ECE 321 - Quiz #3 - Name _____

Filters

1) Assume X and Y are related by the following transfer function

$$Y = \left(\frac{200}{(s+4)(s+6)}\right) X$$

Find y(t) assuming

$$x(t) = 10 + 5\cos(mt) + d\sin(mt)$$

where

- m is your birth month (1..12), and
- d is your birth date (1..31)

m = 5, d = 14

$$x(t) = 10 + 5\cos(5t) + 14\sin(5t)$$

DC Analysis

$$x(t) = 10$$

$$s = 0$$

$$Y = \left(\frac{200}{(s+4)(s+6)}\right)_{s=0} \cdot (10 + j0)$$

$$Y = 83.333$$

AC Analysis

$$x(t) = 5\cos(5t) + 14\sin(5t)$$

$$s = j5$$

$$X = 5 - j14$$

$$Y = \left(\frac{200}{(s+4)(s+6)}\right)_{s=j5} \cdot (5 - j14)$$

$$Y = -56.377 - j18.872$$

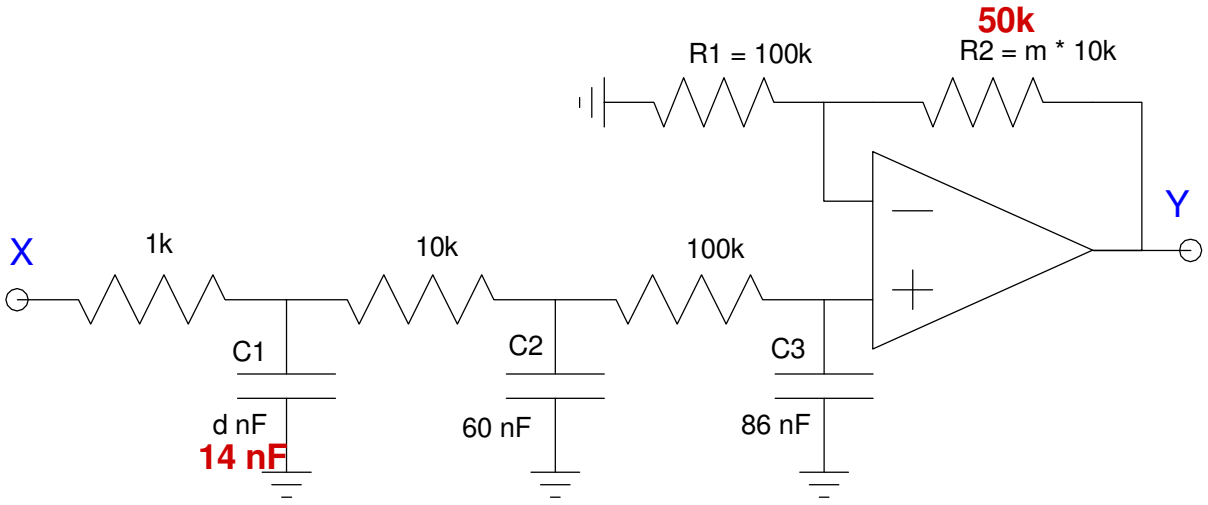
$$y(t) = -56.377\cos(5t) + 18.872\sin(5t)$$

The total answer is DC + AC

$$y(t) = 83.333 - 56.377\cos(5t) + 18.872\sin(5t)$$

2) Determine the transfer function for the following filter. Assume

- m is your birth month (1..12) (Ra = 10k .. 120k Ohms)
- d is your birth date (1..31) (C1 = 1nf .. 31nF)



