ECE 111 - Homework #10

ECE 343 Signals & Systems- Due Due 8am, Tuesday, March 29th

Please submit as a Word or pdf file and email to Jacob_Glower@yahoo.com with header ECE 111 HW#10

1) A filter has the following transfer function

$$Y = \left(\frac{6s+30}{s^2+6s+60}\right)X$$

1a) What is the differential equation relating X and Y?

$$(s^2 + 6s + 60)Y = (6s + 30)X$$

$$y'' + 6y' + 60y = 6x' + 30x$$

or

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 60y = 6\frac{dx}{dt} + 30x$$

1b) Find y(t) assuming $x(t) = 5$

At DC (0 rad/sec)

$$X = 5$$

$$Y = \left(\frac{6s+30}{s^2+6s+60}\right)_{s=0} \cdot (5)$$

$$Y = 2.5$$

1c) Find y(t) assuming $x(t) = 5\cos(7t)$

At 7 rad/sec

$$X = 5 + j0$$ *real = cosine, -imag = sine*

$$Y = \left(\frac{6s+30}{s^2+6s+60}\right)_{s=j7} \cdot (5 + j0)$$

$$Y = 5.5544 - j2.1167$$

meaning

$$y(t) = 5.5544\cos(7t) + 2.1167\sin(7t)$$

2) Plot the gain vs. frequency for this filter from 0 to 50 rad/sec.

