ECE 321 - Homework #3

Calibration & Noise, Active Filters. Due Monday, April 17th

Please email to jacob.glower@ndsu.edu, or submit as a hard copy, or submit on BlackBoard

Filters

1) Assume X and Y are related by the following transfer function:

$$Y = \left(\frac{80}{(s+5)(s+10)}\right)X$$

a) What is the differential equation relating x and y?

Cross multiply

$$(s^2 + 15s + 50)Y = (80)X$$

Note that sY means *the derivative of Y*

$$y'' + 15y' + 50y = 80x$$

b) Determine y(t) assuming

$$x(t) = 6 + 2\cos(4t) + 3\sin(4t)$$

Use superposition:

x(t) = 6

s = 0

X = 6

$$Y = \left(\frac{80}{(s+5)(s+10)}\right)_{s=0} \cdot (6)$$

$$Y = 9.6$$

x(t) = 2 cos(4t) + 3 sin(4t)

s = j4

X = 2 - j3

$$Y = \left(\frac{80}{(s+5)(s+10)}\right)_{s=j4} \cdot (2 - j3)$$

$$Y = -3.7463 - j1.4829$$

$$y(t) = -3.7463\cos(4t) + 1.4829\sin(4t)$$

The total answer is DC + AC

$$y(t) = 9.6 - 3.7463\cos(4t) + 1.4829\sin(4t)$$

Filter Design

2) Give an op-amp circuit to implement the following filter

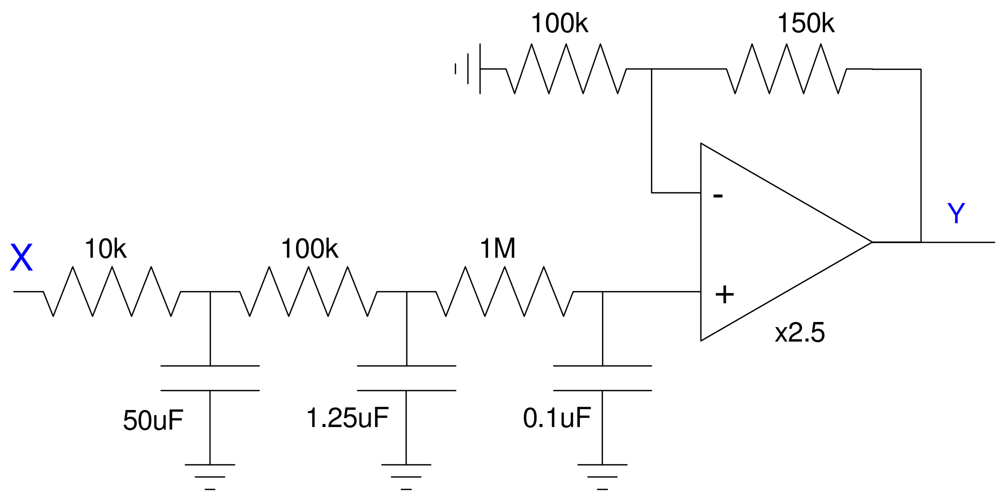
$$Y = \left(\frac{400}{(s+2)(s+8)(s+10)}\right)X$$

Rewrite as

$$Y = \left(\frac{2}{s+2}\right)\left(\frac{8}{s+8}\right)\left(\frac{10}{s+10}\right)(2.5)$$

Build using a 3-stage RC filter along with an amplifier with a gain of 2.5

- note: there are other solutions



3) Give an op-amp circuit to implement the following filter

$$Y = \left(\frac{200}{(s^2+s+20)(s^2+5s+30)} \right) X$$

Design as a two-stage active low-pass filter. Rewrite in polar form

$$Y = \left(\frac{200}{(s+4.4721\angle\pm 83.58^0)(s+5.4772\angle\pm 64.8427^0)} \right) X$$

