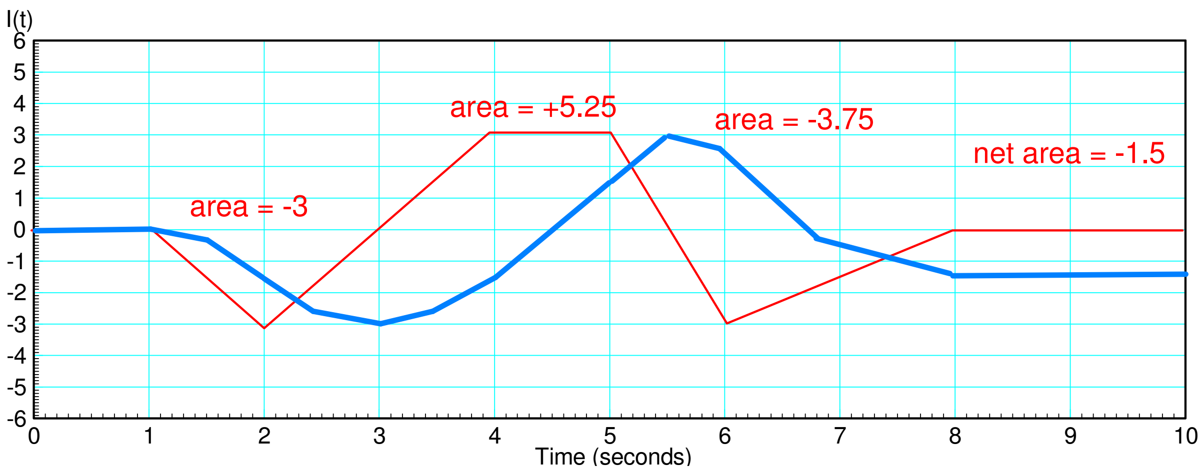
ECE 111 - Homework #7

Week #7: ECE 311 Circuits II - - Due 8am Tuesday, March 1st
Please submit as a Word or pdf file and email to Jacob_Glower@yahoo.com with header ECE 111 HW#7

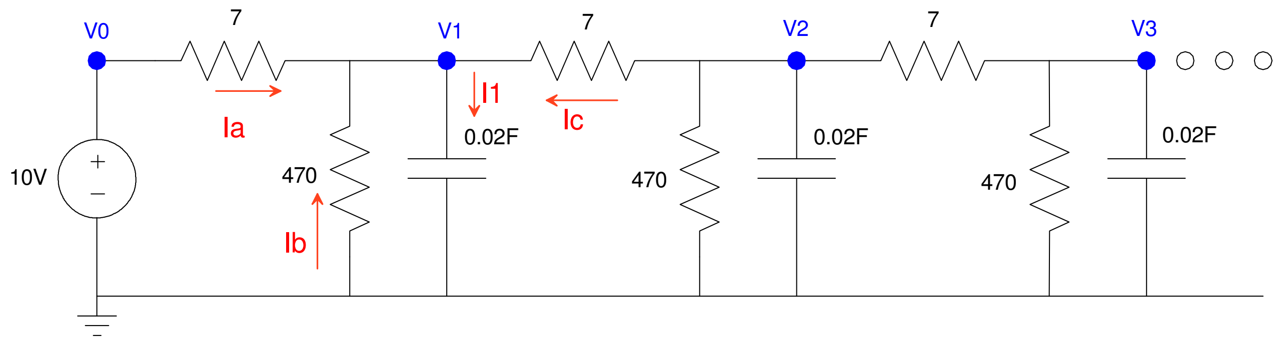
1) Assume the current flowing through a one Farad capacitor is shown below. Sketch the voltage. Assume V(0) = 0. The voltage is the integral of the current (capacitors are integrators)

$$V = \frac{1}{C} \int I \cdot dt$$

The integral is the area under the curve



Problem 2-5: Assume a 10-stage RC filter (V0 .. V10)



