

# ECE 111 - Homework #9

Week #9: ECE 311 Circuits II - Due Tuesday, March 21st

1) Solve for Y

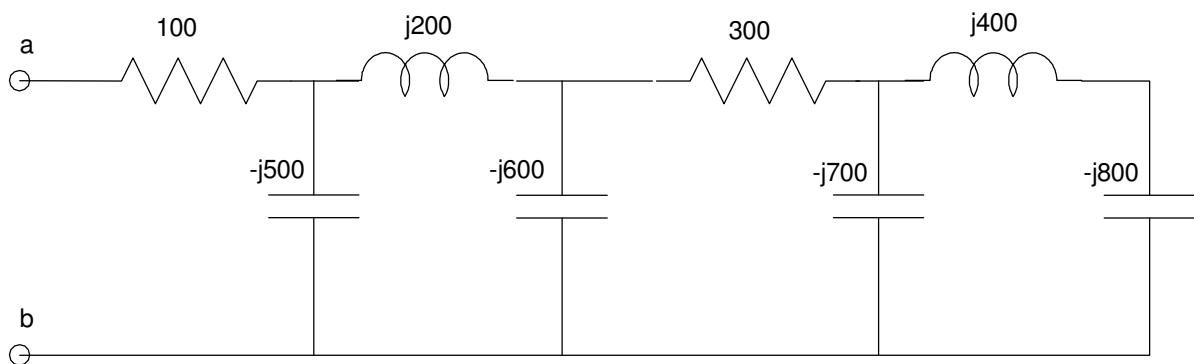
$$\bullet Y = \left( \frac{(4+j12)(2-j13)}{(2-j)} \right)$$

$$Y = 71.200 + j21.600$$

$$\bullet Y = \left( \left( \frac{4+j12}{2-j} \right) + \left( \frac{2-j13}{2+j} \right) \right) \left( \frac{3+j8}{9+j7} \right)$$

$$Y = -1.660 - j1.020$$

2) Determine the impedance  $Z_{ab}$



This is a bunch of series - parallel calculations with complex numbers

$$j400 + (-j800) = -j400 \quad \text{series}$$

$$-j400 \parallel -j700 = -j254.545 \quad \text{parallel}$$

$$(-j254.545) + (300) = 300 - j254.545 \quad \text{series}$$

$$(300 - j254.545) \parallel (-j600) = 131.668 - j224.947 \quad \text{parallel}$$

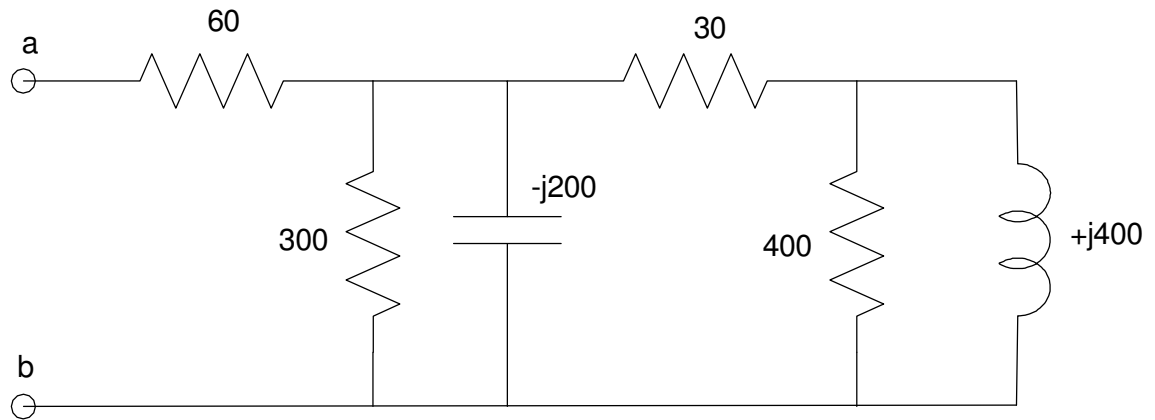
$$(131.668 - j224.947) + (j200) = 131.668 - j24.947$$

