ECE 111 - Homework #12

Week #12: ECE 341 Random Processes. Due 8am April 12th
Please submit as a Word or pdf file to BlackBoard or email to Jacob_Glower@yahoo.com with header ECE 111 HW#12
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Chi-Squared Tests

Problem 1: The following Matlab code generates 60 random die rolls for a six sided die

```
RESULT = zeros(1,6);
for i=1:60
   D6 = ceil( 6*rand );
   RESULT(D6) = RESULT(D6) + 1;
   end
RESULT
```

Determine whether this is a fair or loaded die using a Chi-Squared test.

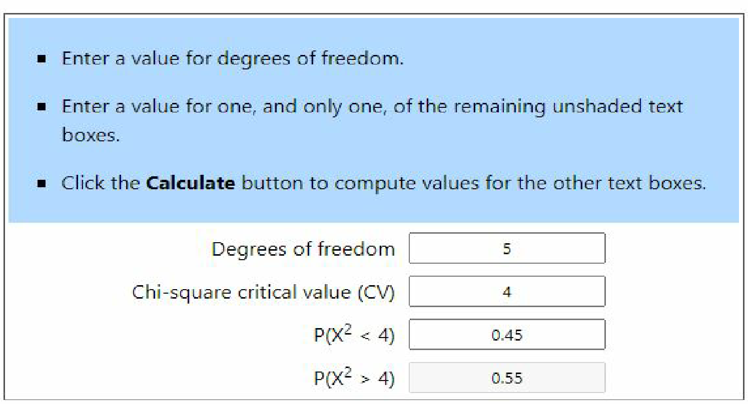
```
RESULT =

    11     7     9     7    14    12
```

Put this into a table and compute the Chi-Squred score

Roll	p	n*p	N	$\chi^2 = \left(\frac{(np-N)^2}{np}\right)$
1	1/6	10	11	0.1
2	1/6	10	7	0.9
3	1/6	10	9	0.1
4	1/6	10	7	0.9
5	1/6	10	14	1.6
6	1/6	10	12	0.4
			Total	4

From StatTrek, a chi-squared score of 4.00 with 5 degrees of freedom corresponds to a probability of 0.45

There is a 45% chance that this die is not fair



Problem 2: The following Matlab code generates 60 rolls of a loaded six-sided die (20% of the time, you roll a 6):

```
RESULT = zeros(1,6);
for i=1:60
   if(rand < 0.2)
      D6 = 6;
   else
      D6 = ceil( 6*rand );
      end
   RESULT(D6) = RESULT(D6) + 1;
   end
RESULT
```

Determine whether this is a fair or loaded die using a Chi-Squared test.

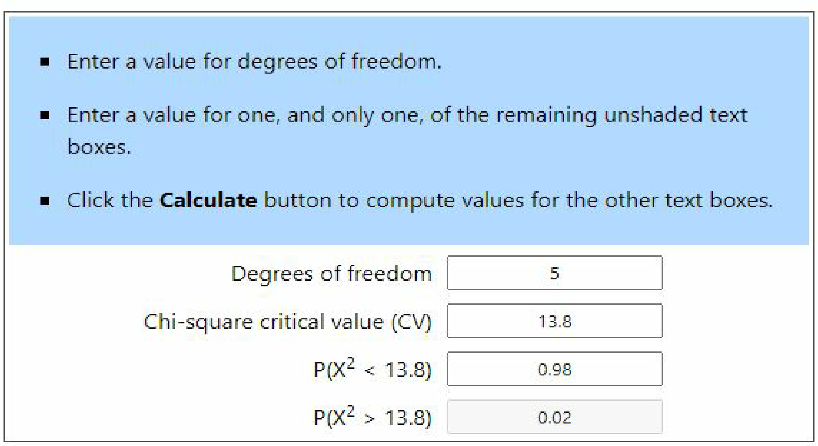
```
RESULT =    15     6     9     6     6    18
```

Place the data in to a table and compute the chi-squared score:

Roll	p	n*p	N	$\chi^2 = \left(\frac{(np-N)^2}{np}\right)$
1	1/6	10	15	2.5
2	1/6	10	6	1.6
3	1/6	10	9	0.1
4	1/6	10	6	1.6
5	1/6	10	6	1.6
6	1/6	10	18	6.4
			Total	13.8

From StatTrek, a chi-squared score of 13.8 with 5 degrees of freedom corresponds to a probability of 0.98

There is a 98.0% chance that this die is not fair



Am I Psychic?

