ECE 321 - Homework #3

Active Filters. Due Wednesday, April 20th

Please make the subject "ECE 321 HW#3" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Filters

1) Assume X and Y are related by the following transfer function:

$$Y = \left(\frac{40}{(s+2)(s+7)}\right) X$$

a) What is the differential equation relating x and y?

Cross multiply

$$(s^2 + 9s + 14) Y = 40X$$

'sy' means 'the derivative of y'

$$\frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 14y = 40x$$

or using prime notation where y' means dy/dt

$$y'' + 9y' + 14y = 40x$$

b) Determine y(t) assuming

$$x(t) = 4 + 5\cos(7t) + 6\sin(7t)$$

Treat this as two problems (x(t) contains two frequencies)

(i) $x(t) = 4$

$$s = 0$$

$$Y = \left(\frac{40}{(s+2)(s+7)}\right)_{s=0} \cdot (4)$$

$$Y = 11.429$$

(ii) $x(t) = 5\cos(7t) + 6\sin(7t)$

$$X = 5 - j6$$

$$s = j7$$

$$Y = \left(\frac{40}{(s+2)(s+7)}\right)_{s=j7} \cdot (5 - j6)$$

$$Y = -4.259 - j0.809$$

$$y(t) = -4.259\cos(7t) + 0.809\sin(7t)$$

The total answer is the sum of the two

$$y(t) = 11.429 - 4.259\cos(7t) + 0.809\sin(7t)$$

Filter Design

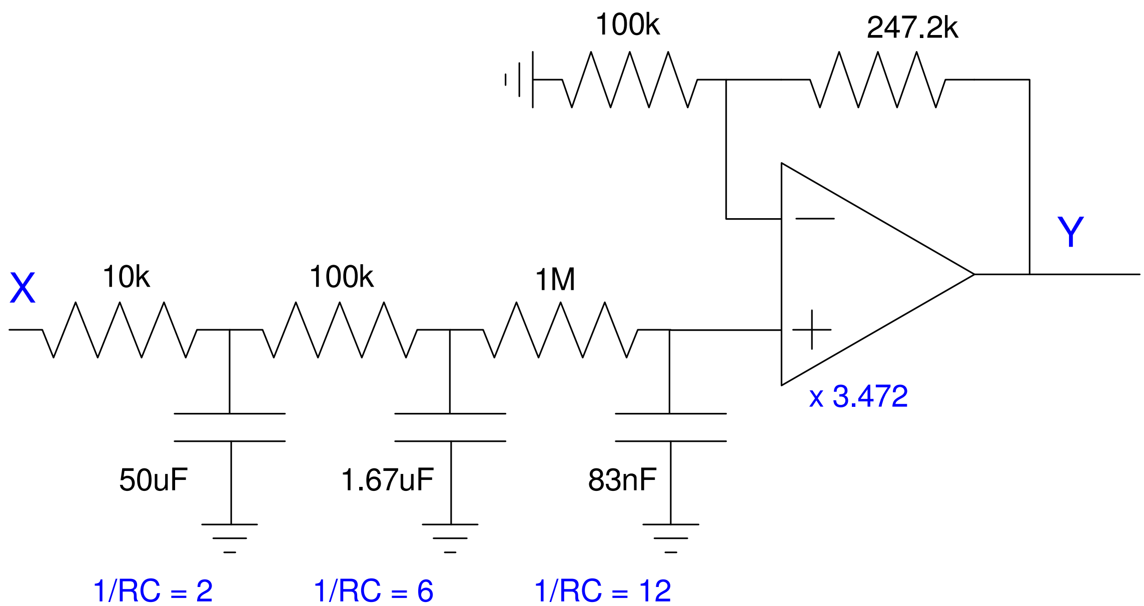
2) Give an op-amp circuit to implement the following filter

$$Y = \left(\frac{500}{(s+2)(s+6)(s+12)}\right) X$$

Rewrite as

$$Y = \left(\frac{2}{s+2}\right)\left(\frac{6}{s+6}\right)\left(\frac{12}{s+12}\right)(3.472) X$$