$$(131.668 - j24.947) \parallel (-j500) = 112.380 - j51.949$$

$$(112.380 - j51.949) + (100) = 212.380 - j51.949$$

ans:  $Z_{ab} = 212.380 - j51.949$

3) Determine the impedance  $Z_{ab}$



$$(400) \parallel (j400) = 200 + j200$$

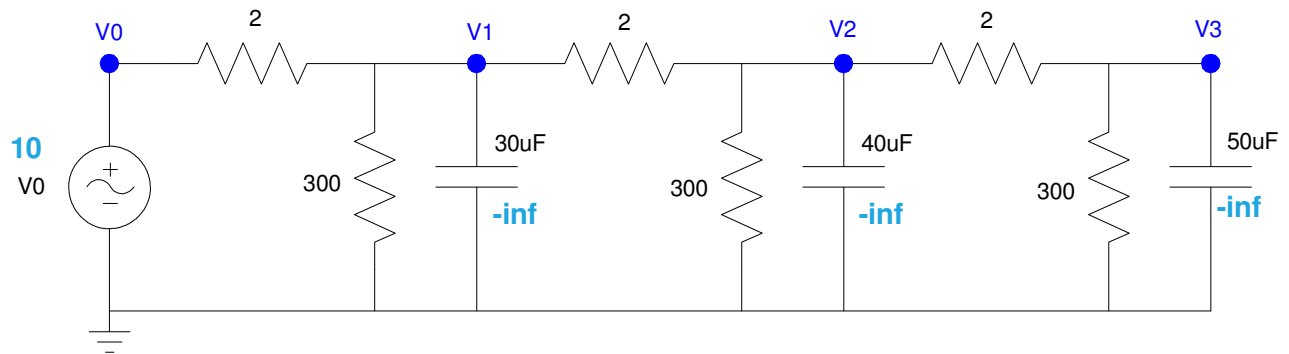
$$(200 + j200) + (30) = 230 + j200$$

$$(230 + j200) \parallel (-j200) \parallel (300) = 138.801 - j68.029$$

$$(138.801 - j68.029) + (60) = 198.801 - j68.029$$

**ans**     **$Z_{ab} = 198.801 - j68.029$**

4) Assume  $V_0 = 10$



a) Determine the impedances of each element at 0 rad/sec

The voltage is just ten:

$$V_0 = 10$$

The capacitors are infinite (which means the current is zero which means you can ignore them)

$$C \rightarrow \frac{1}{j\omega C} = -\infty$$

Capacitors have no affect at DC.

b) Write the votlage node equations

*note: you don't need to include the current through the capacitors since it's zero at DC.*

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{2}\right) + \left(\frac{V_1}{300}\right) + \left(\frac{V_1 - V_2}{2}\right) = 0$$

$$\left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2}{300}\right) + \left(\frac{V_2 - V_3}{2}\right) = 0$$

$$\left(\frac{V_3 - V_2}{2}\right) + \left(\frac{V_3}{300}\right) + \left(\frac{V_3 - V_4}{2}\right) = 0$$

$$\left(\frac{V_4 - V_3}{2}\right) + \left(\frac{V_4}{300}\right) = 0$$

c) Solve for V1, V2, and V3.