Problem #3: Shuffle a deck of 52 playing cards and place it face down on a table.

- Predict the suit of the top card then reveal it. If correct, place the card in one pile (correct). If incorrect, place it in another pile.
- Repeat for all 52 cards.

Use a chi-squared test to test the hypothesis that you're just guessing (probability of being correct is 25%)

Flipping throgh a deck of cards and predicting the suit, I was

- Correct 19 times
- Incorrect 33 times

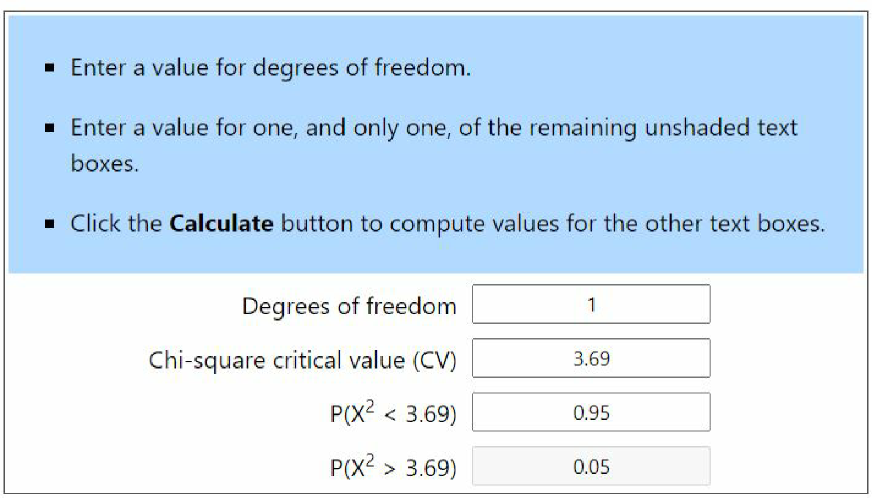
Put this data into a table and compute the chi-squared score

Pediction	p	n*p	N	$\chi^2 = \left(\frac{(np-N)^2}{np}\right)$
Correct	1/4	13	19	2.77
Incorrect	3/4	39	33	0.92
			Total	3.69

From StatTrek, a chi-squared score of 3.69 with 1 degree of freedom corresponds to a probabiliy of 0.95

There is 95% chance that I wasn't just guessing

and a 5% chance I got lucky... before I mortgage the house and go to the cassino, I might want to repeat this test to see if the result is repeatable



Normal Approximation

The mean and standard deviation for a fair 6-sided die and 4-sided die are:

$$\mu_{d6} = 3.5 \qquad \mu_{d4} = 2.5$$

$$\sigma_{d6} = 1.7078 \qquad \sigma_{d4} = 1.118$$

Problem 4: Let Y be the sum of rolling three 6-sided dice (3d6) plus four 4-sided dice (4d4).

Y = 3d6 + 4d4

a) What is the mean and standard deviation of Y?