Implement as three RC filters along with a non-inverting amplifier



3) Give an op-amp circuit to implement the following filter

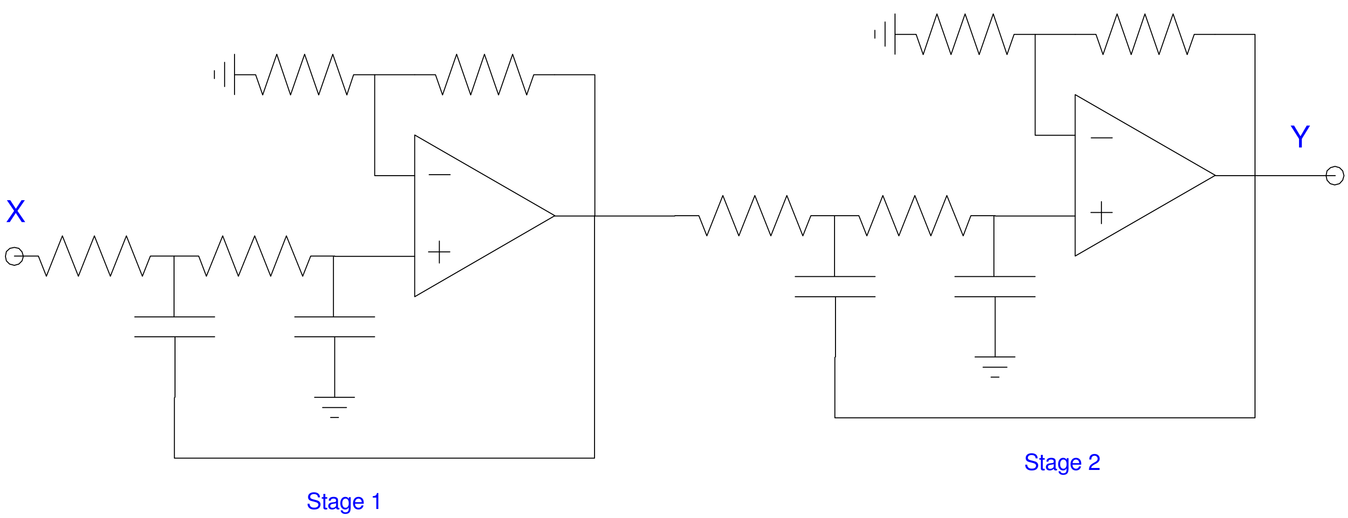
$$Y = \left(\frac{500}{(s^2+2s+10)(s^2+4s+20)} \right) X$$

Rewrite this as

$$Y = \left(\frac{500}{(s+1\pm j3)(s+2\pm j4)} \right) X$$

$$Y = \left(\frac{-}{s+3.162\angle\pm 71.57^0} \right) \left(\frac{-}{\bar{s}+4.472\angle\pm 63.43^0} \right) X$$

Implement this in two stages



Stage 1:

$$G_1(s) = \left(\frac{k}{s+3.162\angle\pm 71.57^0} \right)$$

$$\frac{1}{RC} = 3.162$$

Let R = 100k. C = 3.163uF

$$3 - k = 2\cos(71.57^0)$$

$$k = 2.368$$

Stage 2:

