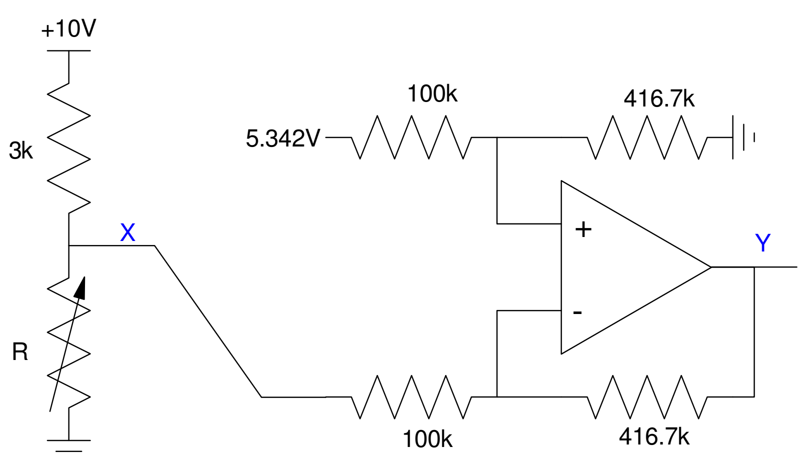
At +20C

- R = 1,251 Ohms
- X = 2.942V

Y = +10V

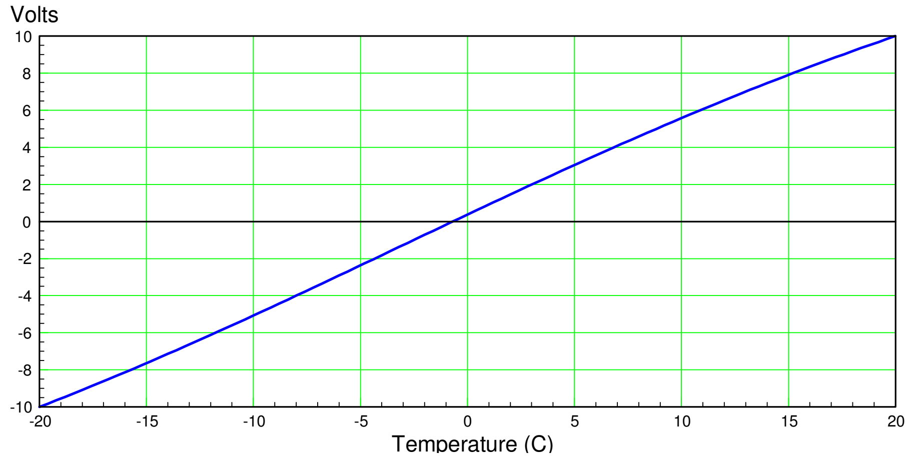
$$gain = \left(\frac{10V-(-10V)}{2.942V-7.742V}\right) = -4.167$$

$$offset = \left(\frac{7.742V+2.942V}{2}\right) = 5.342V$$



Checking in Matlab:

```
T = [-20:0.4:20]';
R = 1000 * exp( 3905 ./ (T + 273)  -  3905/298 );
X = R ./ (3000 + R)  * 10;
Y = 4.167 * ( 5.342 - X );
plot(T,Y);
xlabel('Temperature (C)');
ylabel('Y (Volts)');
```

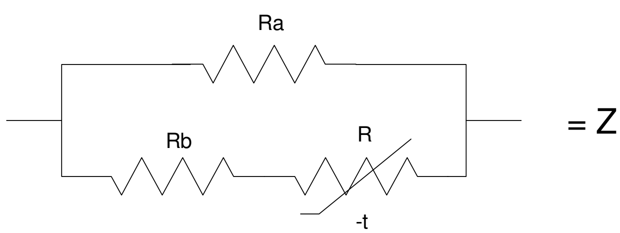


Note:

- With thermistors, a much lower gain was needed than we had with RTD's. This is due to the higher sensitivity.
- This is still a nonlinear voltage - temperature relationship.
  - The thermistor is highly nonlinear, and
  - The voltage divider is nonlinear

Linearizing Circuits:

Since R is strongly nonlinear, a circuit which linearizes the resistance vs. temperature relationship would be nice. One such circuit is



If Ra and Rb are chosen properly, you can sometimes obtain an overall resistance which is linear at two temperatures and the midpoint. For example, if you are using a thermistor with

$$R = 1000 \cdot \exp\left(\frac{3905}{T} - \frac{3905}{298}\right)\ \Omega$$

from 0C to +20C, you may want them resistance at 10C to be half way between the resistance at 0C and +20C:

$$Z_{10C} = \left(\frac{Z_{20C}+Z_{0C}}{2}\right)$$

There are actually an infinite number of solutions for Ra and Rb in this case (one equation with two unknowns). One solution is to constrain Rb to 1000, resulting in Ra=504.44 Ohms. This then results in

