ECE 111 - Homework #11

Week #11 - ECE 343 Signals- Due 8am Tuesday, April 5th

Please submit as a Word or pdf file and email to Jacob_Glower@yahoo.com with header ECE 111 HW#11

Problem 1-5) Let x(t) be a function which is periodic in 2π

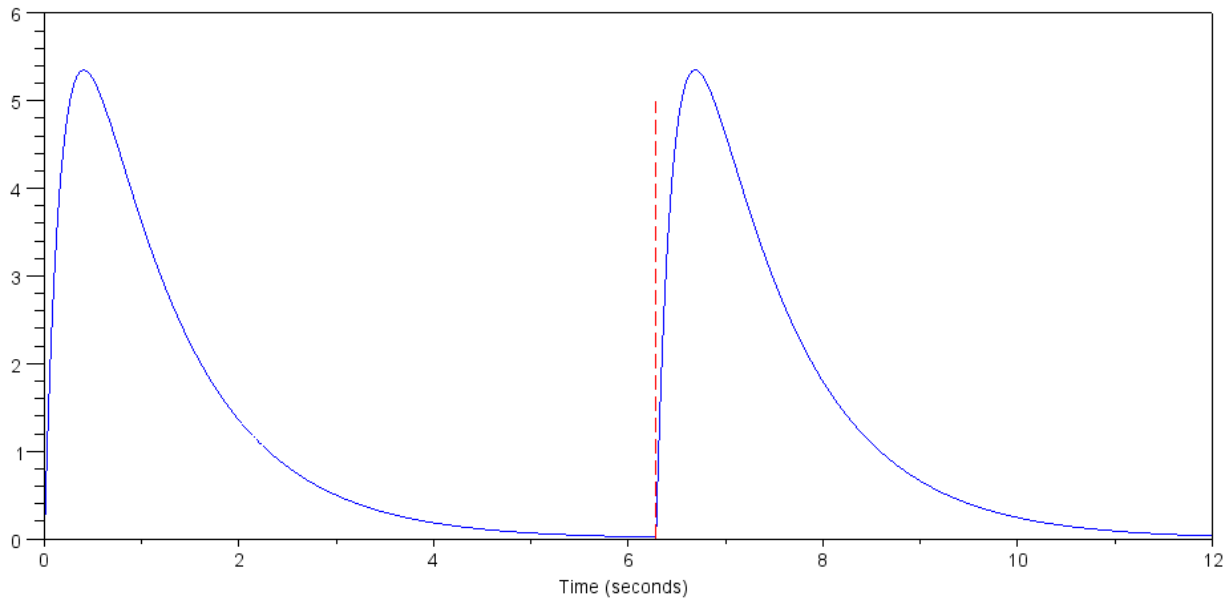
$$x(t) = x(t + 2\pi)$$

Over the interval $(0, 2\pi)$ x(t) is

$$x(t) = 5e^{-t} - 5e^{-5t}$$

or in Matlab:

```
t = [0:0.001:2*pi]';
x = 5 * exp(-t) - 5 * exp(-5*t);
plot(t,x)
```



x(t) Note that x(t) repeats repeats every 2π seconds

Curve Fitting with a power series:

1) Using least squares, approximate x(t) over the interval (0, 2π) as

$$x(t) \approx a + bt + ct^2 + dt^3$$

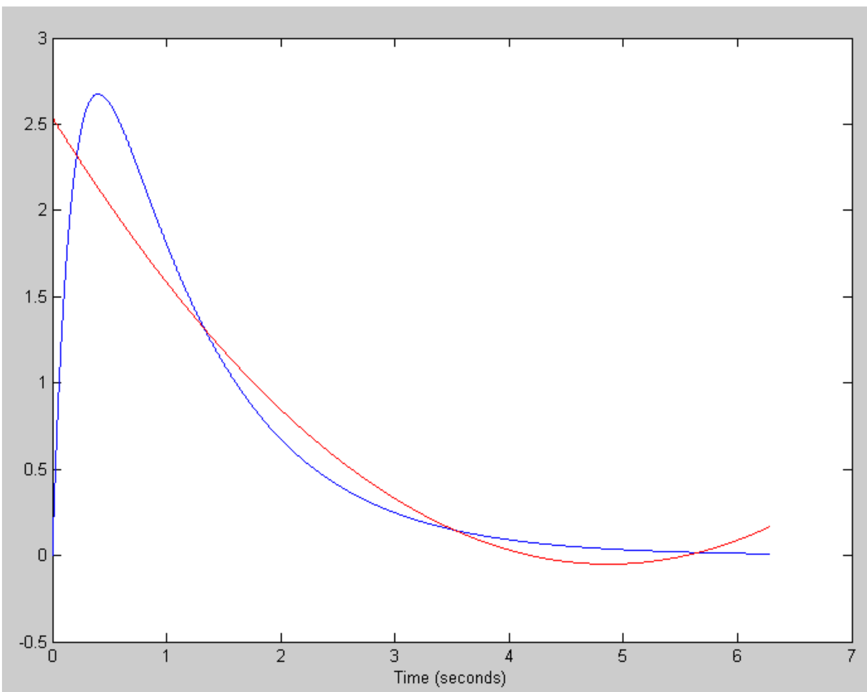
Plot x(t) along with it's approximation.

```
>> t = [0:0.001:2*pi]';
x = 5 * exp(-t) - 5 * exp(-5*t);
plot(t,x)
>>
>> B = [t.^3, t.^2, t, t.^0];
>> A = inv(B'*B)*B'*x

A =

   -0.0001
    0.1096
   -1.0639
    2.5387

>> plot(t,x,'b',t,B*A,'r')
>> xlabel('Time (seconds)');
>>
```