$$Y = \left(\frac{6s+30}{s^2+6s+60} \right) X$$

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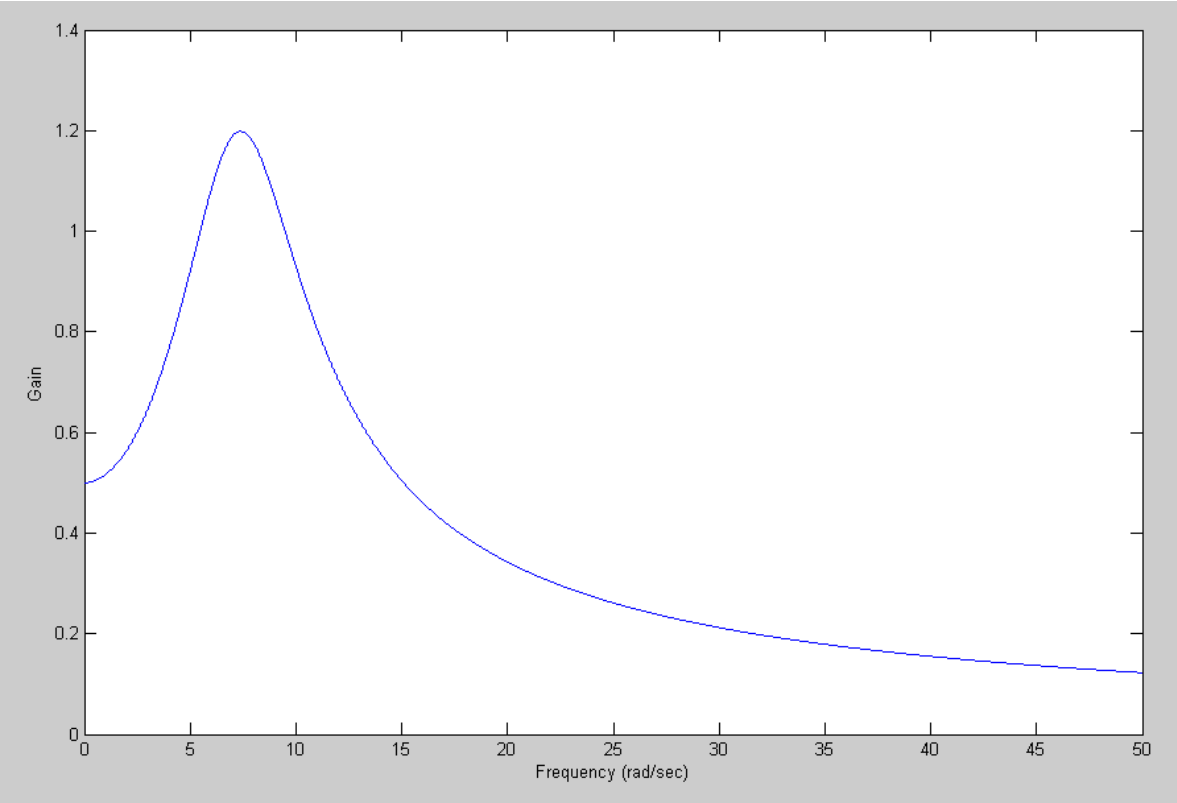
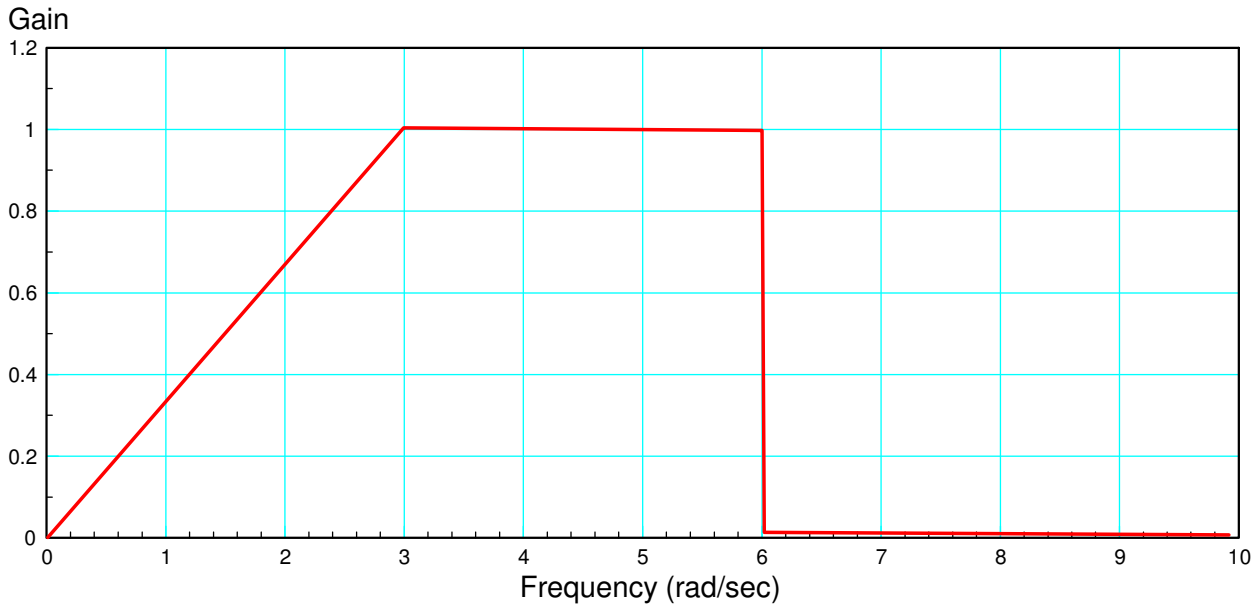
>> w = [0:0.01:50]';
>> s = j*w;
>> G = (6*s+30) ./ ( s.^2 + 6*s + 60);
>> plot(w,abs(G))
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
>>
```



Problem 3-5) Design a filter of the following form so that the gain matches the graph below:

$$G(s) = \left(\frac{as}{\left(s^2+bs+c\right)\left(s^2+ds+e\right)\left(s^2+fs+g\right)} \right)$$



3) Write an m-file, cost.m, which

- Is passed an array, z, with each element representing (a, b, c, d, e, f, g)
- Computes the gain, G(s) for this value of (a, b, c, d, e, f, g)
- Computes the difference between the gain, G, and the target (above), and
- Returns the sum-squared error in the gain