$$G_2(s) = \left(\frac{-}{s+4.472\angle\pm 63.43^0} \right)$$

$$\frac{1}{RC} = 4.472$$

Let R = 100k. C = 2.236uF

$$3 - k = 2\cos(63.43^0)$$

$$k = 2.105$$

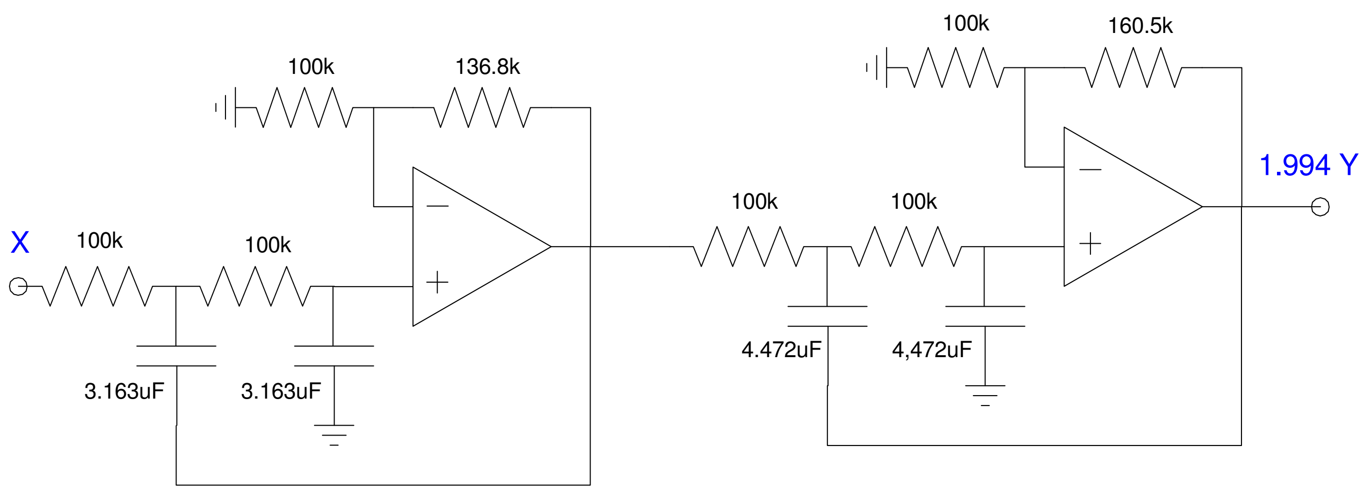
The resulting DC gain is

$$DC = 2.368 \cdot 2.105 = 4.985$$

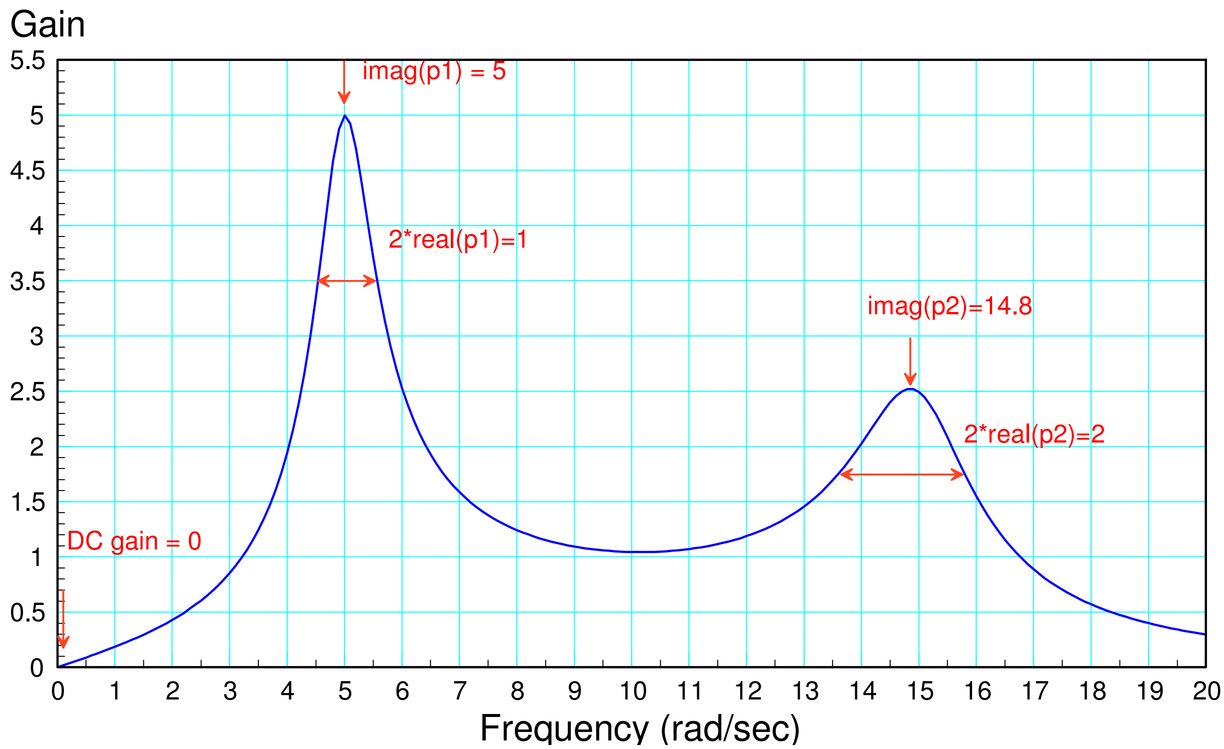
The DC gain should be 2.50

$$\left(\frac{500}{\left(s^2+2s+10\right)\left(s^2+4s+20\right)} \right)_{s=0} = 2.50$$

