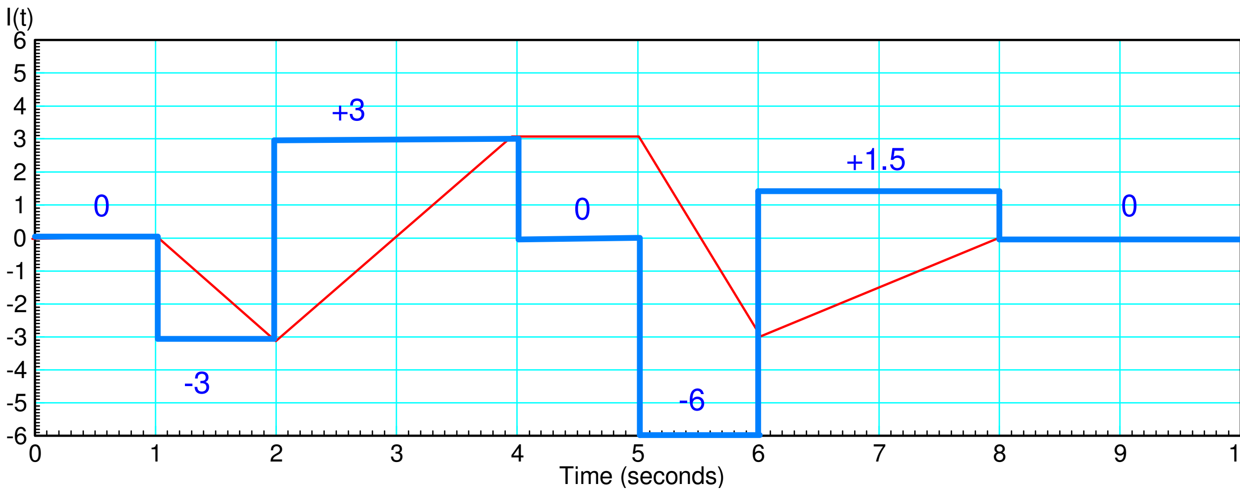
ECE 111 - Homework #8

Week #8: ECE 351 Electromagnetics - - Due 8am Tuesday, March 8th
Please submit as a Word or pdf file and email to Jacob_Glower@yahoo.com with header ECE 111 HW#8

1) Assume the current flowing through a one Henry inductor is shown below. Sketch the voltage.

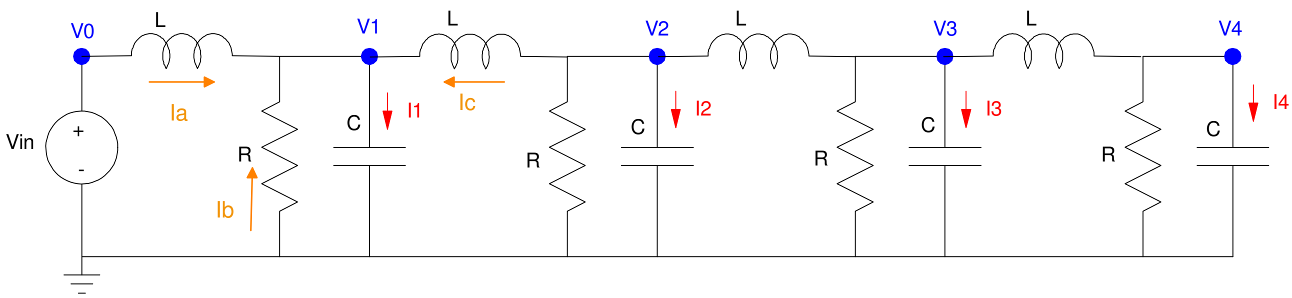
$$V = L\frac{dI}{dt}$$

The derivative is the slope: change in I(t) / change in time



Problem 2-3) 4-Node RLC Circuit

2) Write the dynamic equations for the following 4-stage RLC circuit. (i.e. write the node equations)



