# ECE 111 - Homework #9

Week #9: ECE 311 Circuits II - Due 8am, Tuesday, March 22nd
Please submit as a Word or pdf file and email to Jacob_Glower@yahoo.com with header ECE 111 HW#9

1) Solve for Y

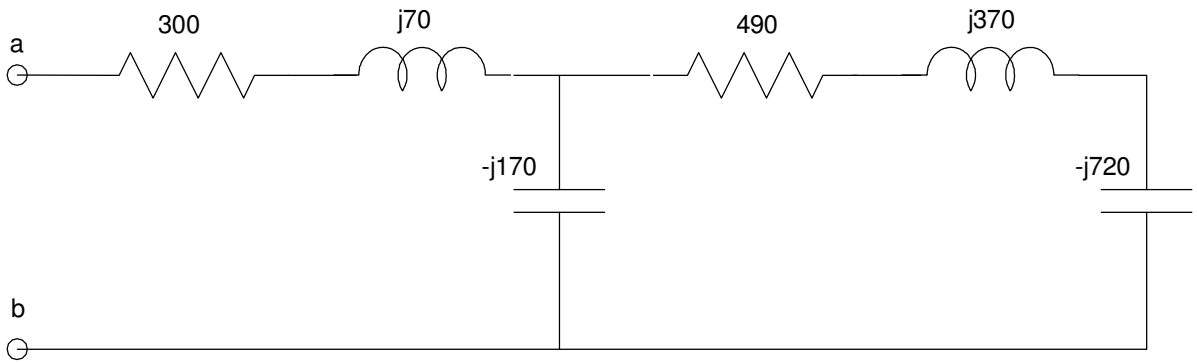
$$Y = \left( \frac{(2+j5)(1+j6)}{(7-j2)} \right)$$

```
>> Y = (2+j*5)*(1+j*6) / (7-j*2)

Y =  -4.3396 + 1.1887i
```

$$Y = \left( \left( \frac{2+j5}{7-j2} \right) + \left( \frac{1+j6}{2-j7} \right) \right) \left( \frac{5+j6}{3+j8} \right)$$

```
>> Y = ( (2+j*5)/(7-j*2) + (1+j*6)/(2-j*7) ) * (5+j*6)/(3+j*8)

Y =   -0.2564 + 1.1491i
```

