ECE 111 - Homework #10

ECE 343 Signals & Systems- Due Tuesday, March 28th

Filter Analysis

1) A filter has the following transfer function

$$Y = \left(\frac{10(s+3)}{(s+1)(s+2)(s+5)}\right)X$$

1a) What is the differential equation relating X and Y?

Cross multiply

$$(s+1)(s+2)(s+5)Y = 10(s+3)X$$

$$(s^3 + 8s^2 + 17s + 10)Y = (10s + 30)X$$

'sY' means 'the derivative of Y'

$$\boxed{\frac{d^3y}{dt^3} + 8\frac{d^2y}{dt^2} + 17\frac{dy}{dt} + 10y = 10\frac{dx}{dt} + 30x}$$

or using prime notation

$$\boxed{y''' + 8y'' + 17y' + 10y = 10x' + 30x}$$

1b) Find y(t) assuming $x(t) = 5$

$$s = j0$$

$$X = 5$$

$$Y = \left(\frac{10(s+3)}{(s+1)(s+2)(s+5)}\right)_{s=j0} \cdot (5)$$

$$Y = 15$$

meaning

$$y(t) = 15$$

1c) Find y(t) assuming $x(t) = 5\sin(4t)$

$$s = j4$$

$$X = 0 - j5$$

$$Y = \left(\frac{10(s+3)}{(s+1)(s+2)(s+5)} \right)_{s=j4} \cdot (0 - j5)$$

$$Y = -1.7360 + j1.2123$$

meaning

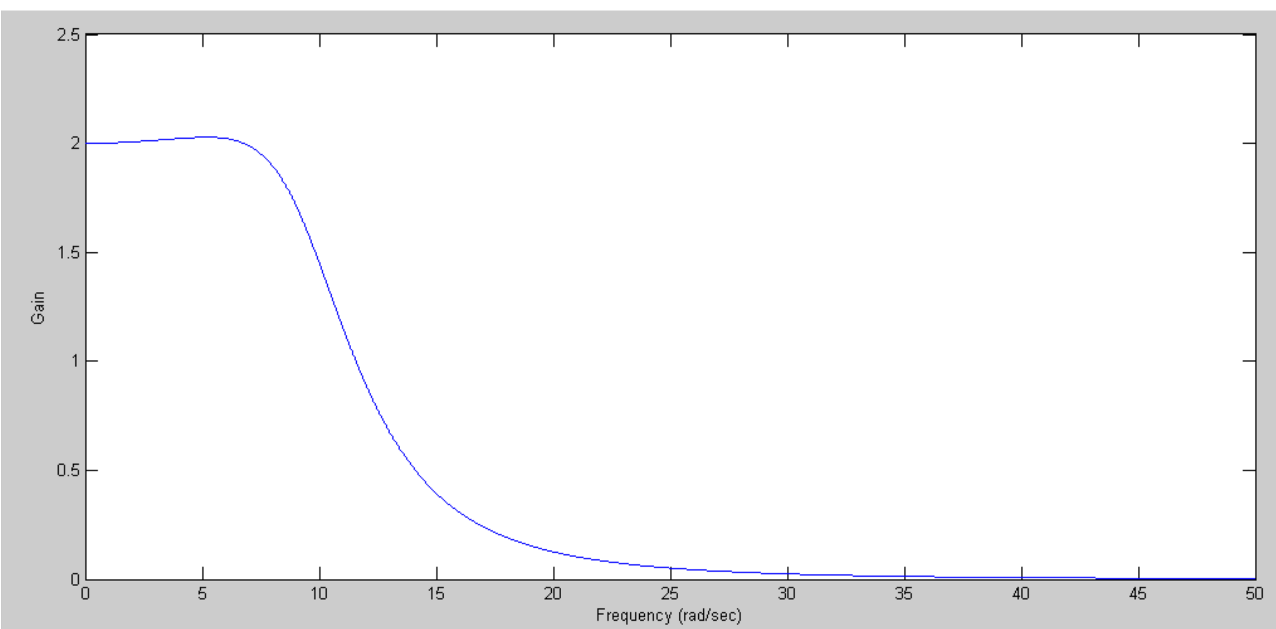
$$y(t) = -1.7360\cos(4t) - 1.2123\sin(4t)$$

2) Plot the gain vs. frequency for this filter from 0 to 50 rad/sec.