R = 400Ω, C = 0.033F, L = 0.2H. Repeat for 30 nodes for problems 4-6

Note

$$I = C\frac{dV}{dt} = C\dot{V}$$

$$V = L\frac{dI}{dt} = L\dot{I}$$

Node V1:

$$I_1 = C\dot{V}_1 = I_a + I_b + I_c$$

$$C\dot{V}_1 = I_a + \left(\frac{0-V_1}{R}\right) + I_c$$

Take the derivative (sY means 'the derivative if Y'):

$$C\ddot{V}_1 = sI_a - \left(\frac{1}{R}\right)\dot{V}_1 + sI_c$$

For an inductor

$$V = L\dot{I}$$

$$V_0 - V_1 = L\dot{I}_a$$

$$\dot{I}_a = \left(\frac{V_0-V_1}{L}\right)$$

Substitute for dIa/dt and dIc/dt

$$C\ddot{V}_1 = \left(\frac{V_0 - V_1}{L}\right) - \left(\frac{1}{R}\right)\dot{V}_1 + \left(\frac{V_2 - V_1}{L}\right)$$

Simplify

$$\ddot{V}_1 = \left(\frac{1}{LC}\right)V_0 - \left(\frac{2}{LC}\right)V_1 + \left(\frac{1}{LC}\right)V_2 - \left(\frac{1}{RC}\right)\dot{V}_1$$

$$\ddot{V}_2 = \left(\frac{1}{LC}\right)V_1 - \left(\frac{2}{LC}\right)V_2 + \left(\frac{1}{LC}\right)V_3 - \left(\frac{1}{RC}\right)\dot{V}_2$$

$$\ddot{V}_3 = \left(\frac{1}{LC}\right)V_2 - \left(\frac{2}{LC}\right)V_3 + \left(\frac{1}{LC}\right)V_4 - \left(\frac{1}{RC}\right)\dot{V}_3$$

$$\ddot{V}_4 = \left(\frac{1}{LC}\right)V_3 - \left(\frac{1}{LC}\right)V_4 - \left(\frac{1}{RC}\right)\dot{V}_4$$

Plugging in numbers:

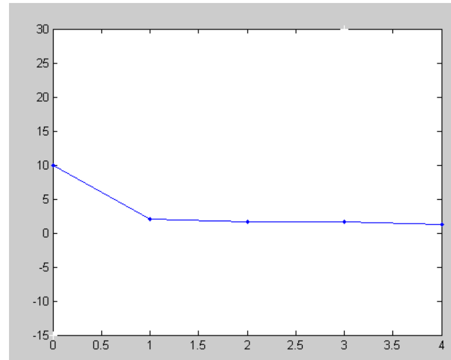
$$\ddot{V}_1 = 151.52V_0 - 303.04V_1 + 1515.2V_2 - 0.076\dot{V}_1$$

$$\ddot{V}_2 = 151.52V_1 - 303.04V_2 + 151.52V_3 - 0.076\dot{V}_2$$

$$\ddot{V}_3 = 151.52V_2 - 303.04V_3 + 151.52V_4 - 0.076\dot{V}_3$$

$$\ddot{V}_4 = 151.52V_3 - 151.52V_4 - 0.076\dot{V}_4$$

3) Assume Vin = 10V and the initial conditions are zero ($V_1 = V_2 = V_3 = V_4 = 0$). Solve for the voltages at t = 3 seconds. *Hint: Solve numerically using Matlab*