meaning the DC gain is 1.994x too much. Call the output 1.994y



4) Give the transfer function of a filter with the following gain vs. frequency



The DC gain is zero.

There must be a zero at s = 0

There is a resonance at 5 rad/sec

imag(pole #1) = resonance frequency = 5 rad/sec

2 * real(pole #1) = bandwidth = 1 rad/sec

real(pole #1) = 0.5

$$p_1 = -0.5 \pm j5$$

There is a resonance at 14.8 rad/sec

imag(pole #2) = resonance frequency = 14.8 rad/sec

2 * real(pole #2) = bandwidth = 2 rad/sec

real(pole #2) = 1 rad/sec

$$p_2 = -1 \pm j14.8$$

So

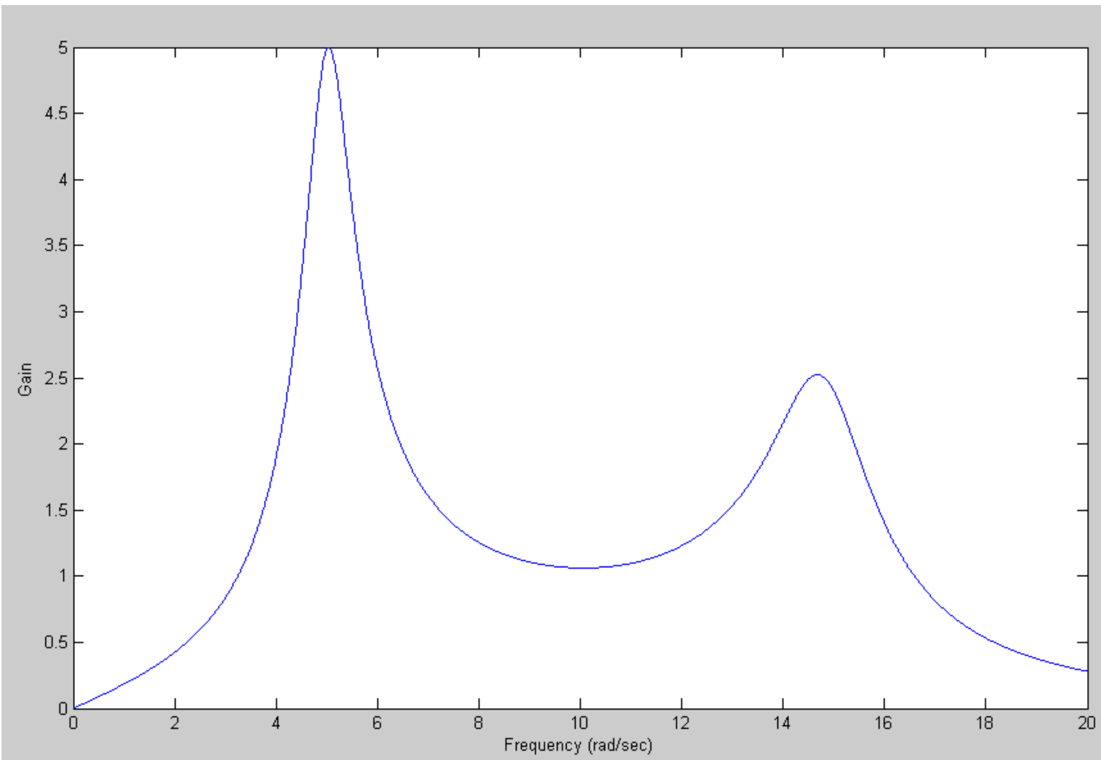
$$G(s) \approx \left(\frac{ks}{(s+0.5 \pm j5)(s+1 \pm j14.8)} \right)$$