Problem 2) Write the dynamics for this system as a set of ten coupled differential equations:

$$I_1 = C\frac{dV_1}{dt} = \sum(\text{current to node}V_1)$$

$$I_c = C\frac{dV_1}{dt} = I_a + I_b + I_c$$

$$0.02\frac{dV_1}{dt} = \left(\frac{V_0 - V_1}{7}\right) + \left(\frac{0 - V_1}{470}\right) + \left(\frac{V_2 - V_1}{7}\right)$$

Grouping terms (same pattern for nodes 1..9)

$$\frac{dV_1}{dt} = 7.143V_0 - 14.392V_1 + 7.143V_2$$

repeat for nodes 2..9

$$\frac{dV_2}{dt} = 7.143V_1 - 14.392V_2 + 7.143V_3$$

$$\frac{dV_3}{dt} = 7.143V_2 - 14.392V_3 + 7.143V_4$$

$$\vdots$$

$$\frac{dV_9}{dt} = 7.143V_8 - 14.392V_9 + 7.143V_{10}$$

Node #10 is slightly different

$$0.02\frac{dV_{10}}{dt} = \left(\frac{V_9 - V_{10}}{7}\right) + \left(\frac{0 - V_{10}}{470}\right)$$

$$\frac{dV_{10}}{dt} = 7.143V_9 - 7.249V_{10}$$

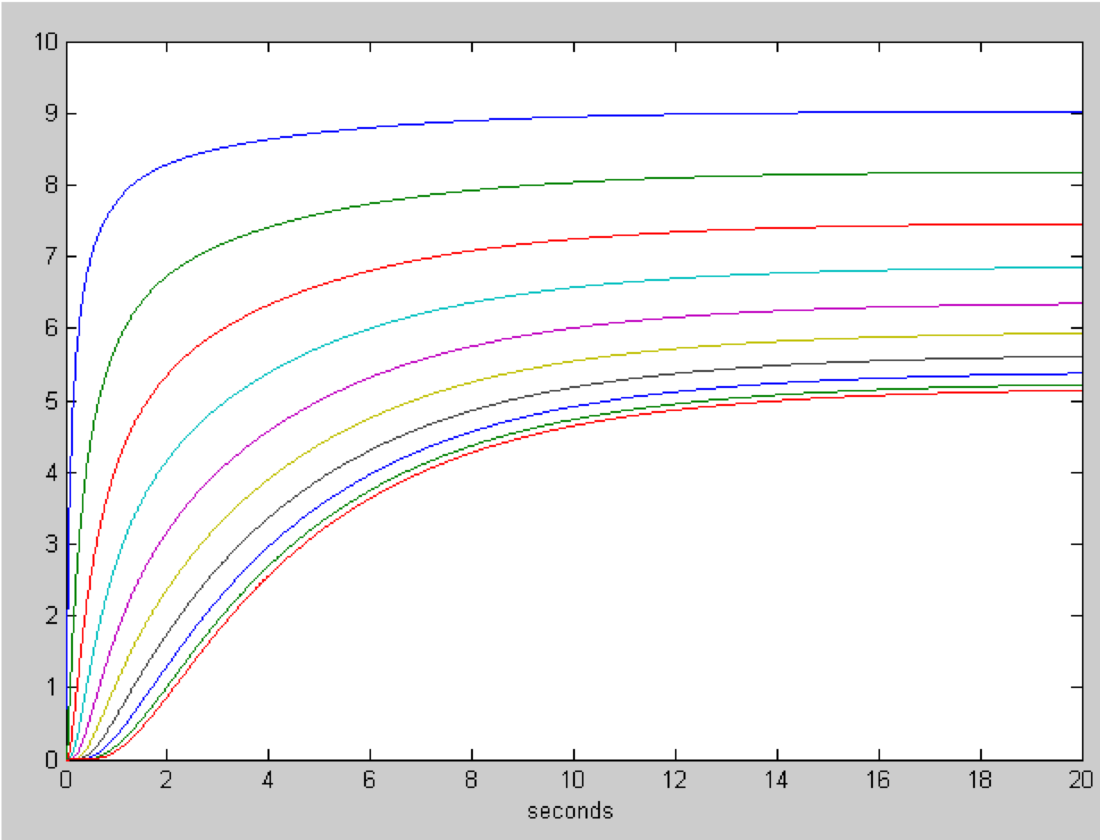
Forced Response for a 10-Node RC Filter (heat.m):