Group terms

$$-\left(\frac{1}{2}\right)V_0 + \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{2}\right)V_1 - \left(\frac{1}{2}\right)V_2 = 0$$

$$-\left(\frac{1}{2}\right)V_1 + \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{2}\right)V_2 - \left(\frac{1}{2}\right)V_3 = 0$$

$$-\left(\frac{1}{2}\right)V_2 + \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{2}\right)V_3 - \left(\frac{1}{2}\right)V_4 = 0$$

$$-\left(\frac{1}{2}\right)V_3 + \left(\frac{1}{2} + \frac{1}{300}\right)V_4 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{2}\right) & \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{2}\right) & \left(\frac{-1}{2}\right) & 0 \\ 0 & \left(\frac{-1}{2}\right) & \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{2}\right) & \left(\frac{-1}{2}\right) \\ 0 & 0 & \left(\frac{-1}{2}\right) & \left(\frac{1}{2} + \frac{1}{300}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> a1 = [1,0,0,0];
>> a2 = [-1/2, 1/2+1/300+1/2, -1/2, 0];
>> a3 = [0, -1/2, 1/2+1/300+1/2, -1/2];
>> a4 = [0, 0, -1/2, 1/2+1/300];
>> A = [a1;a2;a3;a4]
```

```
1.0000    0    0    0
-0.5000    1.0033   -0.5000    0
0   -0.5000    1.0033   -0.5000
0    0   -0.5000    0.5033
```

```
>> B = [10;0;0;0]
```

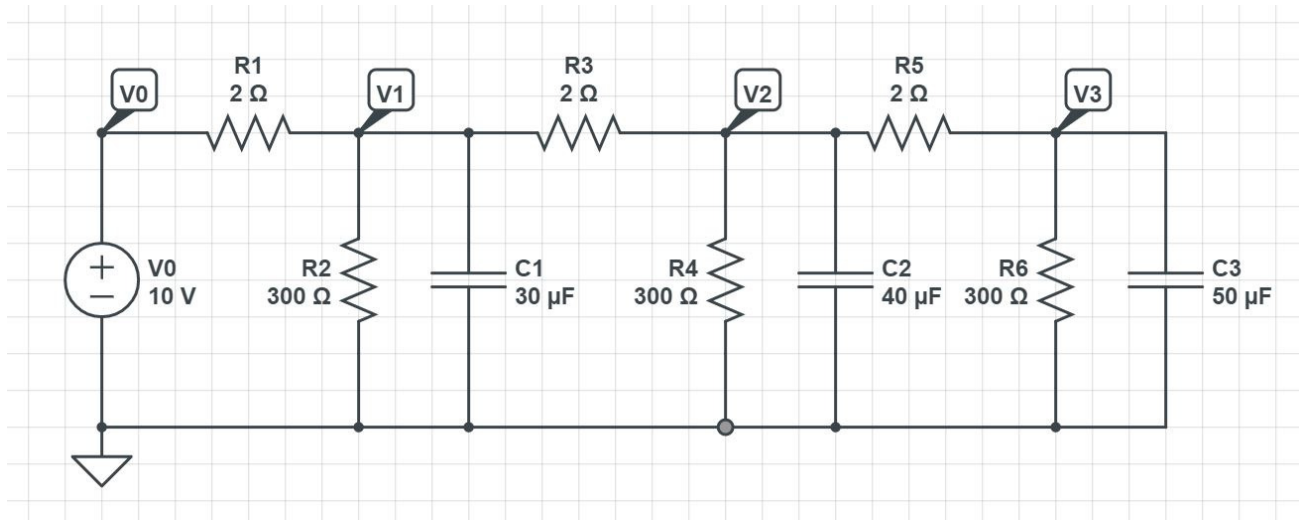
```
10
0
0
0
```