pole 1

$$\left(\frac{1}{RC}\right) = \left(\frac{1}{1k \cdot 14nF}\right) = 71,428$$

pole 2

$$\left(\frac{1}{RC}\right) = \left(\frac{1}{10k \cdot 60nF}\right) = 1667$$

pole 3

$$\left(\frac{1}{RC}\right) = \left(\frac{1}{100k \cdot 86nF}\right) = 116.3$$

DC gain

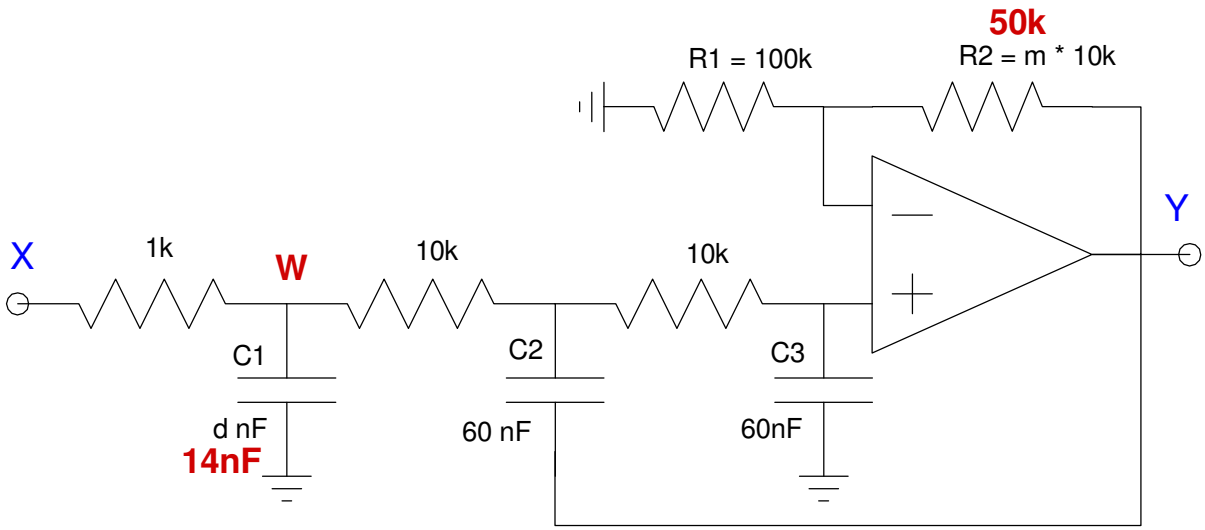
$$1 + \frac{R_2}{R_1} = 1.50$$

so

$$Y = \left(\frac{71428}{s+71428}\right)\left(\frac{1667}{s+1667}\right)\left(\frac{116.3}{s+116.3}\right)(1.50)X$$

3) Determine the transfer function for the following filter. Assume

- m is your birth month (1..12) (Ra = 10k .. 120k Ohms)
- d is your birth date (1..31) (C1 = 1..31 nF)



Note that this is an RC filter cascaded with an active low-pass filter

RC filter

$$\left(\frac{1}{RC}\right) = \left(\frac{1}{1k \cdot 14nF}\right) = 71,428$$

$$W = \left(\frac{71429}{s+71428}\right) X$$

Active Filter

$$\left(\frac{1}{RC}\right) = \left(\frac{1}{10k \cdot 60nF}\right) = 1667$$

$$k = 1 + \frac{R_2}{R_1} = 1.5$$

$$3 - k = 2\cos\theta$$

$$\theta = 41.4^0$$

$$Y = \left(\frac{1.5 \cdot 1667^2}{\left(s+1667\angle \pm 41.4^0\right)}\right) W$$

The total filter is then

$$Y = \left(\frac{1.5 \cdot 1667^2}{\left(s+1667\angle \pm 41.4^0\right)}\right)\left(\frac{71429}{s+71429}\right) X$$

4) Give the transfer function for a filter which meets the following requirements

- 0.9 < gain < 1.1 for frequencies below 30 rad/sec
- gain < 0.2 for frequencies above 50 rad/sec

The number of poles needed is

$$\left(\frac{30}{50}\right)^n < 0.2$$

$$n > 3.151$$

Let n = 4.

Assume a 4th-order Chebychev filter

Assume the corner is 30 rad/sec

A 4th-order Chebychev filter with a corner at 1 rad/sec (from lecture notes) is

$$G(s) = \left(\frac{0.72^2 \cdot 1.11^2}{\left(s+0.72\angle\pm 38.5^0\right)\left(s+1.11\angle\pm 77.8^0\right)} \right)$$

A 4th-order Chebychev filter with a corner at 30 rad/sec is then

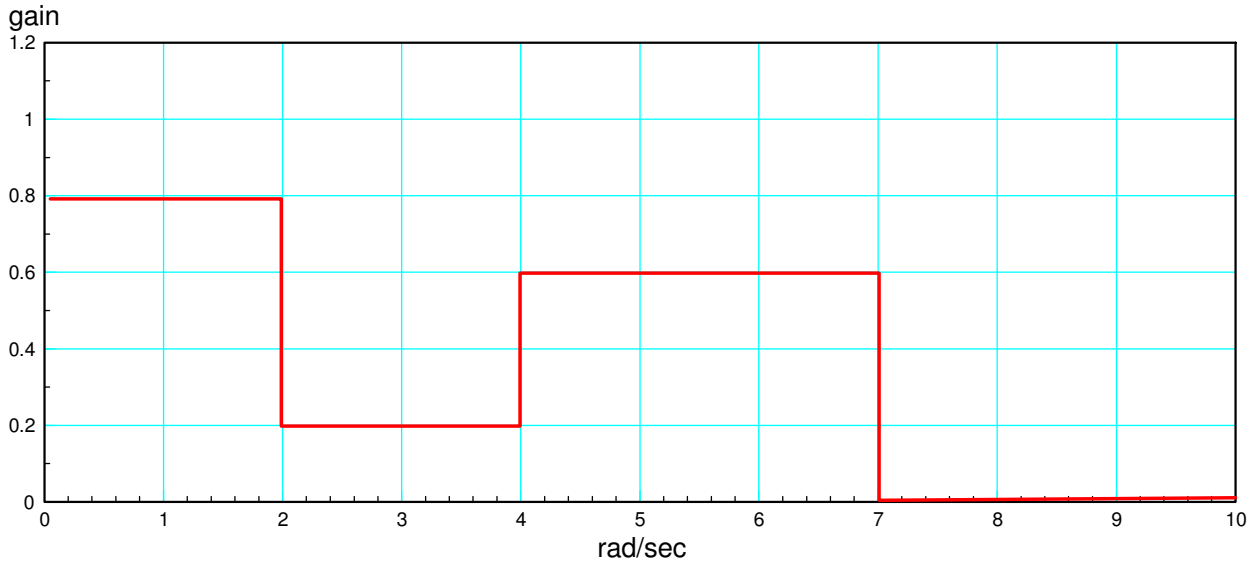
$$G(s) = \left(\frac{21.6^2 \cdot 33.3^2}{\left(s+21.6\angle\pm 38.5^0\right)\left(s+33.3\angle\pm 77.8^0\right)} \right)$$