Stage 1:

$$\frac{1}{RC} = 4.4721$$

$$3 - k = 2\cos\theta$$

$$k = 2.7764$$

Stage 2:

$$\frac{1}{RC} = 5.4772$$

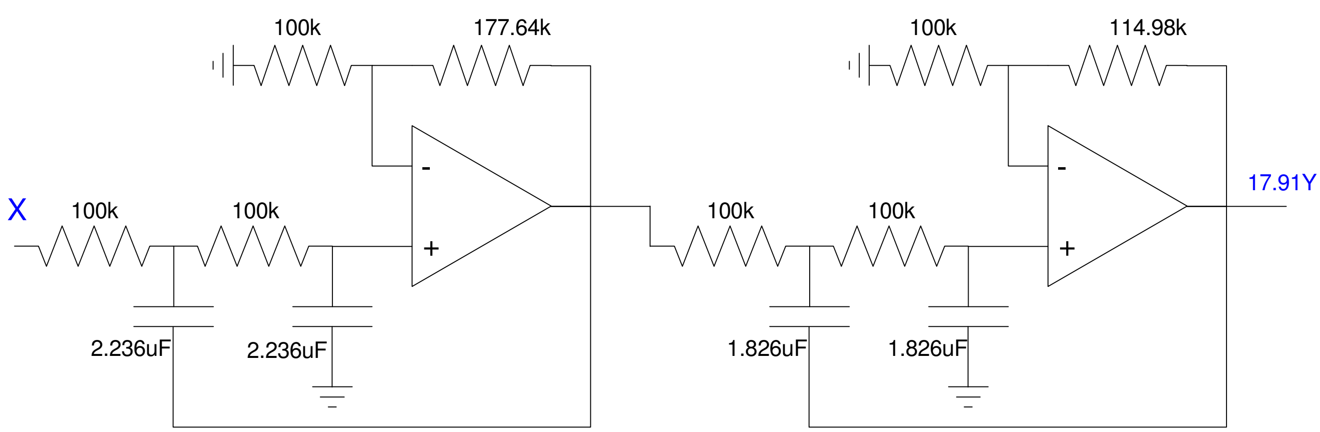
$$k = 2.1498$$

DC gain

$$DC = 0.333 = 2.7764 \cdot 2.1498 \cdot k_3$$

$$k_3 = 0.0558$$

or the output is 17.91Y



4) Give the transfer function of a filter with the following gain vs. frequency

This has three resonances (three sets of complex poles:

Resonance #1

- Frequency = 4 rad/sec = complex part of pole
- Bandwidth = 0.5 rad/sec = 2 x real part of pole

Resonance #2

- Frequency = 11 rad/sec = complex part of pole
- Bandwidth = 1 rad/sec = 2 x real part of pole