- Low-Pass Filter

$$Y = \left(\frac{20{,}000}{\left(s^2+18.5s+100\right)\left(s^2+7.65s+100\right)} \right) X$$

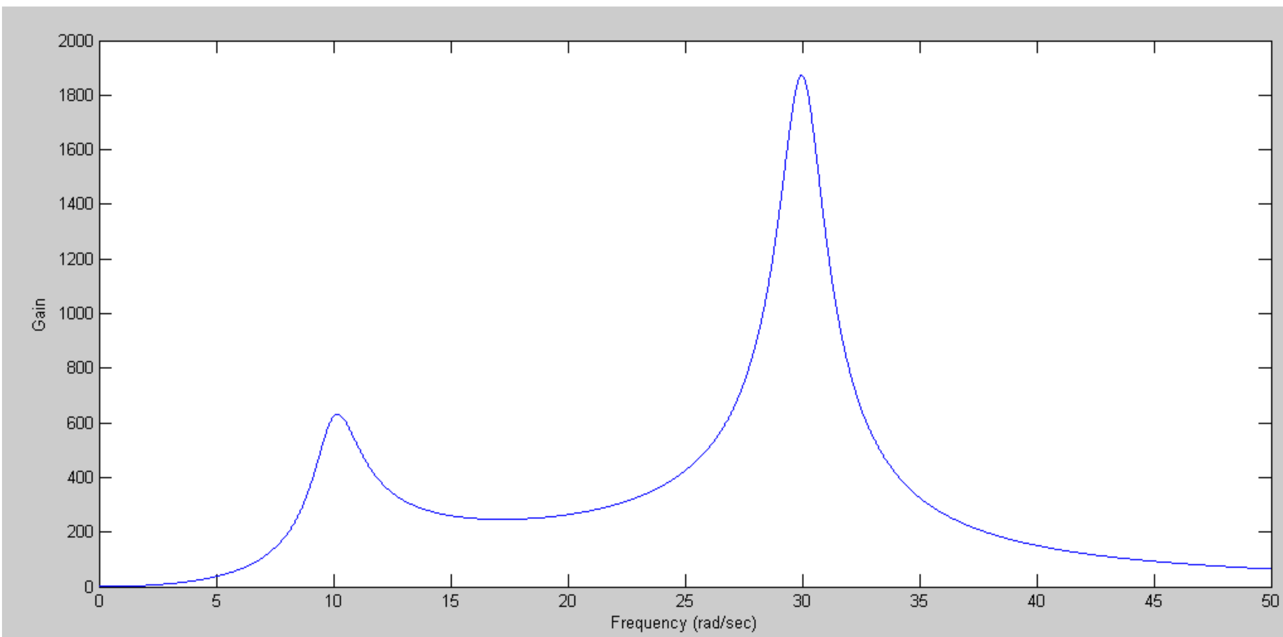
```
>> w = [0:0.01:50]';
>> s = j*w;
>> G = 20000 ./ ( (s.^2 + 18*s + 100) .* (s.^2 + 7.65*s + 100) );
>> plot(w,abs(G));
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
```



3) Plot the gain vs. frequency for this filter from 0 to 50 rad/sec.

$$Y = \left(\frac{100{,}000 \cdot s^2}{(s+1\pm j10)(s+1\pm j30)}\right)X = \left(\frac{100{,}000 \cdot s^2}{\left(s^2+2s+101\right)\left(s^2+2s+901\right)}\right)X$$

```
>> w = [0:0.01:50]';
>> s = j*w;
>> G = 1e5 * s.^2 ./ ( (s.^2 + 2*s + 101) .* (s.^2 + 2*s + 901) );
>> plot(w,abs(G));
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
>>
```



Note: Filter analysis is pretty straight forward if you don't mind using complex numbers.