```
>> V = inv(A)*B
```

```
V0    10.0000
V1     9.8060
V2     9.6774
V3     9.6133
```

5) Check your results in CircuitLab

Input the circuit in CircuitLab using a DC source



Run a DC simulation to find the voltages.

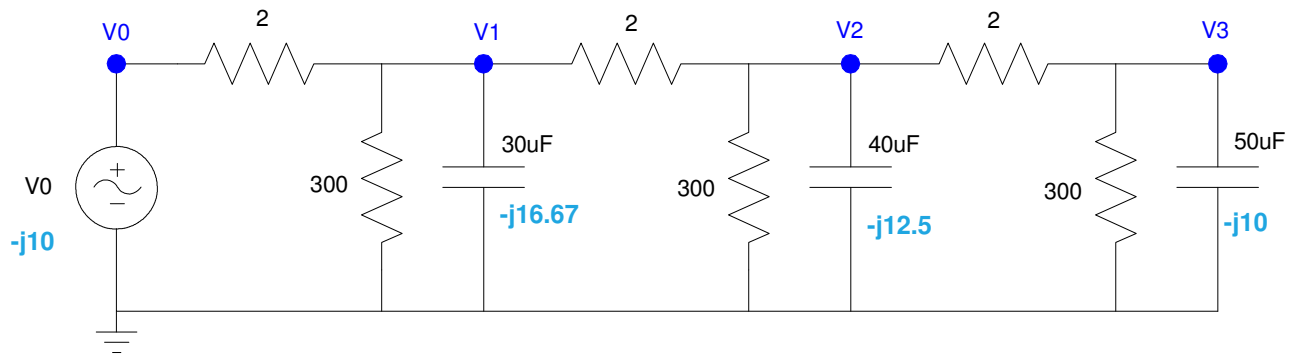
These match Matlab's answers:

<b>V0</b>	<b>10.0000</b>
<b>V1</b>	<b>9.8060</b>
<b>V2</b>	<b>9.6774</b>
<b>V3</b>	<b>9.6133</b>

▼ DC	
V(V0)	10.00 V
V(V1)	9.806 V
V(V2)	9.677 V
V(V3)	9.613 V
+ Add Expression	
Export Results...	
Run DC Solver	
▶ DC Sweep	
▶ Time Domain	
▶ Frequency Domain	

6) Assume  $V_0$  is a 10V, 2000 rad/sec (318.3Hz)

$$V_0 = 10 \sin(2000t)$$



a) Determine the impedances of each element at 2000 rad/sec

$$V_0 = 0 - j10$$

$$\text{cosine} = 0, \text{ sine} = 10$$

$$C \rightarrow \frac{1}{j\omega C} = \frac{1}{j \cdot 2000 \cdot C}$$

$$\frac{1}{j\omega C} = -j16.667$$

$$C = 30\mu\text{F}$$

$$\frac{1}{j\omega C} = -j12.5$$

$$C = 40\mu\text{F}$$

$$\frac{1}{j\omega C} = -j10$$

$$C = 50\mu\text{F}$$

b) Write the voltage node equations

Same equations as before but add a term for the capacitors

$$V_0 = -j10$$

$$\left(\frac{V_1 - V_0}{2}\right) + \left(\frac{V_1}{300}\right) + \left(\frac{V_1}{-j16.667}\right) + \left(\frac{V_1 - V_2}{2}\right) = 0$$

$$\left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2}{300}\right) + \left(\frac{V_2}{-j12.5}\right) + \left(\frac{V_2 - V_3}{2}\right) = 0$$

$$\left(\frac{V_3 - V_2}{2}\right) + \left(\frac{V_3}{300}\right) + \left(\frac{V_3}{-j10}\right) = 0$$

c) Solve for  $V_1$ ,  $V_2$ , and  $V_3$  as complex numbers

Group terms

$$V_0 = -j10$$

$$-\left(\frac{1}{2}\right)V_0 + \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{-j16.67} + \frac{1}{2}\right)V_1 - \left(\frac{1}{2}\right)V_2 = 0$$

$$-\left(\frac{1}{2}\right)V_1 + \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{-j12.5} + \frac{1}{2}\right)V_2 - \left(\frac{1}{2}\right)V_3 = 0$$

$$-\left(\frac{1}{2}\right)V_2 + \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{-j10} + \frac{1}{2}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{2}\right) & \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{-j16.67} + \frac{1}{2}\right) & \left(\frac{-1}{2}\right) & 0 \\ 0 & \left(\frac{-1}{2}\right) & \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{-j12.5} + \frac{1}{2}\right) & \left(\frac{-1}{2}\right) \\ 0 & 0 & \left(\frac{-1}{2}\right) & \left(\frac{1}{2} + \frac{1}{300} + \frac{1}{-j10}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -j10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> a1 = [1,0,0,0];
>> a2 = [-1/2, 1/2+1/300+1/2+1/(-j*16.667), -1/2, 0];
>> a3 = [0, -1/2, 1/2+1/300+1/(-j*12.5)+1/2, -1/2];
>> a4 = [0, 0, -1/2, 1/2+1/300+1/(-j*10)];
>> A = [a1;a2;a3;a4]
```

```
    1.0000         0         0         0
   -0.5000    1.0033 + 0.0600i   -0.5000         0
         0    -0.5000    1.0033 + 0.0800i   -0.5000
         0         0    -0.5000   -0.5000 + 0.1000i
```

```
>> B = [-j*10;0;0;0]
```

```
    0 -10.0000i
    0
    0
    0
```

```
>> V = inv(A)*B
```

```
V0         0 -10.0000i
V1   -2.7247 - 7.7350i
V2   -4.5393 - 5.8485i
V3   -5.4484 - 4.7273i
```