2) Determine the impedance Zab



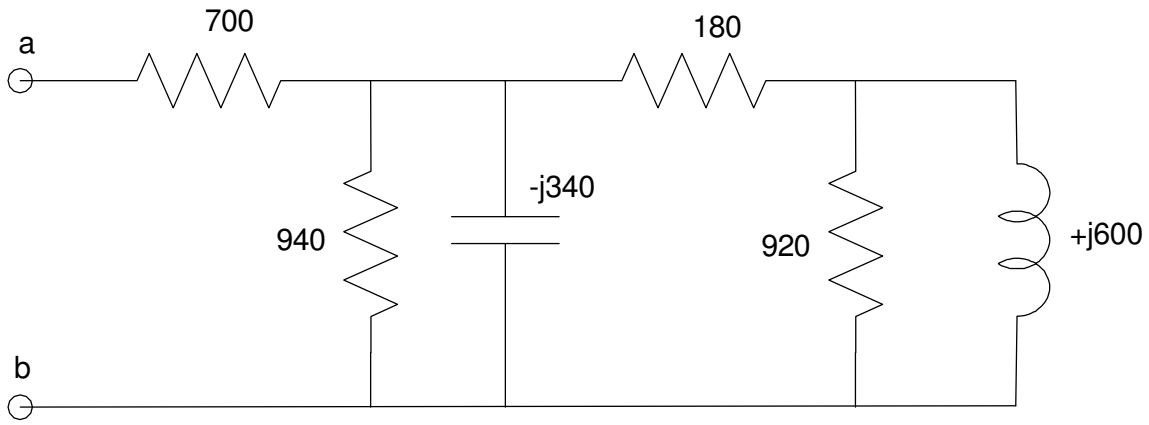

( 490 + j370) + (-j720) = 490 - j350

( 490 - j350 ) || ( -j170 ) = 27.739 - j140.562

( 27.739 - j140.562 ) + (300 + j70 ) = 327.739 - j70.562

**ans: 327.739 - j70.562**

3) Determine the impeadance Zab




( j600 ) ‖ ( 920 ) = 274.536 + j420.955

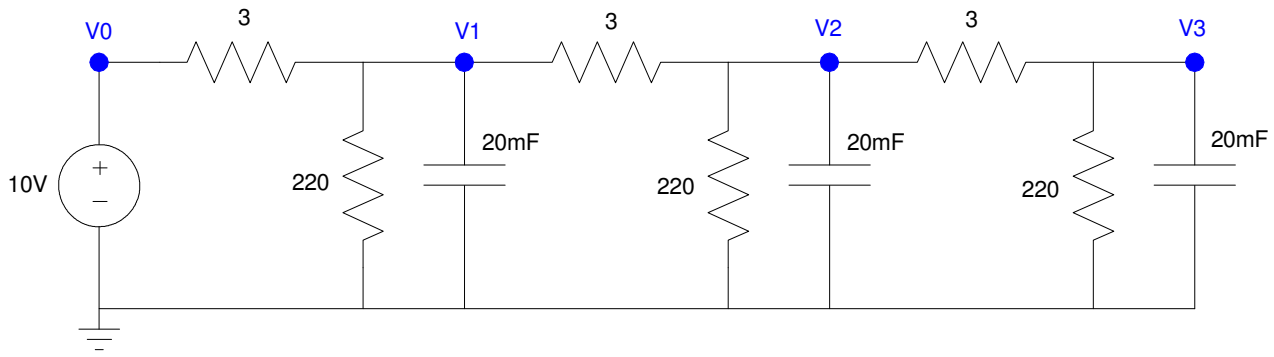
( 274.536 + j420.955 ) + ( 180 ) = 454.536 + j420.955

( 454.536 + j420.955 ) ‖ ( -j340 ) ‖ ( 940 ) = 265.867 - j218.121

( 265.867 - j218.121 ) + ( 700 ) = 965.867 - j218.121

**ans = 965.867 - j218.121**

4) Assume $V_0 = 10$




a) Determine the impedances of each element at 0 rad/sec

V0 = 10

C = infinity (ignore: the current is zero)

b) Write the votlage node equations

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{3}\right) + \left(\frac{V_1}{220}\right) + \left(\frac{V_1 - V_2}{3}\right) = 0$$

$$\left(\frac{V_2 - V_1}{3}\right) + \left(\frac{V_2}{220}\right) + \left(\frac{V_2 - V_3}{3}\right) = 0$$

$$\left(\frac{V_3 - V_2}{3}\right) + \left(\frac{V_3}{220}\right) = 0$$

c) Solve for V1, V2, and V3.

Group terms

$$V_0 = 10$$

$$-\left(\frac{1}{3}\right)V_0 + \left(\frac{1}{3} + \frac{1}{220} + \frac{1}{3}\right)V_1 - \left(\frac{1}{3}\right)V_2 = 0$$

$$-\left(\frac{1}{3}\right)V_1 + \left(\frac{1}{3} + \frac{1}{220} + \frac{1}{3}\right)V_2 - \left(\frac{1}{3}\right)V_3 = 0$$

$$-\left(\frac{1}{3}\right)V_2 + \left(\frac{1}{3} + \frac{1}{220}\right)V_3 = 0$$

Place in matrix form

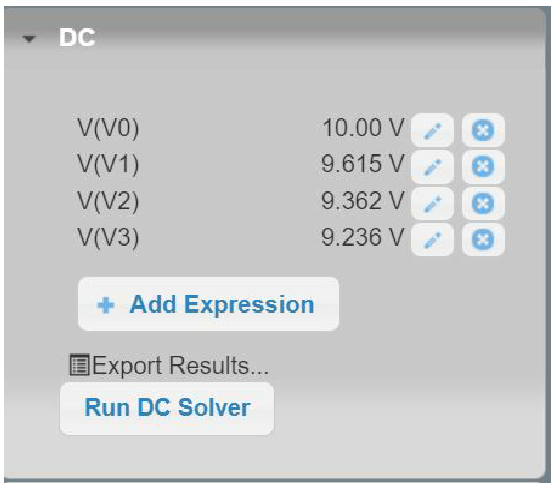
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{3}\right) & \left(\frac{2}{3}+\frac{1}{220}\right) & \left(\frac{-1}{3}\right) & 0 \\ 0 & \left(\frac{-1}{3}\right) & \left(\frac{2}{3}+\frac{1}{220}\right) & \left(\frac{-1}{3}\right) \\ 0 & 0 & \left(\frac{-1}{3}\right) & \left(\frac{1}{3}+\frac{1}{220}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