When adding normal distributions

- The mean adds

$$\mu = 3 \cdot \mu_{d6} + 4 \cdot \mu_{d4}$$

$$\mu = 3 \cdot 3.5 + 4 \cdot 2.5$$

$$\mu = 20.5$$

- The variance adds

$$\sigma^2 = 3 \cdot \sigma_{d6}^2 + 4 \cdot \sigma_{d4}^2$$

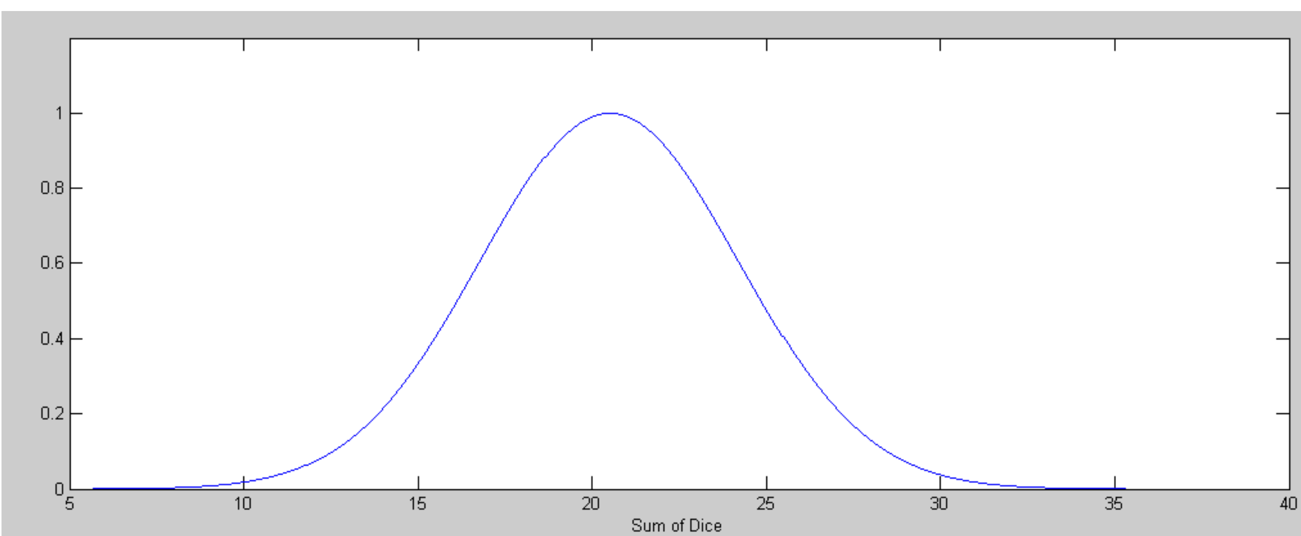
$$\sigma^2 = 3(1.7078)^2 + 4(1.118)^2$$

$$\sigma^2 = 13.7494$$

$$\sigma = 3.7080$$

The normal approximation for the sum of the die rolls is then (not asked for in the homework but informative)

```
>> s = [-4:0.01:4]';
>> p = exp(-s.^2 / 2);
>> plot(s*3.7080 + 20.5,p)
>> xlabel('Sum of Dice')
```



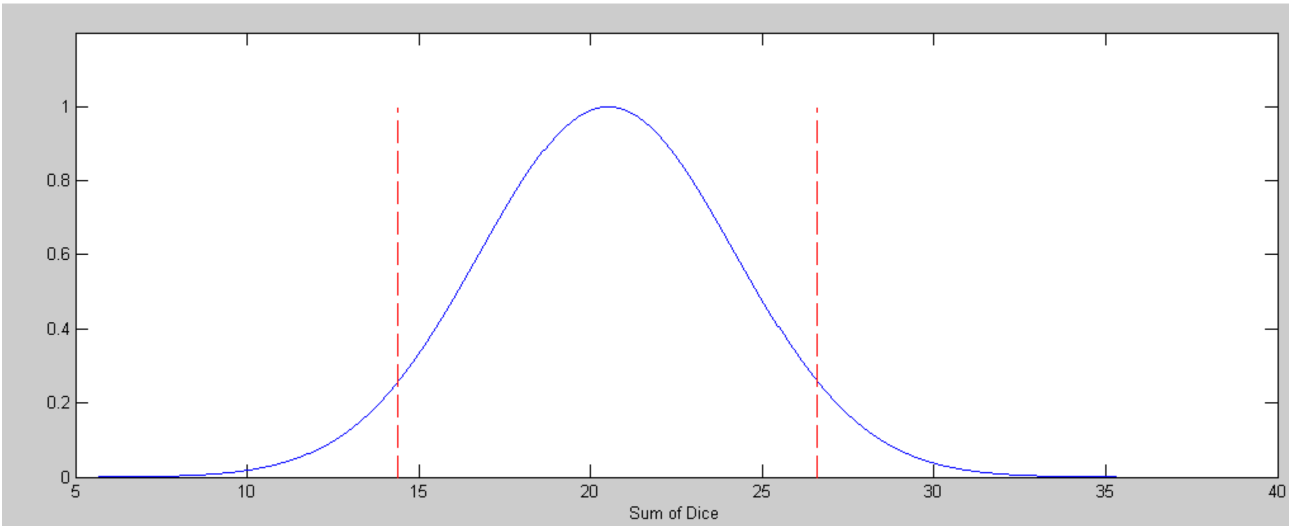
Probability Distribution for the Sum of Dice

b) Using a normal approximation, what is the 90% confidence interval for Y?