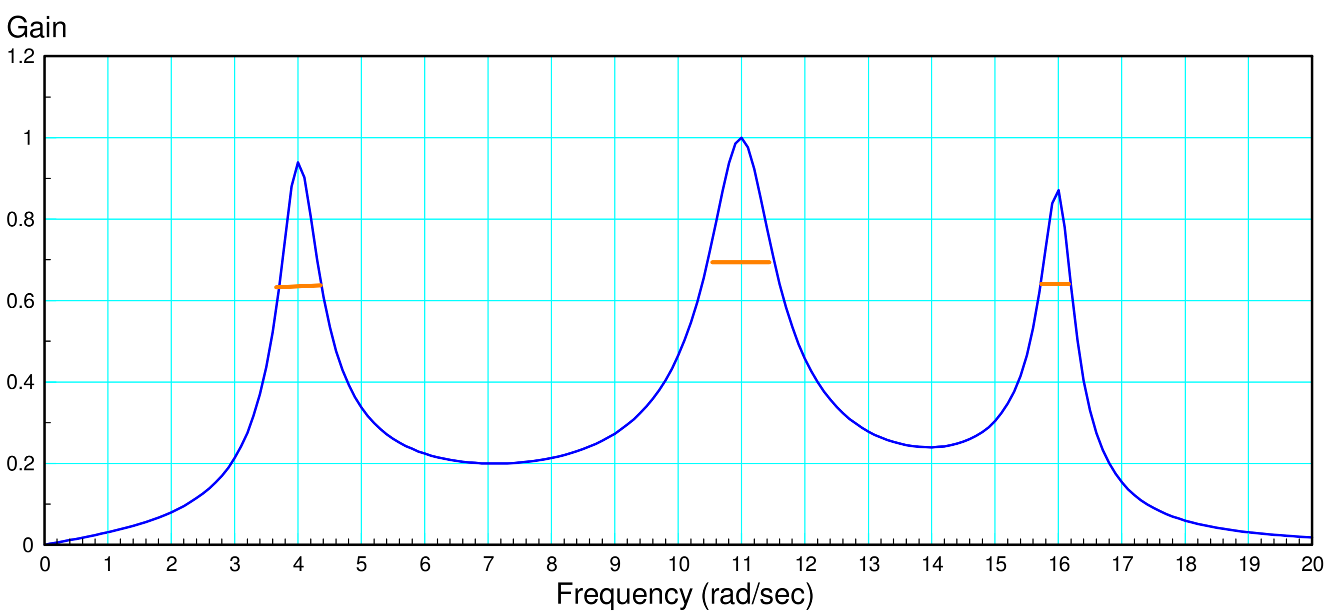
Resonance #3

- Frequency = 16 rad/sec = complex part of pole
- Bandwidth = 0.5 rad/sec = 2 x real part of pole

DC gain = 0 (zero at s = 0)

$$G(s) \approx \left(\frac{ks}{(s+0.25\pm j4)(s+0.5\pm j11)(s+0.25\pm j16)} \right)$$

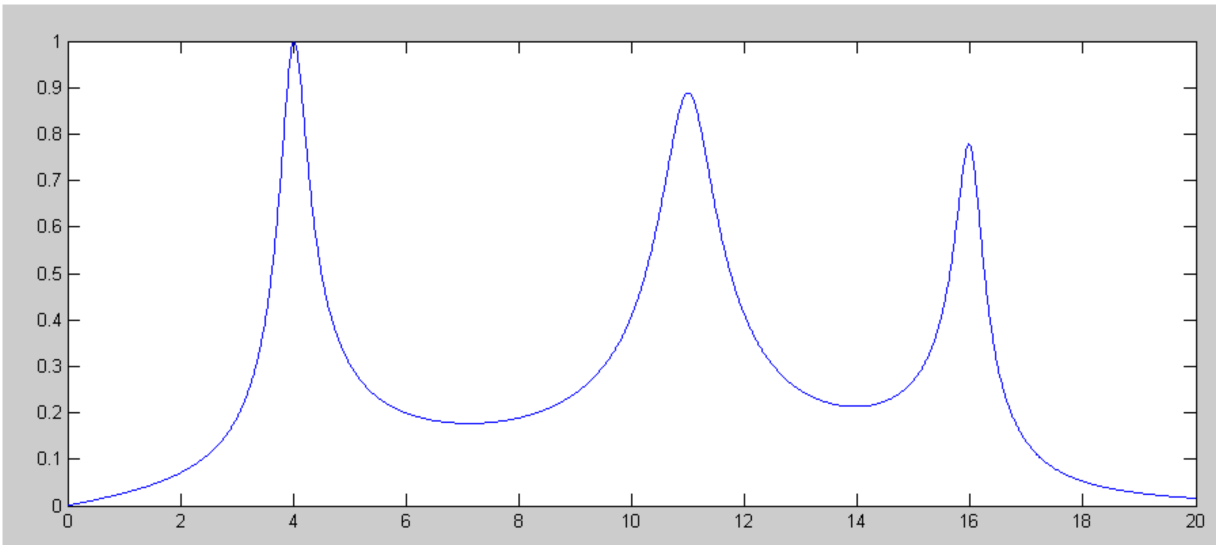
Pick 'k' so that the maximum gain is 1




```
>> p1 = -0.25 + j*4;
>> p2 = conj(p1);
>> p3 = -0.5 + j*11;
>> p4 = conj(p3);
>> p5 = -0.25 + j*16;
>> p6 = conj(p5);
>> w = [0:0.01:20]';
>> s = j*w;
>> G = s ./ ( (s-p1).*(s-p2).*(s-p3).*(s-p4).*(s-p5).*(s-p6) );
>> max(G)

ans =  7.9059e-005 -4.3719e-006i
```

```
>> k = 1/abs(ans)

k =  1.2630e+004

>> G = k* s ./ ( (s-p1).*(s-p2).*(s-p3).*(s-p4).*(s-p5).*(s-p6) );
>> plot(w,abs(G))
>>
```