```
function [ J ] = Filter( z )

 z = abs(z);
 a = z(1);
 b = z(2);
 c = z(3);
 d = z(4);
 e = z(5);
 f = z(6);
 g = z(7);

 w = [0:0.01:10]';
 s = j*w;

 Gideal = (w/3).*(w<3) + (0*w + 1).*(w>=3).*(w<6);

 G = a*s ./ ( (s.^2 + b*s + c) .* (s.^2 + d*s + e ) .* (s.^2 + f*s + g) );
 G = abs(G);
 E = abs(Gideal) - abs(G);

 J = sum(E .^ 2);

 plot(w,Gideal,w,abs(G),'r');
 xlabel('rad/sec');
 ylabel('Gain');
 ylim([0,1.4]);
 pause(0.01);
end
```

4) Use your m-file to determine how 'good' the following filter is:

$$G(s) = \left(\frac{as}{(s^2+bs+c)(s^2+ds+e)(s^2+fs+g)} \right) = \left(\frac{85s}{(s^2+2s+2)(s^2+2s+5)(s^2+2s+17)} \right)$$

```
>> Filter([85,2,2,2,5,2,17])

ans =  239.4441
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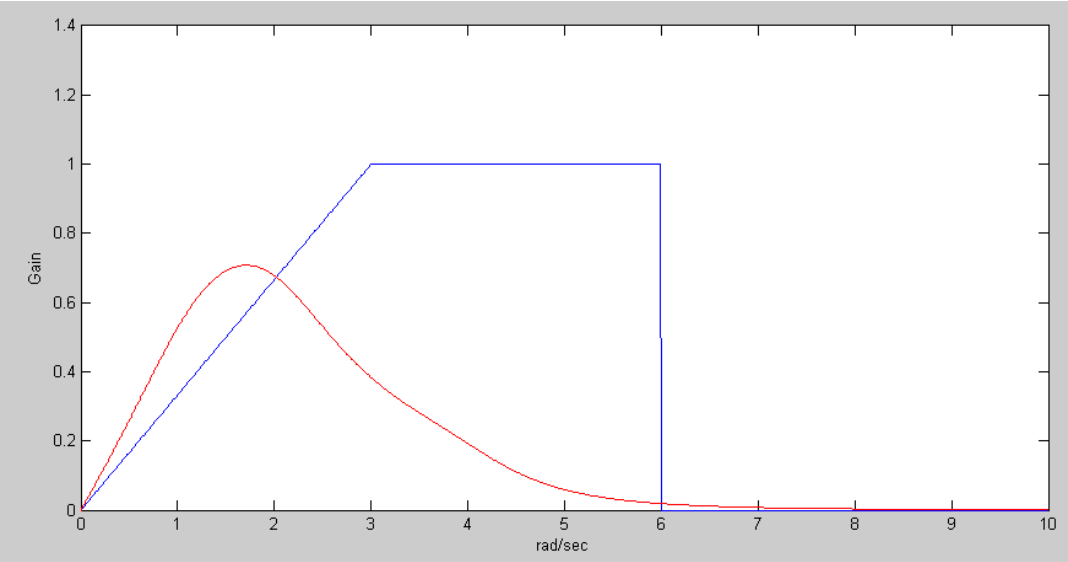

5) Use fminsearch() to find the 'best' filter of the form

$$G(s) = \left(\frac{as}{\left(s^2+bs+c\right)\left(s^2+ds+e\right)\left(s^2+fs+g\right)} \right)$$

5a) Give the resulting (a, b, c, d, e, f, g)

```
>> [Z,e] = fminsearch('Filter',Z)

        a          b          c          d          e          f          g
Z =  431.6913    1.9651    3.9000    0.7258   31.0840    1.8625   14.8573


e =

   11.4704
```

5b) Give the resulting filter, and

$$G(s) = \left(\frac{431.69s}{\left(s^2+1.96s+3.9\right)\left(s^2+0.72s+31.08\right)\left(s^2+1.86s+14.85\right)} \right)$$

5c) Plot the 'optimal' filter's gain vs. frequency