From StatTrek, 5% tails with a uniform distribution corresponds to a z-score of 1.645

$$\mu - 1.645\sigma < roll < \mu + 1.645\sigma \qquad p = 0.9$$

$$14.400 < roll < 26.60$$



90% Condidence Interval for the sum of dice. Each tail has an area of 5%

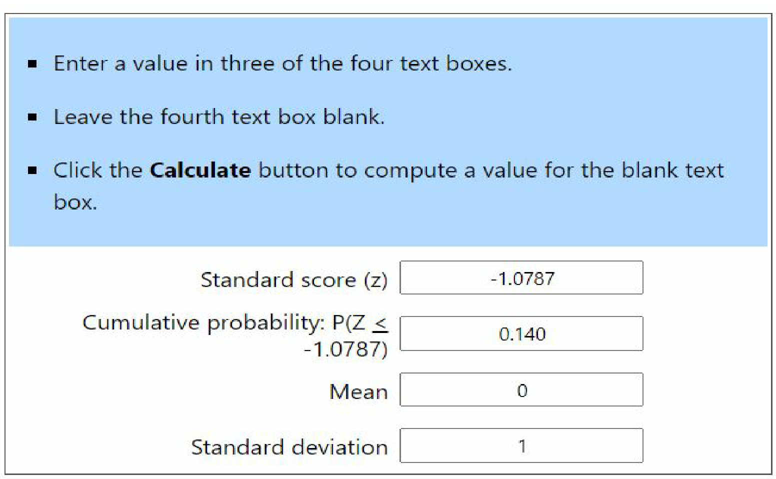
c) Using a normal approximation, what is the probability that the sum the dice will be more than 24.5?

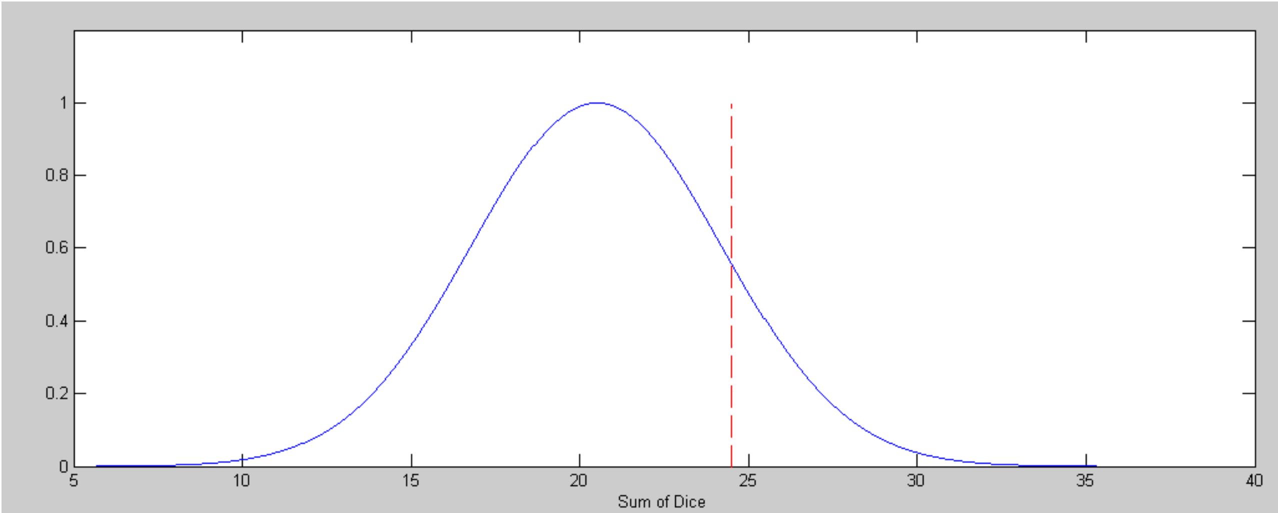
The z-score corresponding to 24.5 is

$$z = \left(\frac{24.5 - \mu}{\sigma}\right) = \left(\frac{24.5 - 20.5}{3.7080}\right) = 1.0787$$

From StatTrek, a z-score of 1,.0787 corresponds to a probability of 0.140

There is a 14.0% chance that the sum will be more than 24.5





The area to the right of 24.5 is 14.0%

Problem 5: Check your answer using a Monte-Carlo simulation in Matlab with one million rolls:

```
N = 0;
for i=1:1e6
   Y = sum( ceil( 6*rand(3,1) ) ) + sum( ceil( 4*rand(4,1) ) );
   if(Y > 24.5)
      N = N + 1;
      end
   end
N / 1e6
```

ans = 0.1437

With one million rolls, 14.37% of the rolls were 25 or higher.