To find k, match the gain at some frequency. Using Matlab, make the maximum gain equal to 5

```
>> w = [0:0.01:20]';
>> s = j*w;
>> p1 = -0.5 + j*5;
>> p2 = -0.5 - j*5;
>> p3 = -1 + j*14.8;
>> p4 = -1 - j*14.8;
>>
>> G = s ./ ( (s-p1).*(s-p2).*(s-p3).*(s-p4) );
>> k = 5 / max(abs(G))

k =

  974.9369

>> plot(w,abs(k*G))
```

and

$$G(s) \approx \left(\frac{974.94s}{(s+0.5\pm j5)(s+1\pm j14.8)}\right)$$



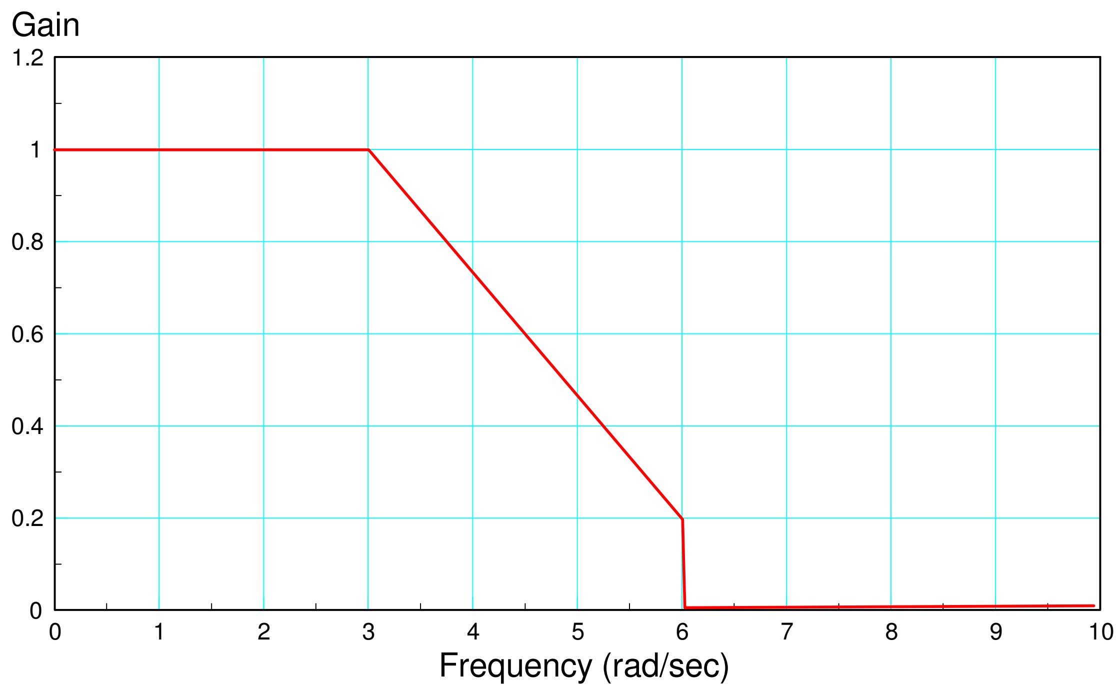
Filter Design using *fminsearch()*

5) Design a filter of the form

$$Y = \left(\frac{ace}{(s+a)\left(s^2+bs+c\right)\left(s^2+ds+e\right)} \right) X$$

to give a gain vs. frequency as close to the following plot as possible over the range of (0, 10) rad/sec.

Plot your filter's actual frequency response vs. it's ideal response (red line).