d) Express  $V_1$ ,  $V_2$ , and  $V_3$  in terms of sine and cosine function:

• hint:  $V_1 = a + jb$  (phasor representation) means  $V_1(t) = a \cos(2000t) - b \sin(2000t)$

$$v_0 \quad 0 - 10.0000i$$

$$v_1 \quad -2.7247 - 7.7350i$$

$$v_2 \quad -4.5393 - 5.8485i$$

$$v_3 \quad -5.4484 - 4.7273i$$

which means

$$V_0 = 10 \sin(2000t)$$

$$V_1 = -2.7247 \cos(2000t) + 7.7350 \sin(2000t)$$

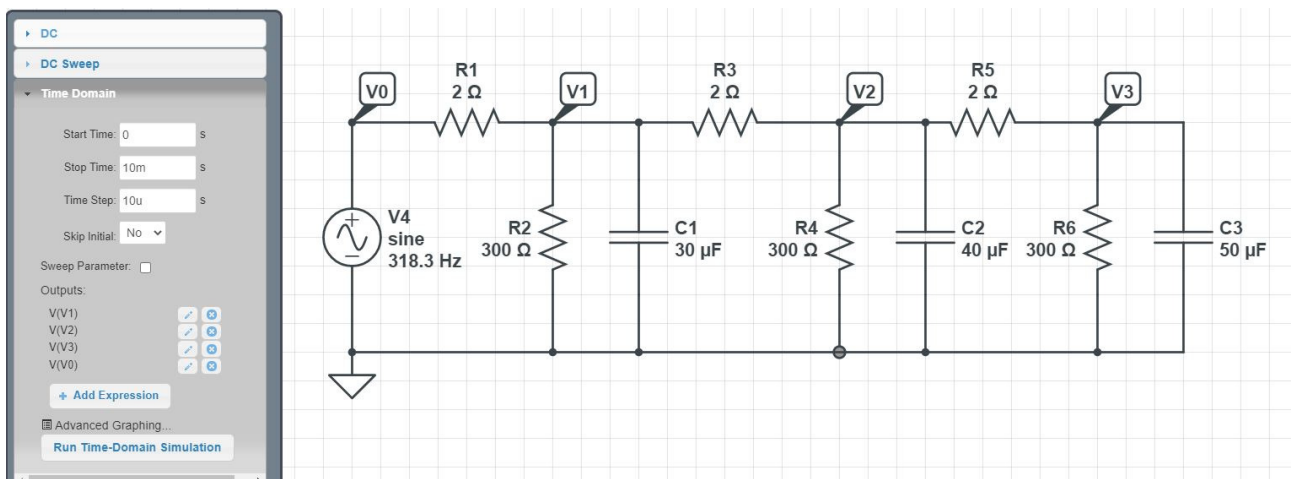
$$V_2 = -4.5393 \cos(2000t) + 5.8485 \sin(2000t)$$