Not exact, but close



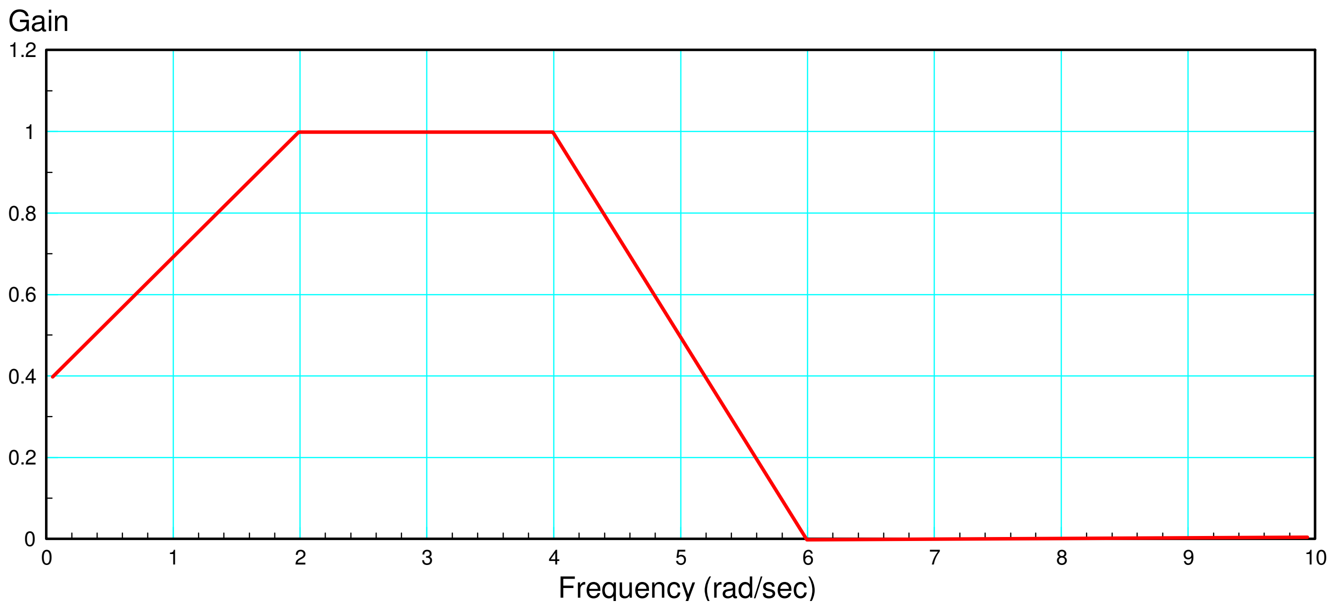
Filter Design using *fminsearch()*

5) Design a filter of the form

$$Y = \left(\frac{100a}{(s+b)\left(s^2+cs+d\right)\left(s^2+es+f\right)} \right) X$$

to give a gain vs. frequency as close to the following plot as possible over the range of (0, 10) rad/sec.

Plot your filter's actual frequency response vs. it's ideal response (red line).