Step 1: Create an m-file where you

- Guess {a,b,c,d,e}
- It computes G(jw)
- It compares it to the desired G(jw), and
- Returns the sum-sqared error

```
function [ J ] = costf( z )
 a = z(1);
 b = z(2);
 c = z(3);
 d = z(4);
 e = z(5);

 w = [0:0.01:10]';
 s = j*w;

 Gideal = (1)*(w<3) + (1.8 - 0.0.267*w).*(w>=3).*(w<6);

 G = a*c*e ./ ( (s+a).*(s.^2 + b*s + c) .* (s.^2 + d*s + e ) );
 G = abs(G);
 E = abs(Gideal) - abs(G);

 J = sum(E .^ 2);

 plot(w,Gideal,w,abs(G),'r');
 ylim([0,1.4]);
 pause(0.01);
end
```

Optimizing using fminsearch

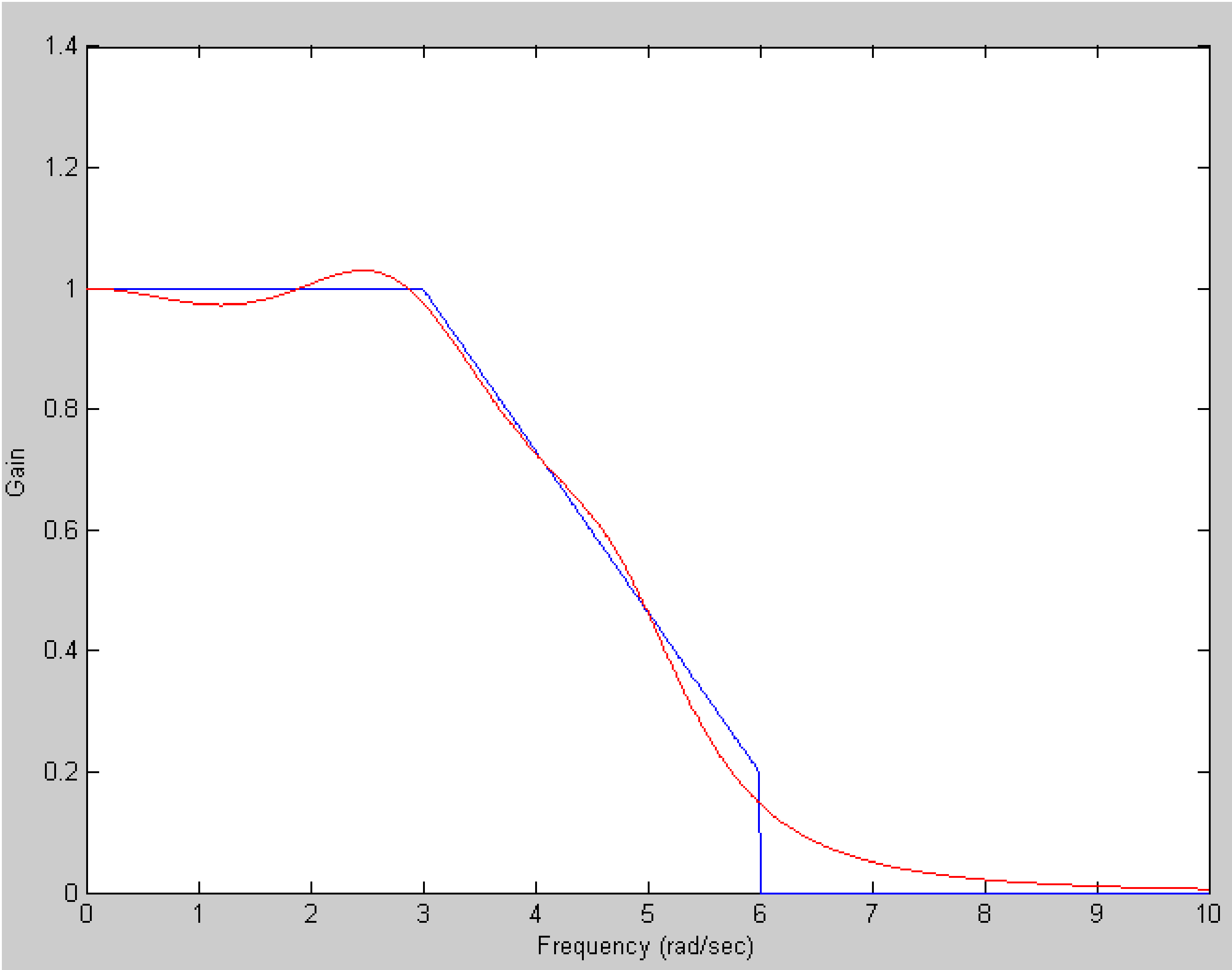
```
>> [Z,e] = fminsearch('costF',[1,2,3,4,5])

            a          b          c          d         e
Z =     1.8061     2.4968     9.5605     1.7538    24.9067

e =     1.4611
```