- Z0 = 451.6976
- Z10 = 431.8880
- Z20 = 412.0783

In MATLAB, Ra and Rb can be found using the following code.:

First, define a cost function which has a minimum for the "correct" values of R1 and R2. For example, call this routine "Thermistor.m" - where you pass Ra (passed in Z) and it returns the square error

$$e = Z_{10C} - \left(\frac{Z_{20C}+Z_{0C}}{2}\right)$$

```
function [ J ] = Thermistor( Z )
   Ra = Z;
   Rb = 1000;

   R0  = 1000*exp(3905/273 - 3905/298);
   R10 = 1000*exp(3905/283 - 3905/298);
   R20 = 1000*exp(3905/293 - 3905/298);

% Z = R1 R2 / (R1 + R2)

   Z0  = (R0  + Rb)*(Ra) / (R0 + Ra + Rb);
   Z10 = (R10 + Rb)*(Ra) / (R10 + Ra + Rb);
   Z20 = (R20 + Rb)*(Ra) / (R20 + Ra + Rb);

   E = Z10 - (Z0 + Z20)/2;

   J = E*E;

 end
```

This routine is then called as

```
>> [Ra, e] = fminsearch('Thermistor',1000)

Ra =  504.4401

e =  1.2991e-015
```

where the initial guess for Ra is 1000. Note that depending upon your initial guess and what you assumed for Rb. You may have to adjust Rb to get a solution that makes sense.

Note that the resistance at 0C, 10C, and 20C is linear with R1 and R2 chosen properly. This does not make the overall resistance linear, however. There will still be some wobble in the resistance vs. temperature relationship - just not as much as before. For example, the resistance of the 3-resistor network from -10C to +30C is shown below along with the error from linear. Note that

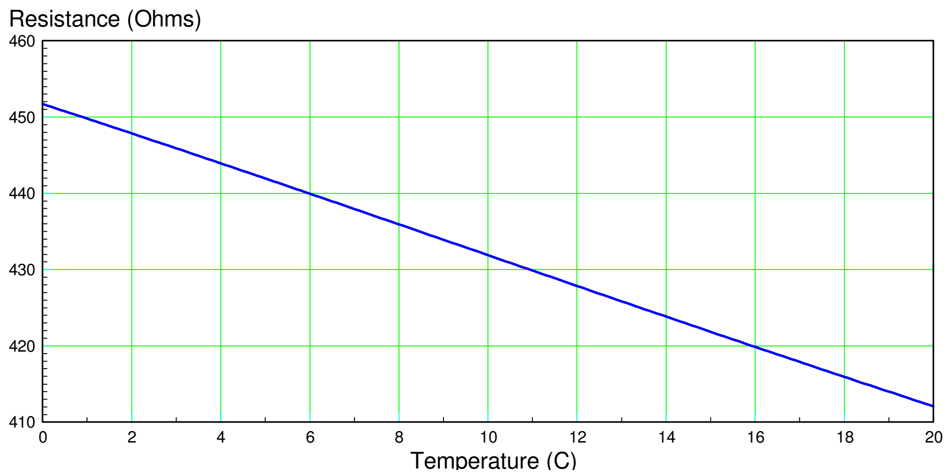
- The error at 0C, 10C, and 20C are zero, since this was the constraint used to find R1 and R2. In addition,
- From 0C to +20C, the resistance is off from linear by as much as 4 Ohms (0.055C) - which is almost as good as you could do using a 2-term calibration model of the form T »(A + B ln(R))-1.

To plot the resulting resistance vs. temperature:

```
T = [0:0.01:20]';
K = T + 273;
R = 1000*exp(3905 ./ K  -  3905/298);

% resistors in parallel
%  Z = R1 R2 / (R1 + R2)

Z = Ra * (R + Rb) ./ (R + Ra + Rb);
plot(T,Z)
```

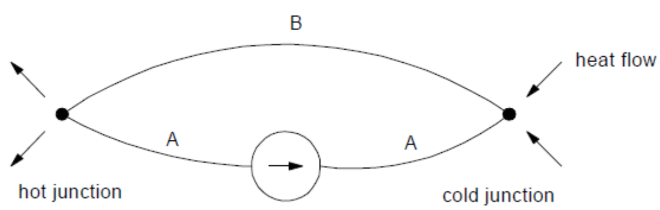


The advantage of using this linearizing circuit is that the resistance is much more linear than before. This makes calibration a simpler process (inverse log square functions don't need to be computed anymore). The disadvantage is that the sensitivity has been reduced. At 0C, the sensitivity is now -0.4%/C instead of -5%/C.