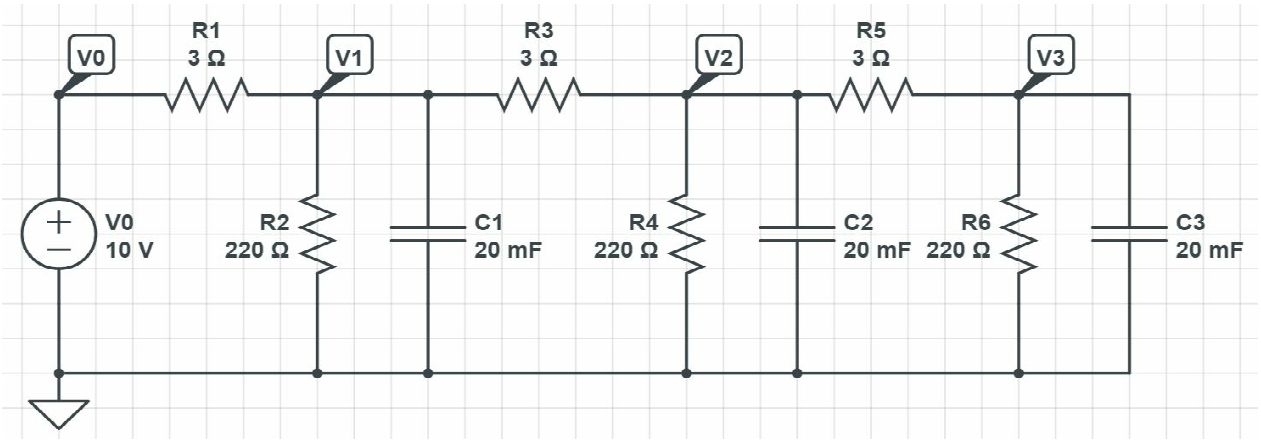
Solve using Matlab

```
A = [1,0,0,0 ; -1/3,2/3+1/220,-1/3,0 ; 0,-1/3,2/3+1/220,-1/3 ; 0,0,-1/3,1/3+1/220]

    1.0000         0         0         0
   -0.3333    0.6712   -0.3333         0
         0   -0.3333    0.6712   -0.3333
         0         0   -0.3333    0.3379

>> B = [10;0;0;0]

    10
     0
     0
     0

>> V = inv(A)*B

   10.0000
    9.6153
    9.3617
    9.2357
```

5) Check your results in CircuitLab

```
>> V = inv(A)*B

   10.0000
    9.6153
    9.3617
    9.2357
```



6) Assume V0 is a 10V, 4 rad/sec ( 0.637Hz )

$$V_0 = 10\sin(4t)$$

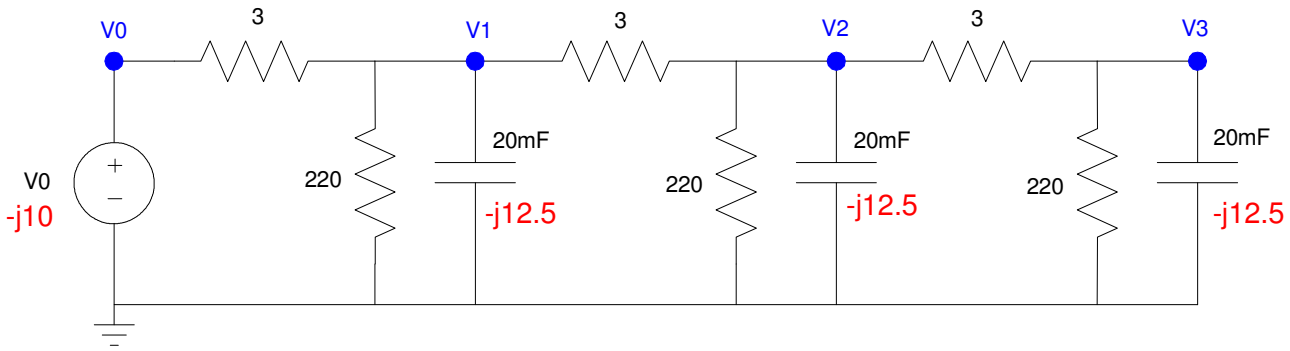
a) Determine the impedances of each element at 4 rad/sec