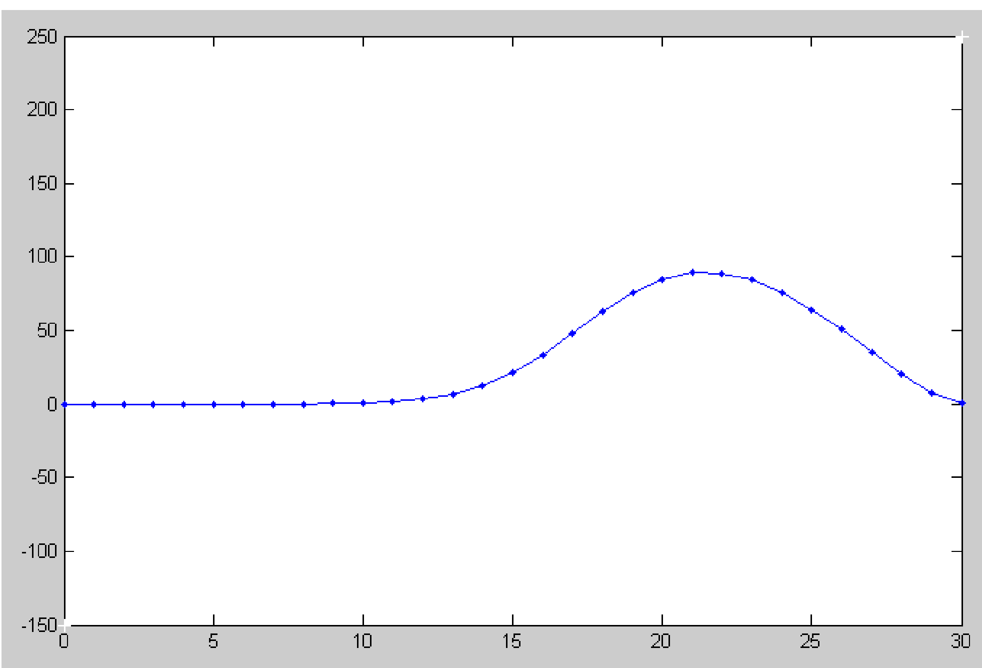



```
V0 = 10;
V1 = 0;
V2 = 0;
V3 = 0;
V4 = 0;

dV1 = 0;
dV2 = 0;
dV3 = 0;
dV4 = 0;

V = [];

t = 0;
dt = 0.01;

while(t < 3)

   ddV1 = 151.52*V0 - 303.04*V1 + 151.52*V2 - 0.076*dV1;
   ddV2 = 151.52*V1 - 303.04*V2 + 151.52*V3 - 0.076*dV2;
   ddV3 = 151.52*V2 - 303.04*V3 + 151.52*V4 - 0.076*dV3;
   ddV4 = 151.52*V3 - 151.52*V4           - 0.076*dV4;

   dV1 = dV1 + ddV1*dt;
   dV2 = dV2 + ddV2*dt;
   dV3 = dV3 + ddV3*dt;
   dV4 = dV4 + ddV4*dt;

   V1 = V1 + dV1*dt;
   V2 = V2 + dV2*dt;
   V3 = V3 + dV3*dt;
   V4 = V4 + dV4*dt;

  t = t + dt;

   hold off
   plot([0,3],[-15,30],'w+');
   hold on
   plot([0,1,2,3,4],[V0,V1,V2,V3,V4],'.-');
   pause(0.01);

   end
```

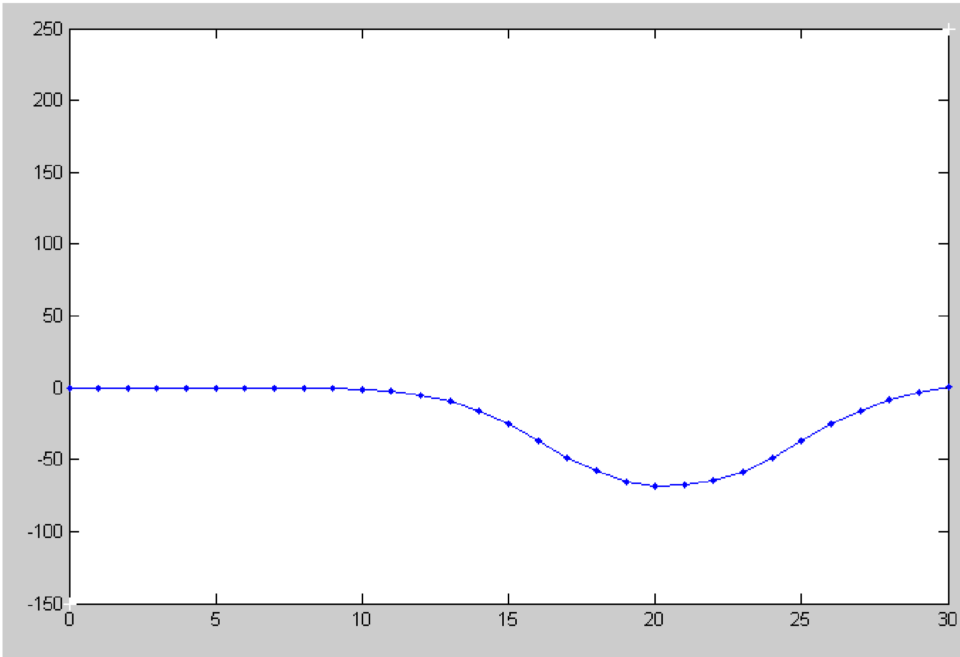
Problem 4-6) 30-Node RLC Circuit (hint: modify the program Wave.m)