Curve Fitting using a Fourier Series

2) Using least squares, approximate x(t) over the interval (0, 2π) as

$$x(t) = a_0 + a_1\cos(t) + b_1\sin(t) + a_2\cos(2t) + b_2\sin(2t) + a_3\cos(3t) + b_3\sin(3t)$$

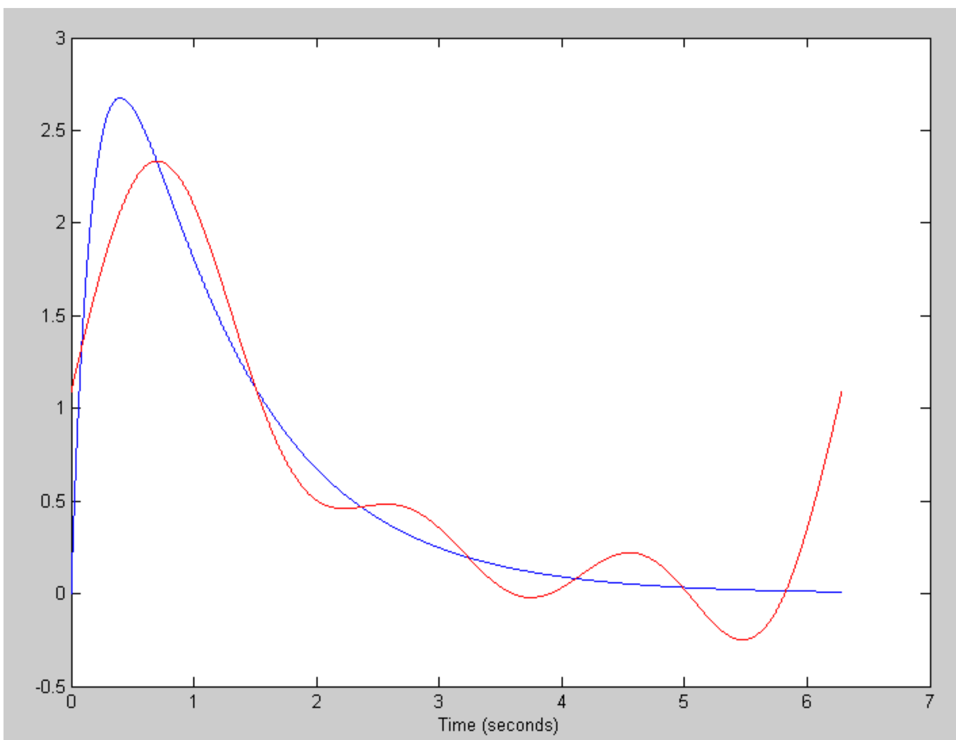
Plot x(t) along with it's approximation.

```
>> B = [t.^0, cos(t), sin(t), cos(2*t), sin(2*t), cos(3*t), sin(3*t)];
>> A = inv(B'*B)*B'*x

A =

    0.6350
    0.4879
    0.7331
    0.0430
    0.5257
   -0.0755
    0.3361

>> plot(t,x,'b',t,B*A,'r')
>> xlabel('Time (seconds)');
>>
```



Superposition

3) Assume X and Y are related by

$$Y = \left(\frac{2}{s^2+2s+2}\right)X$$

3a) Determine x(t) in terms of its Fourier Transform out to 3 rad/sec

```
>> a0 = mean(x)

a0 =    0.6351

>> a1 = 2*mean(x .* cos(t))

a1 =    0.4882

>> b1 = 2*mean(x .* sin(t))

b1 =    0.7330

>> a2 = 2*mean(x .* cos(2*t))

a2 =    0.0433

>> b2 = 2*mean(x .* sin(2*t))

b2 =    0.5256

>> a3 = 2*mean(x .* cos(3*t))

a3 =   -0.0752

>> b3 = 2*mean(x .* sin(3*t))

b3 =    0.3361
```

Note that this is the same result we got for problem #2

```
>> A = inv(B'*B)*B'*x

    0.6350
    0.4879
    0.7331
    0.0430
    0.5257
   -0.0755
    0.3361
```

Fourier Transforms is just least-squares curve fitting with a basis of sine and cosine funcitons.