- A normal approximation gave a 14.0% chance

Note:

- The normal approximation is approximately correct
- This is only the sum of seven dice. If the number of dice increases, the normal approximation becomes more accurate.
- It is a *lot* easier to use a normal approximation than it is to roll dice one million times

t-Tests

Problem 6: Using Matlab, cast six level-10 fireballs (the sum of ten 6-sided dice, or 10d6)

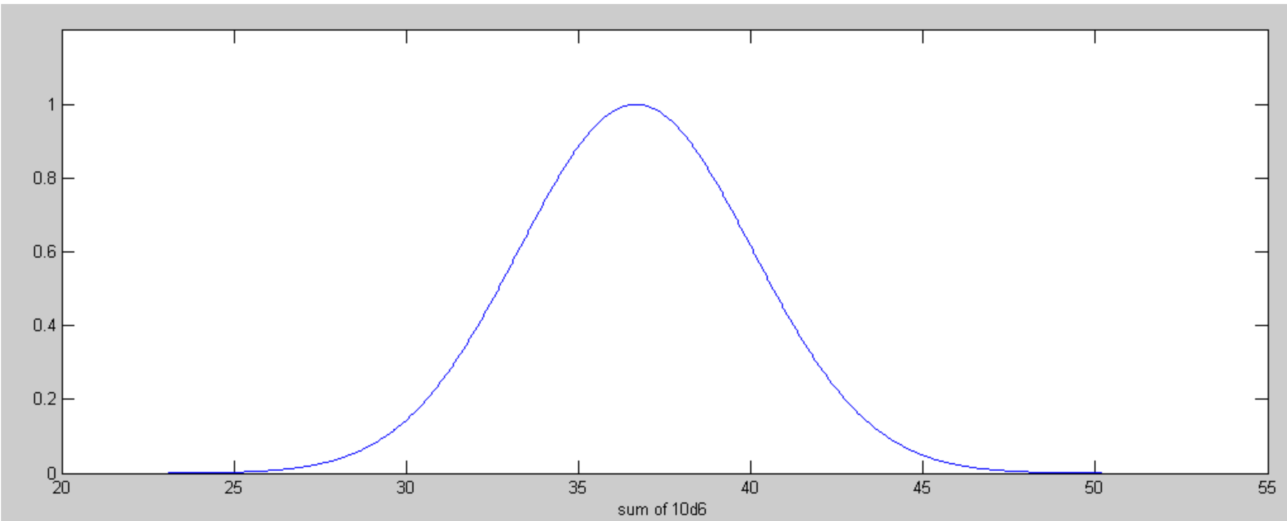
```
damage = [];
for i=1:6
  x = sum( ceil( 6*rand(10,1) ) );
  damage = [damage , x];
  end

damage

    37    35    40    35    41    32
```

From this, determine the mean and standard deviation of your data set.

```
>> x = mean(damage)

x =   36.6667

>> s = std(damage)

s =    3.3862
```

Just for fun, the probability distribution for a level-10 fireball is then

```
>> s1 = [-4:0.01:4]';
>> p = exp(-s1.^2 / 2);
>> plot(s*s1 + x,p)
>> xlabel('sum of 10d6')
```

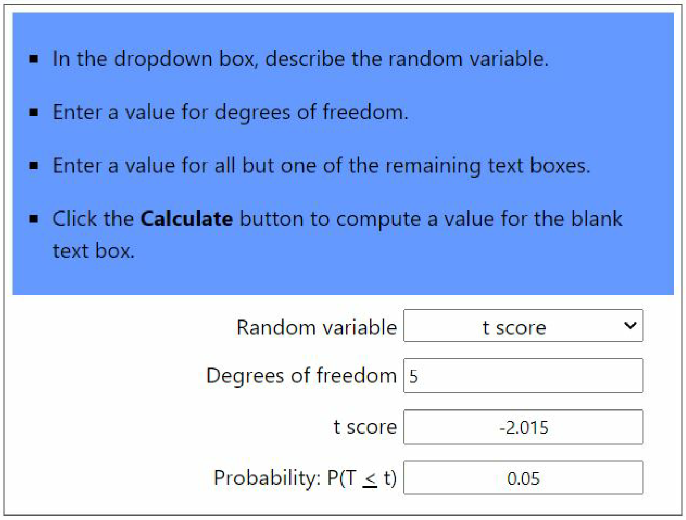


Probability Distribution for the sum of 10d6 (level-10 fireball)

Problem 7: Use a t-test to determine

The 90% confidence interval for a level 10 fireball.