Problem 3) Using Matlab, solve these ten differential equations for $0 < t < 20$ s assuming

- The initial voltages are zero, and
- V0 = 10V.

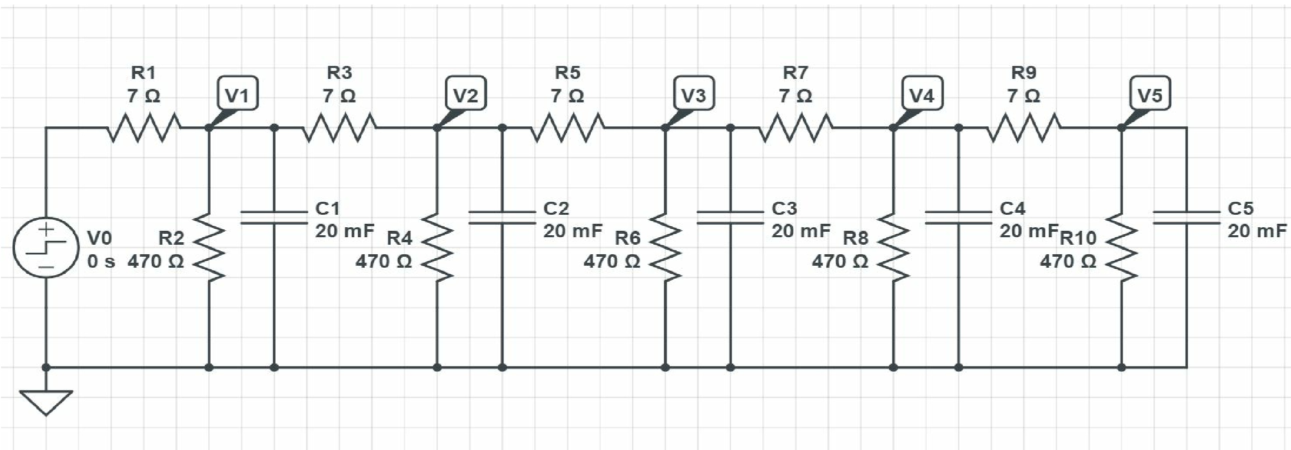
Code:

```
% 10-stage RC Filter

V = zeros(10,1);
dV = 0*V;

dt = 0.01;
t = 0;
V0 = 10;

DATA = [];

while(t < 20)

   dV(1) = 7.143*V0   - 14.392*V(1) + 7.143*V(2);
   dV(2) = 7.143*V(1) - 14.392*V(2) + 7.143*V(3);
   dV(3) = 7.143*V(2) - 14.392*V(3) + 7.143*V(4);
   dV(4) = 7.143*V(3) - 14.392*V(4) + 7.143*V(5);
   dV(5) = 7.143*V(4) - 14.392*V(5) + 7.143*V(6);
   dV(6) = 7.143*V(5) - 14.392*V(6) + 7.143*V(7);
   dV(7) = 7.143*V(6) - 14.392*V(7) + 7.143*V(8);
   dV(8) = 7.143*V(7) - 14.392*V(8) + 7.143*V(9);
   dV(9) = 7.143*V(8) - 14.392*V(9) + 7.143*V(10);
   dV(10) = 7.143*V(9) - 7.249*V(10);

   V = V + dV * dt;
   t = t + dt;

   plot([0:10], [V0;V], 'b.-');
   ylim([0,12]);
   xlim([0,10]);
   pause(0.01);
   DATA = [DATA ; V'];
end

pause(2);
t = [1:length(DATA)]' * dt;
plot(t,DATA)
```