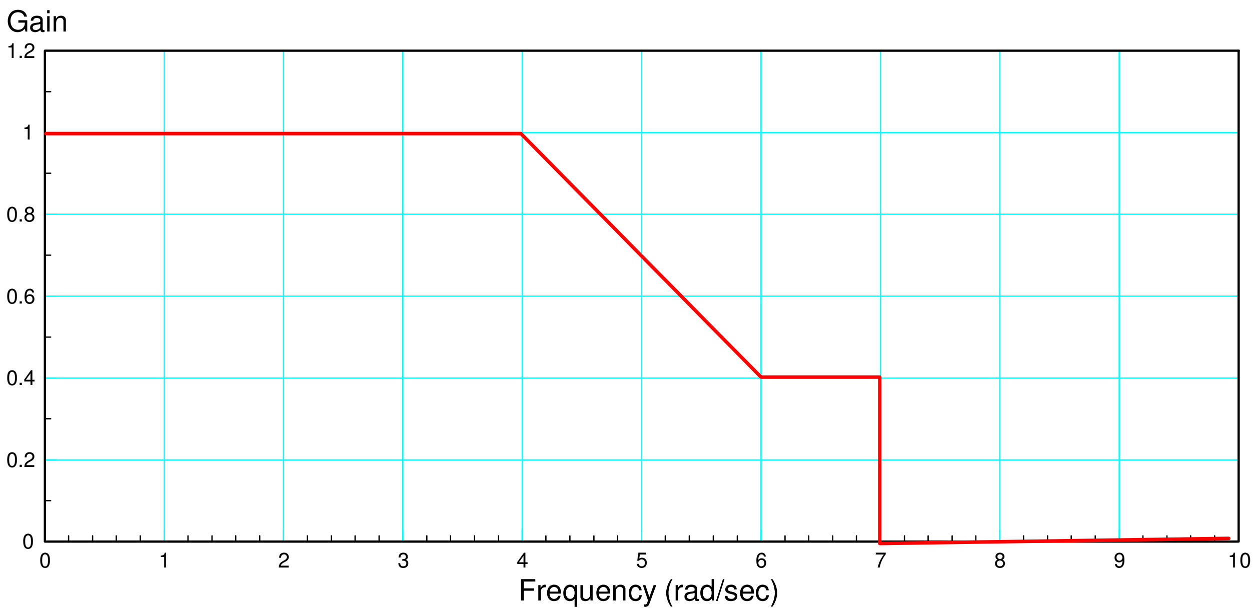
Analyze the gain of the filter at the frequency of the input

Output = Gain * Input

Filter Design

Problem 4-6) Design a filter of the following form so that the gain matches the graph below:

$$G(s) = \left(\frac{a\left(s^2+b\right)}{\left(s^2+cs+d\right)\left(s^2+es+f\right)\left(s^2+gs+h\right)} \right)$$



4) Write an m-file, cost.m, which

- Is passed an array, z, with each element representing (a, b, c, d, e, f, g,h)
- Computes the gain, G(s) for this value of (a, b, c, d, e, f, g,h)
- Computes the difference between the gain, G, and the target (above), and
- Returns the sum-squared error in the gain

Read in eight values for a..h

Compute Gideal as a piecewise linear function

$$G_{ideal} = \begin{cases} 1 & \omega < 4 \\ 2.2 - 0.3\omega & 4 < \omega < 6 \\ 0.4 & 6 < \omega < 7 \\ 0 & otherwise \end{cases}$$

Matlab File:

```
function [ J ] = costF( z )

   a = z(1);
   b = z(2);
   c = z(3);
   d = z(4);
   e = z(5);
   f = z(6);
   g = z(7);
   h = z(8);

   w = [0:0.01:10]';
   s = j*w;

   Gideal = 1 * (w<4) + (2.2 - 0.3*w) .* (w >= 4).*(w<6) + 0.4 * (w>=6) .* (w<7);

   G = a*(s.^2 + b) ./ ( (s.^2 + c*s + d).*(s.^2 + e*s + f) .* (s.^2 + g*s + h) );

   e = abs(Gideal) - abs(G);

   J = sum(e .^ 2);

   plot(w,abs(Gideal),w,abs(G));
   ylim([0,1.2]);
   pause(0.01);

end
```