V0:

$$0\cos(4t) + 10\sin(4t) \Rightarrow (0 - j10)$$

4mF

$$C \Rightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(0.02)} = -j12.5\Omega$$




b) Write the votlage node equations

- same as before just add one term for the capacitor

$$V_0 = -j10$$

$$\left(\frac{V_1 - V_0}{3}\right) + \left(\frac{V_1}{220}\right) + \left(\frac{V_1}{-j12.5}\right) + \left(\frac{V_1 - V_2}{3}\right) = 0$$

$$\left(\frac{V_2 - V_1}{3}\right) + \left(\frac{V_2}{220}\right) + \left(\frac{V_2}{-j12.5}\right) + \left(\frac{V_2 - V_3}{3}\right) = 0$$

$$\left(\frac{V_3 - V_2}{3}\right) + \left(\frac{V_3}{220}\right) + \left(\frac{V_3}{-j12.5}\right) = 0$$

c) Solve for V1, V2, and V3 as complex numbers

- same as before just add the capacitor term

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{3}\right) & \left(\frac{2}{3}+\frac{1}{220}+\frac{1}{-j12.5}\right) & \left(\frac{-1}{3}\right) & 0 \\ 0 & \left(\frac{-1}{3}\right) & \left(\frac{2}{3}+\frac{1}{220}+\frac{1}{-j12.5}\right) & \left(\frac{-1}{3}\right) \\ 0 & 0 & \left(\frac{-1}{3}\right) & \left(\frac{1}{3}+\frac{1}{220}+\frac{1}{-j12.5}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
A = [1,0,0,0 ];
A = [A ; -1/3,2/3+1/220-1/(j*12.5),-1/3,0 ];
A = [A ; 0,-1/3,2/3+1/220-1/(j*12.5),-1/3 ];
A = [A ; 0,0,-1/3,1/3+1/220-(1/(j*12.5))]
```

```
   1.0000                  0                  0                  0
  -0.3333            0.6712 + 0.0800i  -0.3333                  0
        0           -0.3333            0.6712 + 0.0800i  -0.3333
        0                 0           -0.3333            0.3379 + 0.0800i
```