Thermocouples

Thermocouples are based upon the Peltier and Thomson effects. These two combined are called the Seebeck effect.

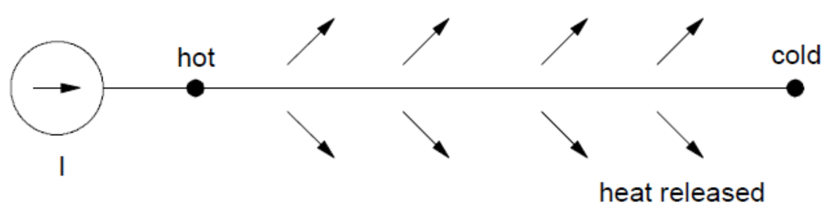
Pentier Effect: The junction of two dissimilar metals will heat and cool as current flows. This effect is reversible and does not depend upon the surface area. It only depends upon the metals used.



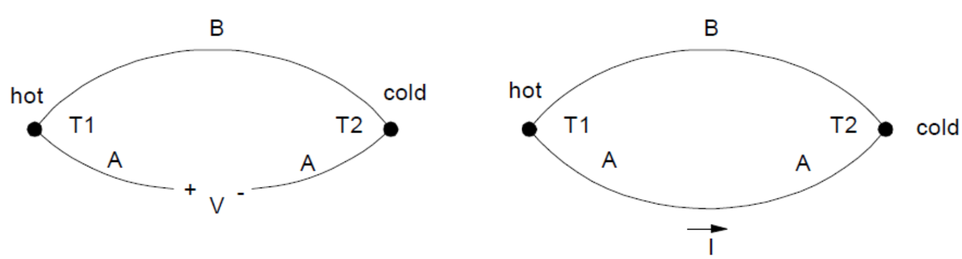
Thomson Effect: If a current is applied to a conductor with a temperature gradient,

- Heat is absorbed if the current flows from cold to hot, and
- Heat is released if the current flows from hot to cold.

Note that heat is released proportional to I instead of I^2 as in Joule heating (and thus is reversible).



Seebeck Effect: The junction of two dissimilar metals at two temperatures produces a current or voltage proportional to the difference in temperature:



The Seebeck coefficient is a measure of the sensitivity of the thermocouple:

$$S_{AB} = \frac{dV_{AB}}{dT_{AB}} = \frac{V_{AB}}{T_1 - T_2}$$

In order to use a thermocouple, you likewise need to...

- Keep the current small to minimize I2R heating and to minimize the Peltier effect, and
- Keep one node at a known temperature (such as an ice water bath).

If this is done, then the voltage across the thermocouple should be proportional to the temperature at the second node.

The advantages of using thermocouples over thermisters or RTDs are

- A wide operating range. thermocouples which operate from -270C to +3000C are commercially available.
- Long term stability. As long as the purity of the metal wires are not changed, the thermocouple's properties won't change.
- Good operating characteristics at low temperatures. Metals conduct better as temperature decreases.
- Fast response / Low thermal inertia. Fine wires can be used since the voltage does not depend upon the surface area at the junction.
- Ease of use. A voltage is produced, which is easily amplified.

The disadvantages of using a thermocouple are

- Limited maximum temperature. You cannot measure temperatures which will melt the probe.
- Current must be kept small,
- The temperature at one junction must be known, and
- Low sensitivity. You need sizable gains to convert the output to 0-5V.

Thermocouple Circuits:

Some typical thermocouples are

Type	Thermocouple	Operating Range	Full Range V
C	W (5%) / rhenium - W(26%) / rhenium	0C to 2,300C	37mV
B	Pt (13%) / rhodium - Pt	0C to 1,593C	18.7mV
N	Nicrosil - Nisil	-270C to +1,300C	51.8mV

Note that the voltage generated is not large (i.e. the sensitivity is small). Using endpoint linearization, the sensitivity of the first thermocouple is

$$\text{Sensitivity} = \frac{37mV}{2300C} = 16.08\text{mV/C}.$$

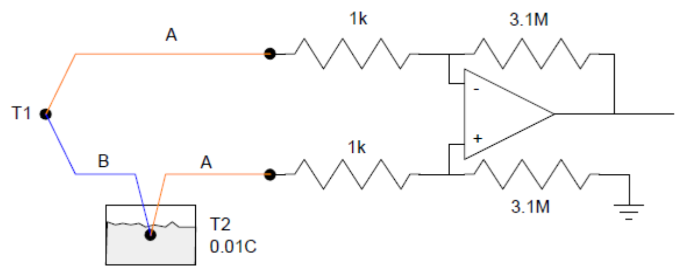
Hence, you need to be able to measure voltages in the sub-microvolt range in order to have good resolution using a thermocouple or else use a high gain amplifier. The latter tends to amplify noise as well, however.