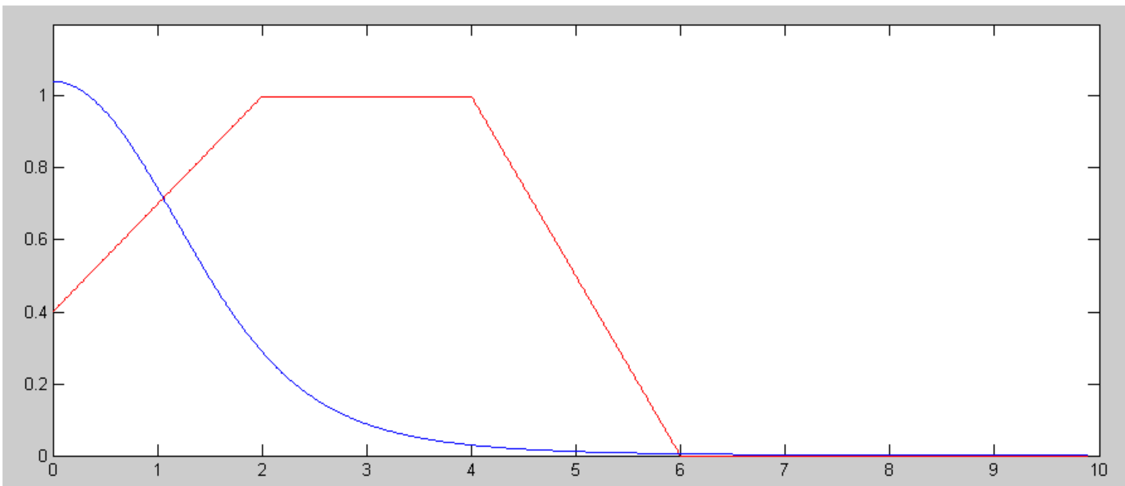
First, create an m-file which

- Is passed your guess for {a..f},
- Computes G(s),
- Computes Gd(s), and
- Returns the sum-squared difference.

```
% ECE 321 Homework #3
% Cost function for a filter

function [ J ] = costF( z )
 a = z(1);
 b = z(2);
 c = z(3);
 d = z(4);
 e = z(5);
 f = z(6);
 w = [0:0.01:9.9]' + 1e-6;
 s = j*w;
 Gideal = (0.3*w+0.4).* (w < 2) + 1*(w>2).*(w<4) + (3-0.5*w).*(w>4).*(w<6);
 G = 100*a ./ ( (s+b).*(s.^2 + c*s + d).*(s.^2 + e*s + f) );
 e = abs(Gideal) - abs(G);
 J = sum(e .^ 2);
 plot(w,abs(Gideal),'r',w,abs(G),'b');
 ylim([0,1.2]);
 pause(0.01);
end
```

Check this from the command window:

```
>> costF([0.5,2,3,4,5,6])

ans =

  257.5852
```



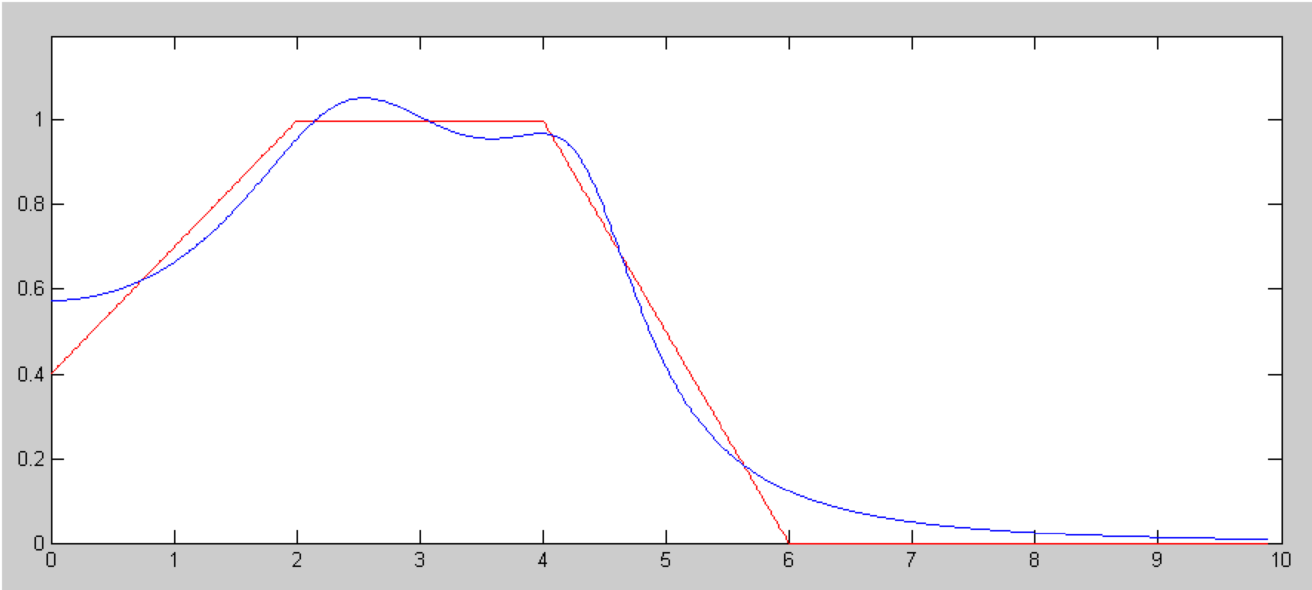
Now let Matlab optimize the parameters:

```
>> [Z,e] = fminsearch('costF',[0.5,2,3,4,5,6])

          a          b          c          d          e          f
Z =   10.8526   14.2080    2.0776    6.8006    1.2423   19.5714
e =    2.7239
```