$$V_3 = -5.4484 \cos(2000t) + 5.7273 \sin(2000t)$$

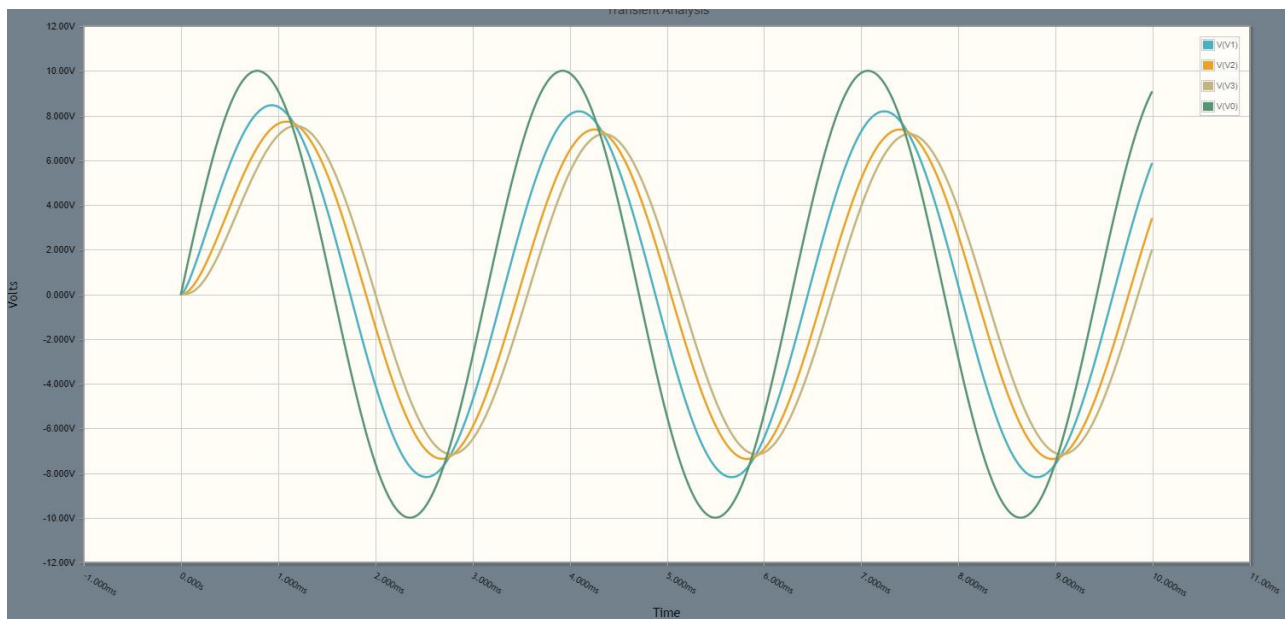


7) Check your results in CircuitLab using a transient simulation for 10ms (step size 10us).

Replace the source with a sine wave:



Run for couple of cycles (10ms) with 1000 points on the plot (time step is 1000x smaller or 10us)



b 7

Checking Results: (comparing with matlab calculations)

In lab (or circuitlab), it's easier to use polar form. The magnitude of V from Matlab is:

```
>> abs(V)
```

```
ans =
```

```
10.0000  
8.2009  
7.4034  
7.2134
```

```
>>
```

The peaks of the sine waves from CircuitLab are

- $V_0 = 10.000V$
- $V_1 = 8.186V$
- $V_2 = 7.371V$
- $V_3 = 7.168V$

These should match dead on. They're a little off due to problems reading a graph and numerical errors resulting from numerical integration.

In polar form, there is also an angle. The angle tells you the delay of each of the sine waves