10
9
8
7
6
5
4
3
2
1
0
0
2
4
6
8
10
12
14
16
18
20
seconds

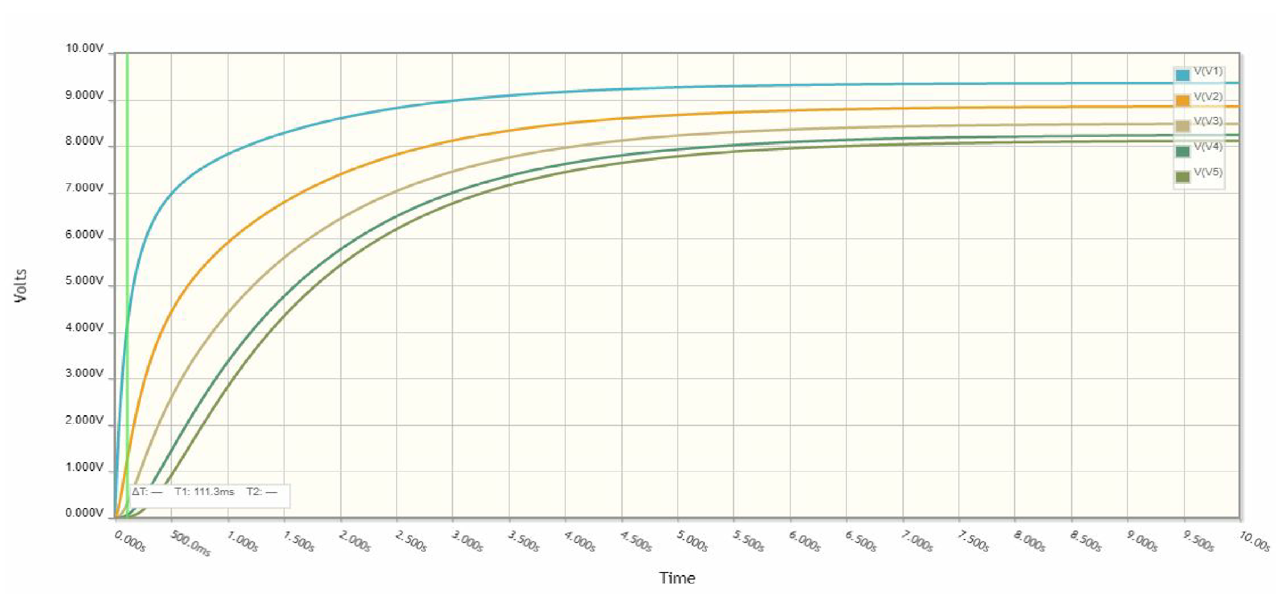
Problem 4) Using CircuitLab, find the response of this circuit to a 10V step input. *note: It's OK if you only build this circuit to 3 nodes...*

Building the circuit (note: V0 is a step input)



Running a Timer Domain simulation for 10 seconds with a time-step of 0.01 second

- generate 1000 points for the plot
- result is similar to what Matlab produced



Natural Response

Problem 5) Assume V0 = 0V. Determine the initial conditions of V1..V10 so that

- The maximum voltage is 10V and
- 5a) The voltages go to zero as slow as possible
- 5b) The voltages go to zero as fast as possible.

Simulate the response for these initial conditions in Matlab.