5) Give the Matlab code for an m-file you would use to have Matlab's *fminsearch()* design a filter with the following gain vs. frequency

$$G(s) = \left(\frac{a\left(s^2+b\right)}{\left(s^2+cs+d\right)\left(s^2+es+f\right)} \right)$$

The m-file should

- Receive parameters {a,b,c,d,e,f,g}
- Compute G(jw)
- Return the sum squared error between G(jw) and the graph below

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function [ J ] = cost_problem5( z )
 a = z(1);
 b = z(2);
 c = z(3);
 d = z(4);
 e = z(5);
 f = z(6);

 w = [0:0.1:10]';
 s = j*w;

 Gideal = 0.8*(w<2) + 0.2*(w>=2).*(w<4) + 0.6*(w>=4).*(w<7);

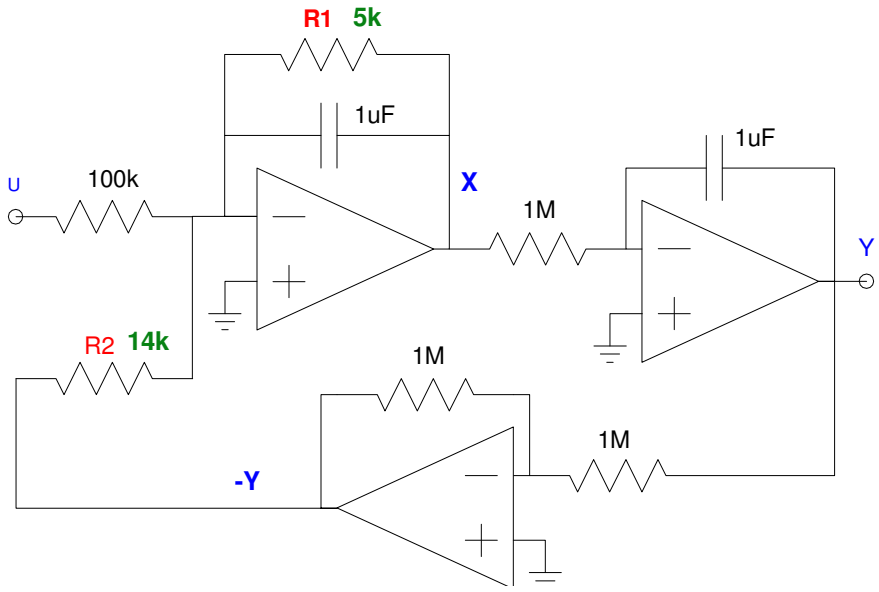
 G = a*(s.^2 + b) ./ ( (s.^2 + c*s + d).*(s.^2 + e*s + f) );

 e = abs(Gideal) - abs(G);
 J = sum(e .^ 2);
 plot(w,abs(Gideal),'r',w,abs(G),'b');
 ylim([0,1.2]);
 pause(0.01);
end
```

6) What is the transfer function for the following analog computer?

Assume

- R1 = your birth month (1..12) k Ohms
- R2 = your birth data (1..31) k Ohms



These are two summing integrators

$$X = -\left(\frac{1}{s}\right)\left(\left(\frac{1}{1\mu F \cdot 100k}\right)U + \left(\frac{1}{1\mu F \cdot 14k}\right)(-Y) + \left(\frac{1}{1\mu F \cdot 5k}\right)X\right)$$

$$-sX = 10U - 71.43Y + 200X$$

$$Y = -\left(\frac{1}{s}\right)X$$

$$-sY = X$$

$$-s(-sY) = 10U - 71.43Y + 200(-sY)$$

$$s^2Y + 200sY + 71.43Y = 10U$$

$$\boxed{Y = \left(\frac{10}{s^2+200s+71.43}\right)U}$$