4) Expand the RLC circuit from problem #2 to 30 nodes. Plot the voltage at t = 4 seconds (just after the reflection) for $1 / R_{30}C = 0.01$

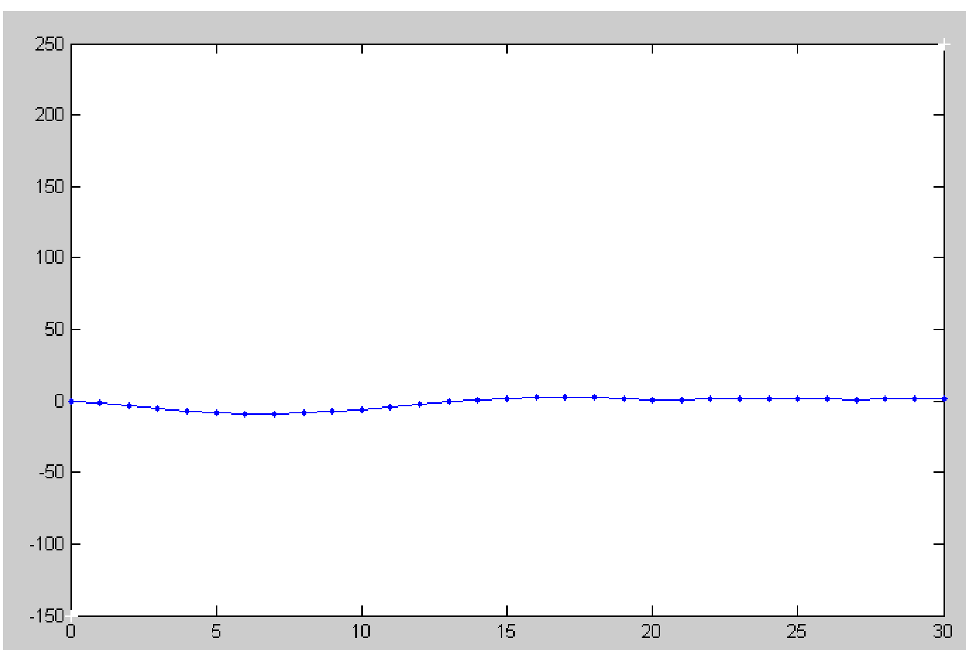
```
V = zeros(30,1);
dV = zeros(30,1);
t = 0;
dt = 0.01;
while(t < 5)
   if (t < 1.5) V0 = 100* ( sin(2*t).^2);
      else V0 = 0;
      end
   ddV(1) = 151.52*V0 - 303.04*V(1) + 151.52*V(2) - 0.076*dV(1);
   for i=2:29
      ddV(i) = 151.52*V(i-1) - 303.04*V(i) + 151.52*V(i+1) - 0.076*dV(i);
      end
   ddV(30) = 151.52*V(29) - 151.52*V(30) - 0.076*dV(30);
   for i=1:30
      dV(i) = dV(i) + ddV(i)*dt;
      V(i) = V(i) + dV(i)*dt;
      end
   t = t + dt;

 hold off
 plot([0,30],[-150,250],'w+');
 hold on
 plot([0:30],[V0;V],'.-');
 pause(0.01);
 end
```



5) Plot the voltage at t = 4 seconds for $1 / R_{30}C = 100$



6) Determine experimentally R_{30} so that the reflection is almost zero



1/R30*C = 15