```
B = [-j*10;0;0;0]
```

```
        0 -10.0000i
        0
        0
        0
```

```
V = inv(A)*B
```

```
V =
```

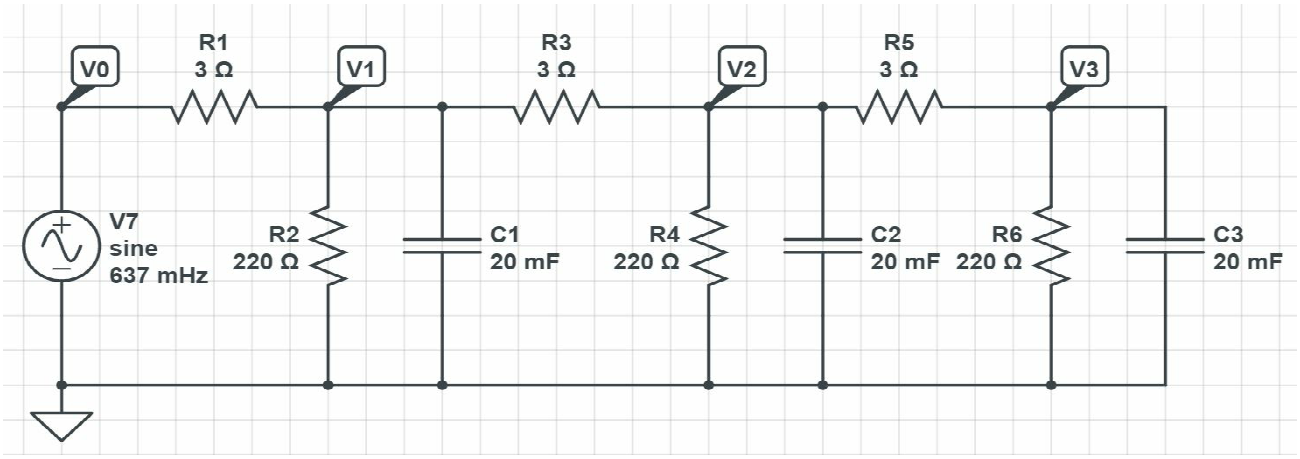
```
V0        0 -10.0000i
V1  -3.1170 - 6.6724i
V2  -4.6750 - 4.1839i
V3  -5.2927 - 2.8745i
```

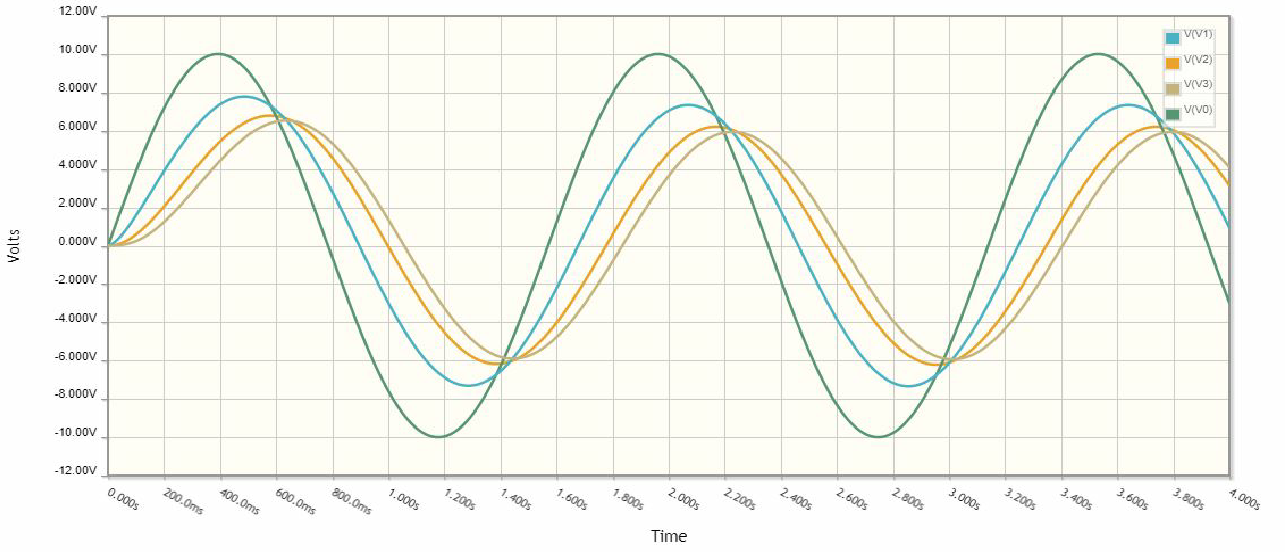
d) Express V1, V2, and V3 in terms of sine and cosine function:

- hint: V1 = a + jb (phasor representation) means $V_1(t) = a\cos(4t) - b\sin(4t)$

```
V0 =       0 cos(4t) + 10.0000 sin(4t)
V1 = -3.1170 cos(4t) + 6.6724 sin(4t)
V2 = -4.6750 cos(4t) + 4.1839 sin(4t)
V3 = -5.2927 cos(4t) + 2.8745 sin(4t)
```

7) Check your results in CircuitLab using a transient simulation for 6 seconds




The peak matches the magniture of the voltages computed in Matlab

```
>> abs(V)

V0   10.0000
V1    7.3646
V2    6.2738
V3    6.0229
```