Run again using the previous answer (just incase Matlab wasn't done yet)

```
>> [Z,e] = fminsearch('costF',Z)

          a          b          c          d          e          f
Z =   10.8526   14.2080    2.0776    6.8006    1.2423   19.5714
e =    2.7239
```

This looks like the best matlab can do



$$Y = \left(\frac{100a}{(s+b)\left(s^2+cs+d\right)\left(s^2+es+f\right)} \right) X$$

```
          a         b          c          d          e          f
Z =    10.8526   14.2080    2.0776     6.8006     1.2423    19.5714
```

6) Design circuit to implement the filter you designed in problem #7

Do this in three stages

$$Y = \left(\frac{1085.26}{(s+14.2080)\left(s^2+2.0776s+6.8006\right)\left(s^2+1.2423s+19.5714\right)} \right) X$$

$$Y = \left(\frac{k_1}{s+14.2080} \right) \left(\frac{k_2}{s^2+2.0776s+6.8006} \right) \left(\frac{k_3}{s^2+1.2423s+19.5714} \right) X$$

$$Y = \left(\frac{k_1}{s+14.2080} \right) \left(\frac{k_2}{s+2.6078\angle\pm 66.53^0} \right) \left(\frac{k_3}{s+4.424\angle\pm 81.929^0} \right) X$$

Stage #1:

1/RC = 14.208

Stage #2

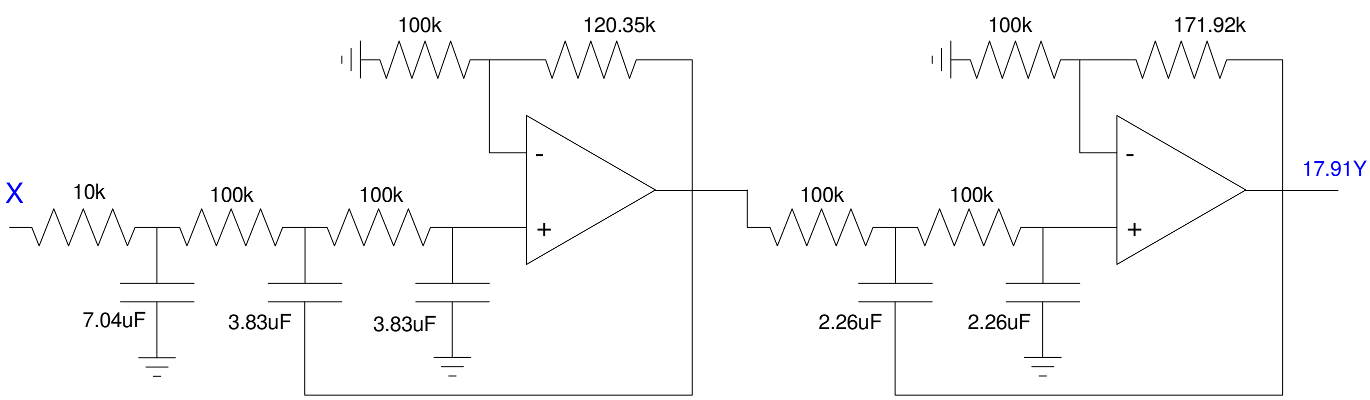
1/RC = 2.6078

k = 2.2035

Stage #3

1/RC = 4.424

k = 2.7192



7) Check your filter using CircuitLab

Kind of hard to tell since the x-axis is on a log-scale. Sort of looks the same though...