3b) Plot x(t) and its Fourier approximation taken out to 3 rad/sec

same answer as problem #2

4) Determine the gain of this filter at each frequency present in problem #2 (i.e. 0, 1, 2, 3 rad/sec)

- *note: You should get a complex number for the gain at each frequency*

```
>> s = 0;
>> G0 = 2 / (s^2 + 2*s + 2)

G0 =     1

>> s = j*1;
>> G1 = 2 / (s^2 + 2*s + 2)

G1 =   0.4000 - 0.8000i

>> s = j*2;
>> G2 = 2 / (s^2 + 2*s + 2)

G2 =  -0.2000 - 0.4000i

>> s = j*3;
>> G3 = 2 / (s^2 + 2*s + 2)

G3 =  -0.1647 - 0.1412i
```

5a) Determine the phasor representation for Y(jw) at each frequency.

- *note: You should get a complex number for Y - the phasor representation for y(t) at 0, 1, 2, and 3 rad/sec*

Output = Gain * Input

Y = G * X

```
>> % DC
>> s = 0;
>> y0 = 2 / (s^2 + 2*s + 2) * a0

y0 =    0.6351

>> % 1 rad/sec
>> s = j*1;
>> y1 = 2 / (s^2 + 2*s + 2) * ( a1 - j*b1 )

y1 =  -0.3911 - 0.6837i

>> % 2 rad/sec
>> s = j*2;
>> y2 = 2 / (s^2 + 2*s + 2) * ( a2 - j*b2 )

y2 =  -0.2189 + 0.0878i

>> % 3 rad/sec
>> s = j*3;
>> y3 = 2 / (s^2 + 2*s + 2) * ( a3 - j*b3 )

y3 =  -0.0351 + 0.0660i
```

5b) From this, determine y(t)

- real = cosine, -imag = sine

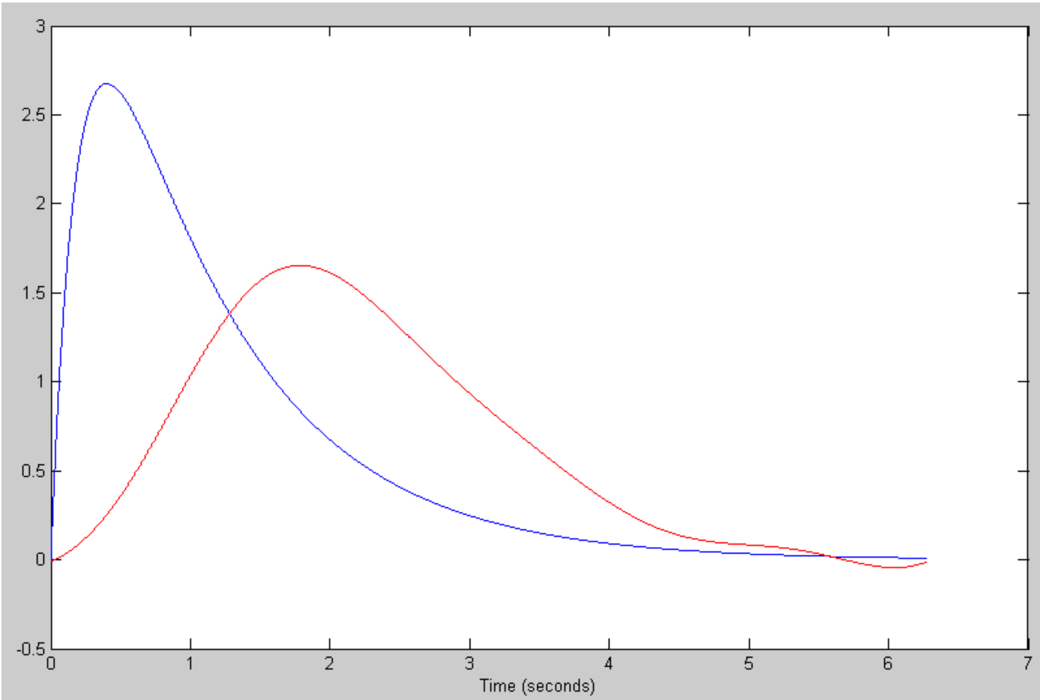
$$
\begin{aligned}
y(t) = \quad & 0.6351 \\
& -0.3911\cos(t) + 0.6831\sin(t) \\
& -0.2189\cos(2t) - 0.0878\sin(2t) \\
& -0.0351\cos(3t) - 0.0660\sin(3t)
\end{aligned}
$$

Note:

- In theory, you need an infinite number of terms
- In practice, the terms go to zero fairly quickly. Truncating after the 3rd harmonic gives a close approximation to y(t)

6) Plot x(t) and y(t).

```
>> y = y0 + real(y1)*cos(t) - imag(y1)*sin(t);
>> y = y  + real(y2)*cos(2*t) - imag(y2)*sin(2*t);
>> y = y  + real(y3)*cos(3*t) - imag(y3)*sin(3*t);
>> plot(t,x,'b',t,y,'r')
>> xlabel('Time (seconds)');
>>
```



x(t) (blue) and y(t) (red)