meaning

$$G(s) = \left(\frac{1.8061 \cdot 9.5605 \cdot 24.9067}{(s+1.8061)\left(s^2+2.4968s+9.5605\right)\left(s^2+1.7538s+24.9067\right)} \right)$$

or

$$G(s) = \left(\frac{430.069}{(s+1.8061)\left(s^2+2.4968s+9.5605\right)\left(s^2+1.7538s+24.9067\right)} \right)$$



1.4
1.2
1
0.8
0.6
0.4
0.2
0
Gain
0
1
2
3
4
5
6
7
8
9
10
Frequency (rad/sec)

6) Design circuit to implement the filter you designed in problem #5

$$G(s) = \left(\frac{430.069}{(s+1.8061)\left(s^2+2.4968s+9.5605\right)\left(s^2+1.7538s+24.9067\right)} \right)$$

Write this as

$$G(s) = \left(\frac{1.8061}{(s+1.8061)} \right) \left(\frac{9.5605}{s^2+2.4968s+9.5605} \right) \left(\frac{24.9067}{s^2+1.7538s+24.9067} \right)$$

$$G(s) = \left(\frac{1.8061}{(s+1.8061)} \right) \left(\frac{9.5605}{s+3.092\angle\pm66.19^0} \right) \left(\frac{24.9067}{s+4.99\angle79.88^0} \right)$$

Implement as

- An RC filter
- An active low-pass filer, and
- An active low-pass filter

Stage 1

$$\frac{1}{RC} = 1.8061$$

Let R = 10k. C = 55.36uF

Stage 2

$$\frac{1}{RC} = 3.092$$

Let R = 100k, C = 3.234uF

$$3 - k = 2\cos(66.19^0)$$

$$k = 2.193$$

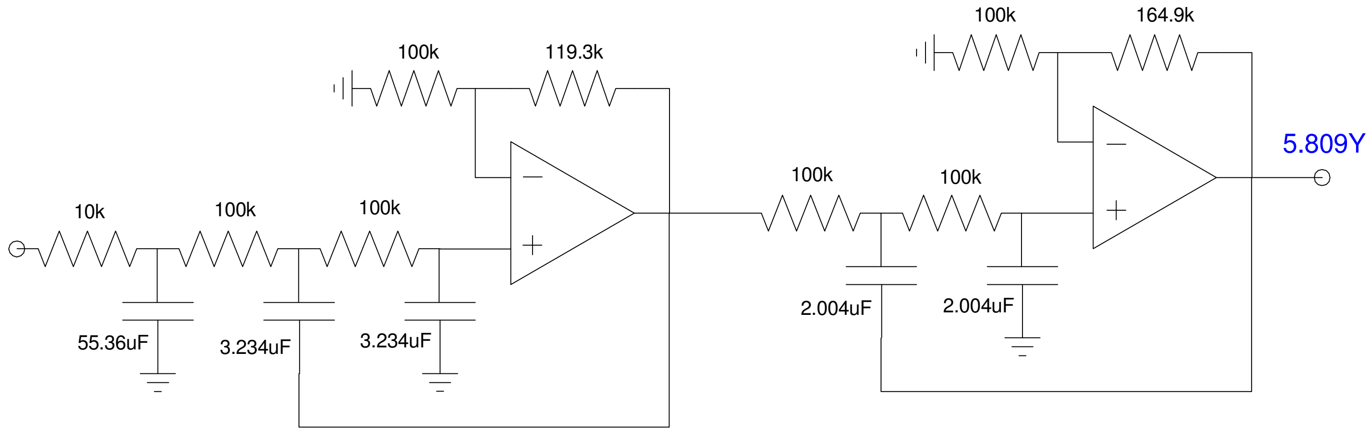
Stage 3

$$\frac{1}{RC} = 4.99$$

Let R = 100k, C = 2.004uF

$$3 - k = 2\cos(79.88^0)$$

$$k = 2.649$$



7) Check your filter using CircuitLab