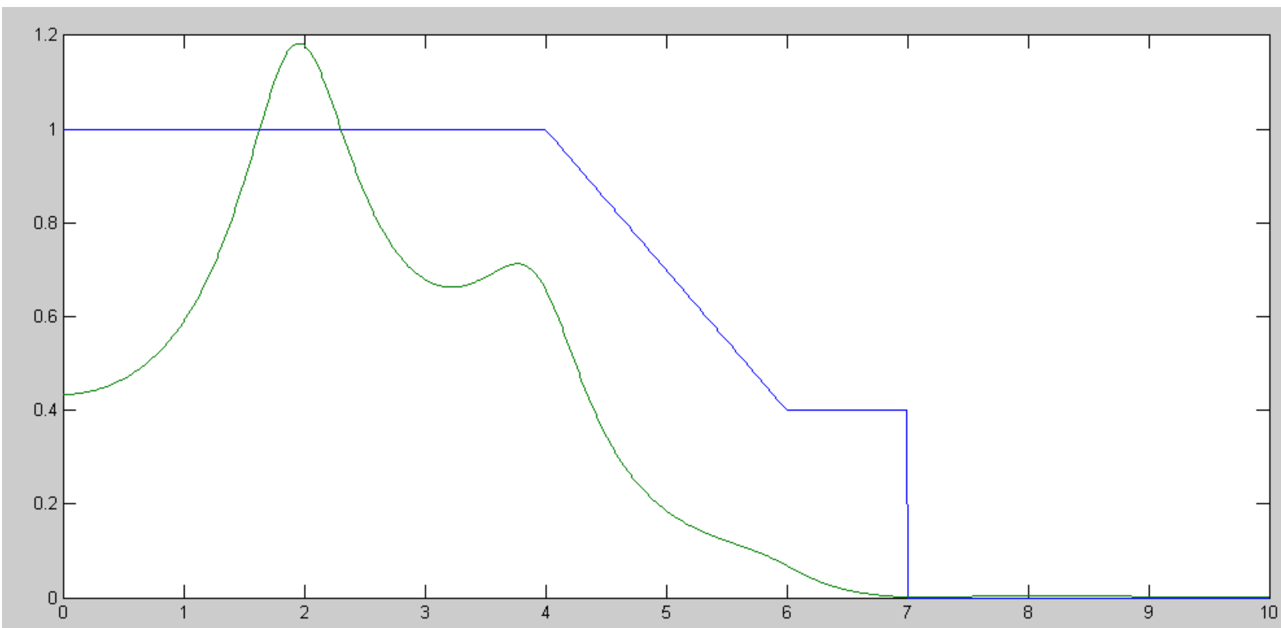
5) Use your m-file to determine how 'good' the following filter is:

$$G(s) = \left(\frac{a\left(s^2+b\right)}{\left(s^2+cs+d\right)\left(s^2+es+f\right)\left(s^2+gs+h\right)} \right) = \left(\frac{20\left(s^2+50\right)}{\left(s^2+s+4\right)\left(s^2+s+16\right)\left(s^2+s+36\right)} \right)$$

From the command window:

```
              a  b   c  d   e  f   g  h
>> costF([20,50, 1, 4, 1,16, 1,36])

ans =  101.4488
```



Note:

- It's not a very good approximation of the desired filter
- If you adjust the parameters, you can do better.

6) Use fminsearch() to find the 'best' filter of the form

$$G(s) = \left(\frac{a\left(s^2+b\right)}{\left(s^2+cs+d\right)\left(s^2+es+f\right)\left(s^2+gs+h\right)} \right)$$

```
>> [Z,e] = fminsearch('costF',[20,50,1,4,1,16,1,36])

Exiting: Maximum number of function evaluations has been exceeded
         - increase MaxFunEvals option.
         Current function value: 0.956713

Z =
   a          b          c          d        e         f          g          h
155.6800   56.9156    4.4910    8.8357    2.3300   22.0895    0.4311   46.4282

e =    0.9567
```

Let it run a litte longer (starting at the result) to see if it kicked out due to