This is an eigenvalue / eigenvector problem. Placing the dynamics in matrix form

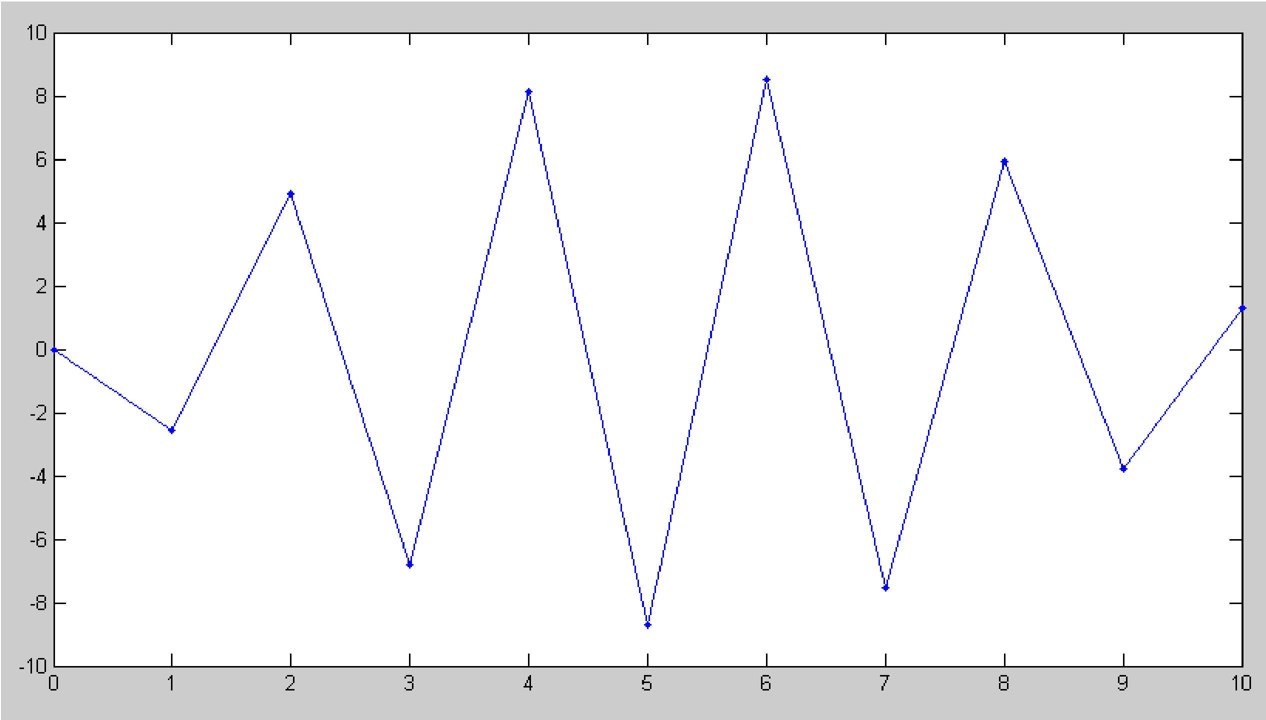
$$\frac{dV}{dt} = AV + BV_0$$

gives

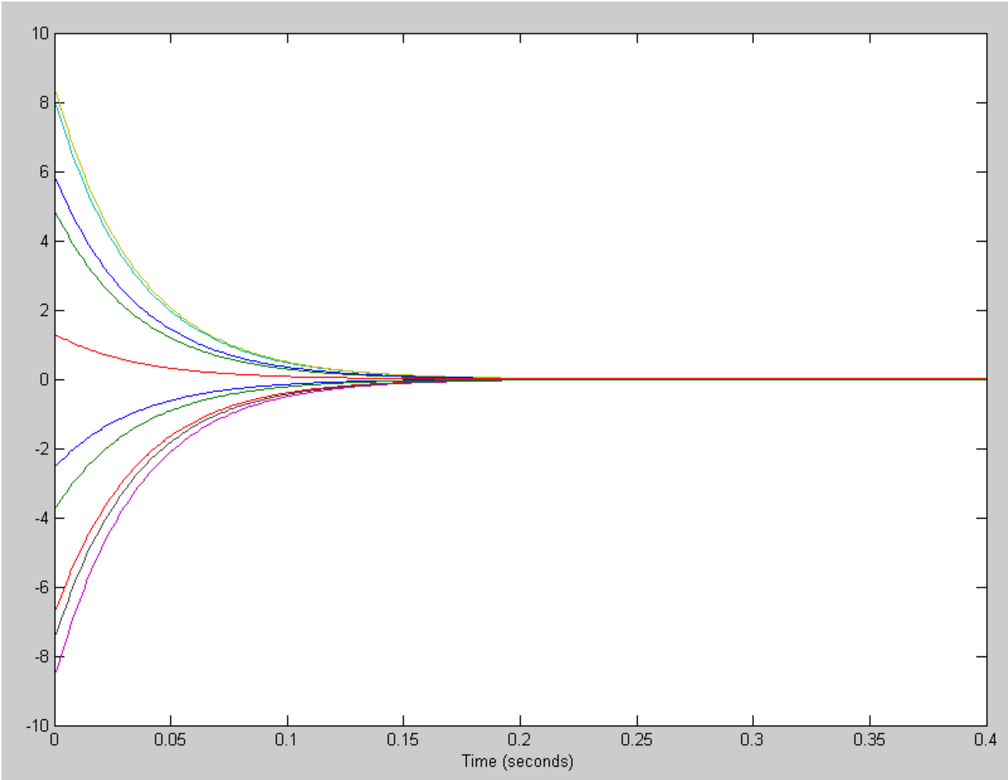
```
>> A = zeros(10,10);
>> for i=1:9
A(i,i) = -14.392;
A(i+1,i) = 7.143;
A(i,i+1) = 7.143;
end
>> A(10,10) = -7.249;
>> A

A =

  -14.3920    7.1430         0         0         0         0         0         0         0         0
    7.1430  -14.3920    7.1430         0         0         0         0         0         0         0
         0    7.1430  -14.3920    7.1430         0         0         0         0         0         0
         0         0    7.1430  -14.3920    7.1430         0         0         0         0         0
         0         0         0    7.1430  -14.3920    7.1430         0         0         0         0
         0         0         0         0    7.1430  -14.3920    7.1430         0         0         0
         0         0         0         0         0    7.1430  -14.3920    7.1430         0         0
         0         0         0         0         0         0    7.1430  -14.3920    7.1430         0
         0         0         0         0         0         0         0    7.1430  -14.3920    7.1430
         0         0         0         0         0         0         0         0    7.1430   -7.2490