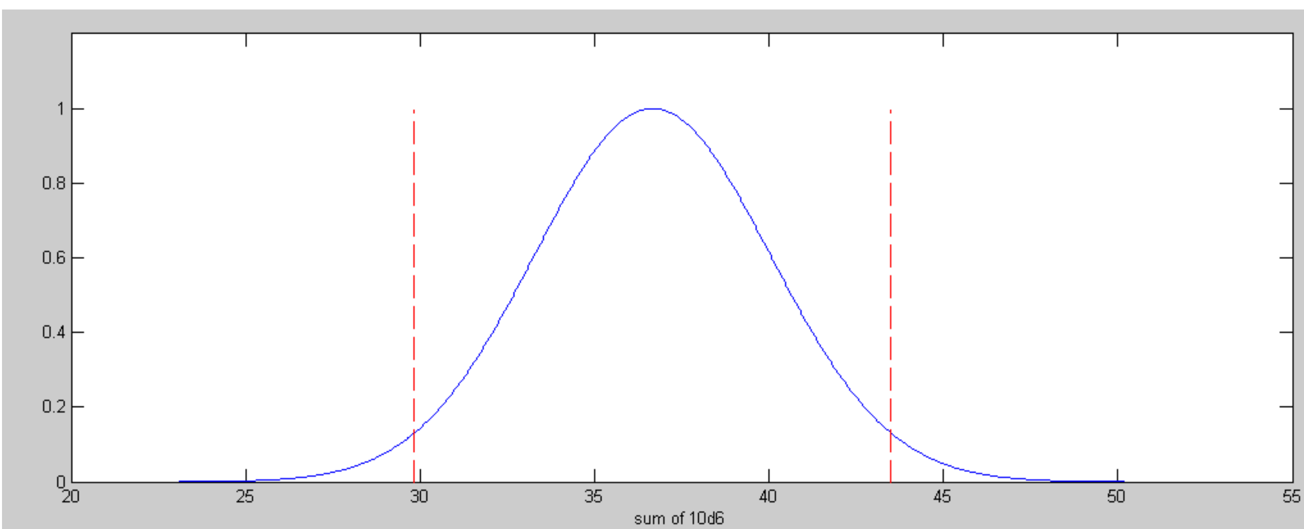
The t-score that corresponds to 5% tails with 5 degrees of freedom (sample size = 6) is 2.015



The 90% confidence interval is then

$$\bar{x} - 2.015s < roll < \bar{x} + 2.015s$$

$$29.84 < roll < 43.49$$



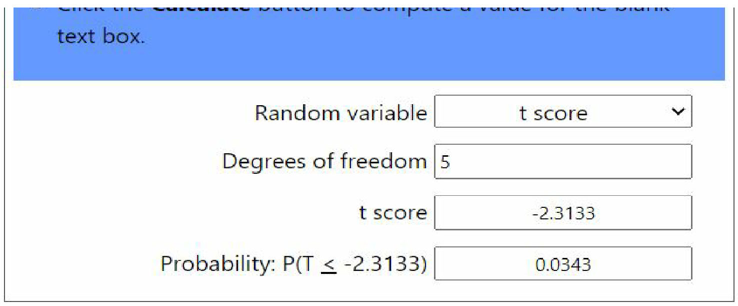
90% confidence interval for a level-10 fireball

The probabillity of doing 45 or more damage with a level-10 fireball

The t-score for 44.5 (rounded to 45) is

$$t = \left(\frac{44.5 - \bar{x}}{s}\right) = 2.3133$$

From StatTrek, this corresponds to a probability of 3.43%



Problem 8) Check your answer using a Monte-Carlo simulation in Matlab by casting 100,000 level-10 fireballs:

```
N = 0;

for i=1:1e6
   damage = sum( ceil( 6*rand(10,1) ) );
   if( damage >= 45)
      N = N + 1;
      end
   end
N / 1e6
```

ans = 0.0387

From a Monte-Carlo simulation, there is a 3.87% chance of rolling 45 or more

- A t-test predicts 3.43% predicted with a t-test

Note:

- You don't need a very large sample size to get pretty good estimates
- If you know the mean and standard deviation, use a normal appproximation
- If you estimate the mean and standard deviation from the data, use a t-test