- Reaching maximum number of itterations, or
- It found the answer

```
>> [Z,e] = fminsearch('costF',Z)

Z =
   a          b          c          d        e         f          g          h
  206.1037   54.8419    5.4227   10.9554    2.5649   22.1712    0.3987   46.8436


e =

    0.8515
```

Trying again one more time

```
>> [Z,e] = fminsearch('costF',Z)

Z =
   a          b          c          d        e         f          g          h
  206.1037   54.8419    5.4227   10.9554    2.5649   22.1712    0.3987   46.8436


e =

    0.8515
```

Looks like this is the best Matlab can do...

a) Give the resulting (a, b, c, d, e, f, g,h)

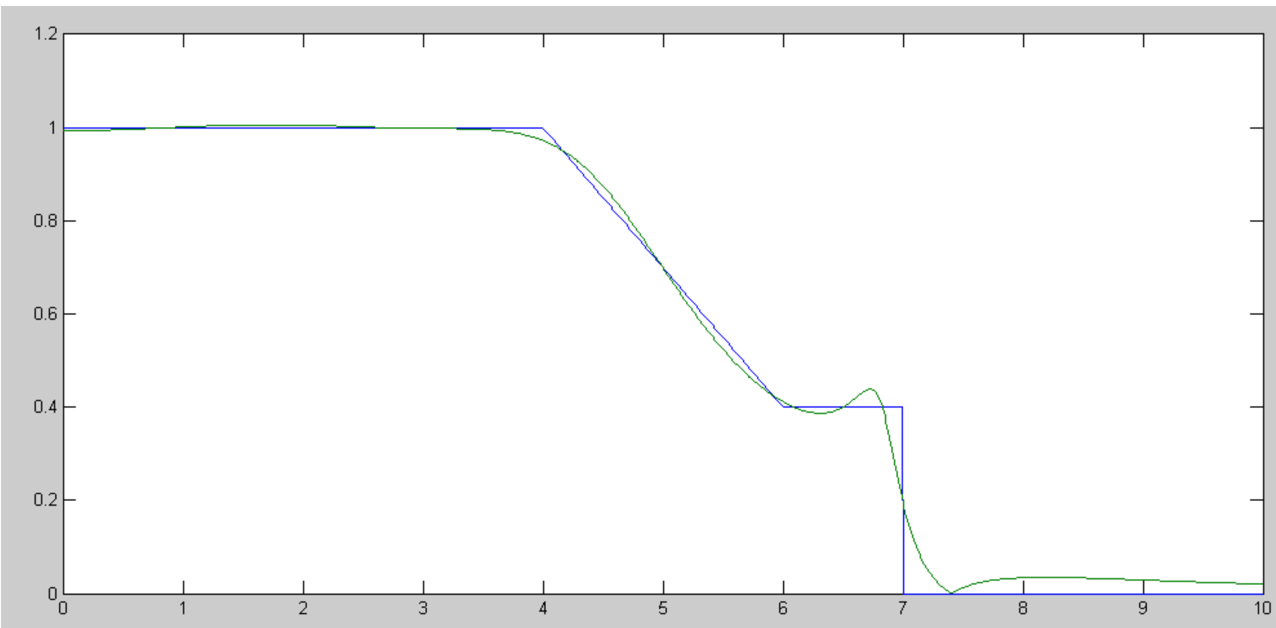
```
Z =
   a          b          c          d        e         f          g          h
  206.1037   54.8419    5.4227   10.9554    2.5649   22.1712    0.3987   46.8436
```

b) Give the resulting filter, and

$$G(s) = \left(\frac{a\left(s^2+b\right)}{\left(s^2+cs+d\right)\left(s^2+es+f\right)\left(s^2+gs+h\right)} \right)$$

$$G(s) = \left(\frac{206.1\left(s^2+54.84\right)}{\left(s^2+5.42s+10.95\right)\left(s^2+2.56s+22.17\right)\left(s^2+0.39s+46.84\right)} \right)$$

c) Plot the 'optimal' filter's gain vs. frequency



Note:

- When you take ECE 321 and ECE 311, you'll cover other ways to design filters
- With Matlab, you can design pretty good filters even if you know nothing about filter design