Example: Design a circuit which measures 0C to +100C using a Type C thermocouple.

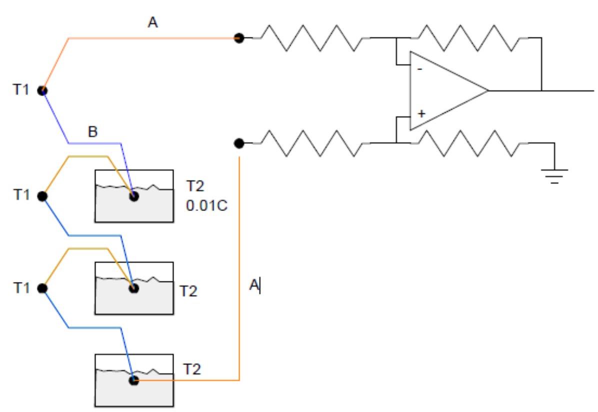
Solution: Assume a ice-water bath is available at the cold junction and that its temperature is 0.01C. At 0C, the voltage will be approximately 0V. At 100C, the voltage across the thermocouple will be

$$V = (16.08\text{mV/C})(100C) = 1.608\text{mV}$$

A gain of 3,108 is thus required to raise this to 5V. One circuit to amplify this is as follows:



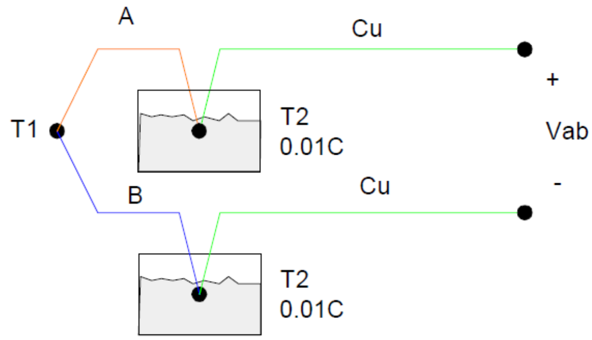
In order to increase the sensitivity, several thermocouples can be placed in series. This is called a thermo-pile.



Several hundred such connections are often used in a single package. The exact number is often times not listed on the package, but can be derived based upon the sensitivity of the thermopile and sensitivity for a single connection given in the previous table.

One sidelight for using thermocouples, since the materials used are often times expensive, you can replace the leads with a less expensive metal, such as copper, using the Law of Intermediate Metals:

Law of Intermediate Metals: The net EMF around a circuit is zero if all junctions are at the same temperature.



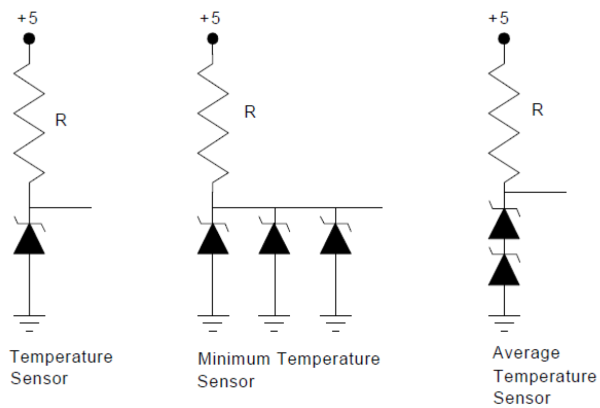
This law allows you to use a third metal in the thermocouple without altering the voltage or current, providing that both junctions are at the same temperature.

LM-335 Precision Temperature Sensor

The LM-335 is essentially a Zener diode where the breakdown voltage is proportional to temperature as

$$V_z = 0.01T + V_o$$

and T is measured in degrees C (another version has a sensitivity of 0.01V per degree F). This allows you to design a temperature sensor using very few elements as follows:



The resistor limits the current flow to the Zener diode. The voltage across the diode is then proportional to temperature.

Problem: Design a thermometer using the LM-335 where the output is -5V at -50C and +5V at +50C

Solution: The gain is 10V out divided by 1V in (100C spread times 0.01V/C), or a gain of 10. A DC offset is required to

compensate for Vo is the Zener voltage equation (or to set 0V at 0C). This can be accomplished with the following circuit: