# ECE 321 - Quiz #4 - Name ____________________

Filters, Filter Design, Analog Computers. Due midnight, March 29th

1) X and Y are related by

$$Y = \left( \frac{20s+30}{(s+M)(s+D)} \right) X$$

where

- M is your birth month (1..12), and
- D is your birth date (1..31)

Determine y(t) assuming

$$x(t) = 3 + 4\sin(5t)$$

Let M = 5, D = 14

$$Y = \left( \frac{20s+30}{(s+5)(s+14)} \right) X$$

Using superposition

x(t) = 3

$$s = 0$$

$$Y = \left( \frac{20s+30}{(s+5)(s+14)} \right)_{s=0} \cdot 3$$

$$Y = 1.2857$$

meaning y(t) = 1.2857.

x(t) = 4 sin(5t)

$$Y = \left( \frac{20s+30}{(s+5)(s+14)} \right)_{s=j5} \cdot (0 - j4)$$

$$Y = 0.5973 - j3.9276$$

meaning

$$y(t) = 0.5973\cos(5t) + 3.9276\sin(5t)$$

The total answer is DC + AC

$$y(t) = 1.2857 + 0.5973\cos(5t) + 3.9276\sin(5t)$$

2) Design an op-amp circuit (a.k.a. an analog computer) to implement

$$Y = \left( \frac{20s+30}{(s+M)(s+D)} \right) X$$

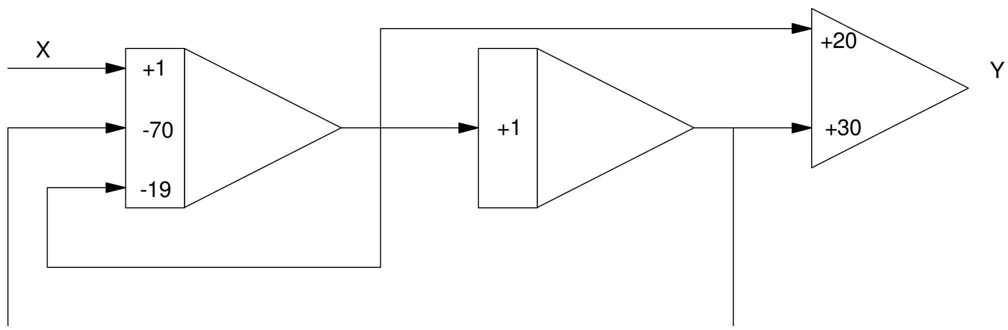
where

- M is your birth month (1..12), and
- D is your birth date (1..31)

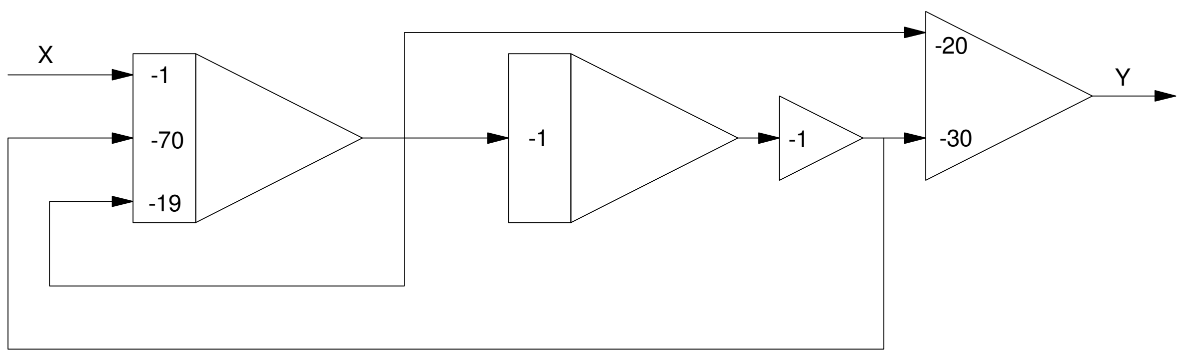
$$Y = \left( \frac{20s+30}{(s+5)(s+14)} \right) X = \left( \frac{20s+30}{s^2+19s+70} \right) X$$

$$s^2 Y = -19sY - 70Y + 20sX + 30X$$

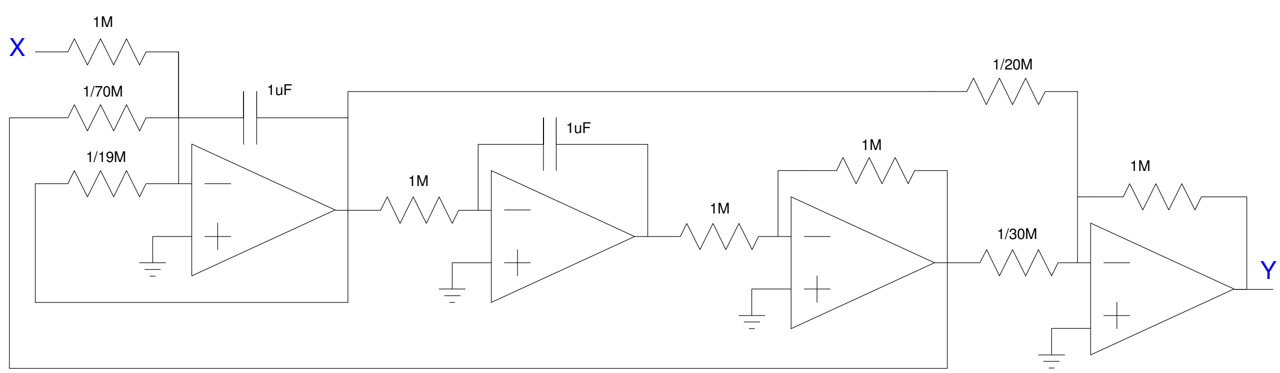
using analog comptuer notation




Adjusting the gains so they are all negative. Add an inverter when needed




Replace with op-amp ciruits



3) The transfer function for a 6th-order Chebychev filter with a corner at 1 rad/sec is

$$G(s) = \left( \frac{0.1593}{\left(s+0.4722\angle\pm36.10^0\right)\left(s+0.8100\angle\pm69.83^0\right)\left(s+1.0436\angle\pm84.38^0\right)} \right)$$

Give the transfer function for a 6th-order Chebychev filter with

- A DC gain of 1.000 and
- A corner at X rad/sec

where

- X = 1000 + 100*(your birth month) + (your birth date)

X = 1514

1514 * 0.4722 = 714.9

1514 * 0.81 = 1226.3

1514 * 1.0436 = 1580

$$G(s) = \left( \frac{0.1593 \cdot 1514^6}{\left(s+714.9\angle\pm36.10^0\right)\left(s+1226.3\angle\pm69.83^0\right)\left(s+1580\angle\pm84.38^0\right)} \right)$$

4) Give the transfer function for a 7th-order Butterworth filter with

- A DC gain of 1.000 and
- A corner at X rad/sec

where

- X = 1000 + 100*(your birth month) + (your birth date)

The angle between poles is

$$\theta = \frac{180^0}{7} = 25.71^0$$

so

$$G(s) = \left( \frac{1514^7}{(s+1514)\left(s+1514\angle\pm25.71^0\right)\left(s+1514\angle\pm51.42^0\right)\left(s+1514\angle\pm77.14^0\right)} \right)$$

5) Specify a filter to meet the following requirements:

- 0.9 < gain < 1.1          0 < w < 300 rad/sec
- gain < 0.1          w > 450 rad/sec

5a) How many poles does the filter need?

5b) Give the transfer function of a filter, G(s), which meets these requirements

5c) What is the gain of your filter at 300 and 450 rad/sec?

| # poles needed | G(s) | Gain at 300 rad/sec | Gain at 450 rad/sec |
|---|---|---|---|
| 6 | $\left(\frac{0.1593\cdot 300^6}{(s+141\angle\pm 36.10^0)(s+243\angle\pm 69.83^0)(s+313\angle\pm 84.38^0)}\right)$ | 0.9925 | 0.0313 |

# poles needed

$$\left(\frac{300}{450}\right)^n < 0.1$$

$$n > 5.67$$

Let n = 6 (so I can use the filter from problem #2)

Let the corner be 300 rad/sec

$$G(s) = \left(\frac{0.1593\cdot 300^6}{(s+141\angle\pm 36.10^0)(s+243\angle\pm 69.83^0)(s+313\angle\pm 84.38^0)}\right)$$

At 300 rad/sec (using Matlab)

```
>> p1 = 141 * exp(j*36.1*pi/180);
>> p2 = conj(p1);
>> p3 = 243*exp(j*69.83*pi/180);
>> p4 = conj(p3);
>> p5 = 313*exp(j*84.38*pi/180);
>> p6 = conj(p5);
>> num = abs(p1*p2*p3*p4*p5*p6);
>> G = zpk([],[p1,p2,p3,p4,p5,p6],num);
>> DC = evalfr(G,0)
    1.0000

>> G300 = abs(evalfr(G,j*300))
    0.9925

>> G450 = abs(evalfr(G,j*450))
    0.0313
```

6) The difference between a square wave and a sine wave is a square wave has a 3rd harmonic. Design a filter to remove the 3rd harmonic (make it 30x smaller in amplitude than the 1st harmonic)

- $0.9 < gain < 1.1$ $\quad$ $0 < w < 200$ rad/sec
- $gain < 0.1$ $\quad$ $w > 300$ rad/sec

6a) How many poles does the filter need?

6b) Give the transfer function of a filter, G(s), which meets these requirements

6c) What is the gain of your filter at 200 and 300 rad/sec?

| # poles needed | G(s) | Gain at 200 rad/sec | Gain at 300 rad/sec |
|---|---|---|---|
| 6 | $\left(\frac{0.1593 \cdot 200^6}{(s+94\angle\pm 36.10^0)(s+162\angle\pm 69.83^0)(s+209\angle\pm 84.38^0)}\right)$ | 0.9882 | 0.0315 |

$$\left(\frac{200}{300}\right)^n < 0.1$$

$$n > 5.67$$

Let n = 6 so I can reuse the previous filter

Adjust the corner frequency to 200 rad/sec

$$G(s) = \left(\frac{0.1593 \cdot 200^6}{(s+94\angle\pm 36.10^0)(s+162\angle\pm 69.83^0)(s+209\angle\pm 84.38^0)}\right)$$

```
>> p1 = 94 * exp(j*36.1*pi/180);
>> p2 = conj(p1);
>> p3 = 162*exp(j*69.83*pi/180);
>> p4 = conj(p3);
>> p5 = 209*exp(j*84.38*pi/180);
>> p6 = conj(p5);
>> num = abs(p1*p2*p3*p4*p5*p6);
>> G = zpk([],[p1,p2,p3,p4,p5,p6],num);
>> G0 = evalfr(G,0)
    1.0000

>> G200 = abs(evalfr(G,j*200))
    0.9882

>> G300 = abs(evalfr(G,j*300))
    0.0315
```