>> [M,N] = eig(A);
>> M

   -0.1286   -0.2459    0.3412    0.4063    0.4352    0.4255    0.3780    0.2969   -0.1894    0.0650
    0.2459    0.4063   -0.4255   -0.2969   -0.0650    0.1894    0.3780    0.4352   -0.3412    0.1286
   -0.3412   -0.4255    0.1894   -0.1894   -0.4255   -0.3412    0.0000    0.3412   -0.4255    0.1894
    0.4063    0.2969    0.1894    0.4352    0.1286   -0.3412   -0.3780    0.0650   -0.4255    0.2459
   -0.4352   -0.0650   -0.4255   -0.1286    0.4063    0.1894   -0.3780   -0.2459   -0.3412    0.2969
    0.4255   -0.1894    0.3412   -0.3412   -0.1894    0.4255   -0.0000   -0.4255   -0.1894    0.3412
   -0.3780    0.3780   -0.0000    0.3780   -0.3780    0.0000    0.3780   -0.3780   -0.0000    0.3780
    0.2969   -0.4352   -0.3412    0.0650    0.2459   -0.4255    0.3780   -0.1286    0.1894    0.4063
   -0.1894    0.3412    0.4255   -0.4255    0.3412   -0.1894    0.0000    0.1894    0.3412    0.4255
    0.0650   -0.1286   -0.1894    0.2459   -0.2969    0.3412   -0.3780    0.4063    0.4255    0.4352

>> eig(A)'

  -28.0433  -26.1956  -23.2992  -19.6113  -15.4596  -11.2131   -7.2490   -3.9196   -1.5208   -0.2656
```

Fast Mode (red): Make the initial condition proportational to the fast eigenvector and the transient decays as exp(-28.04t)

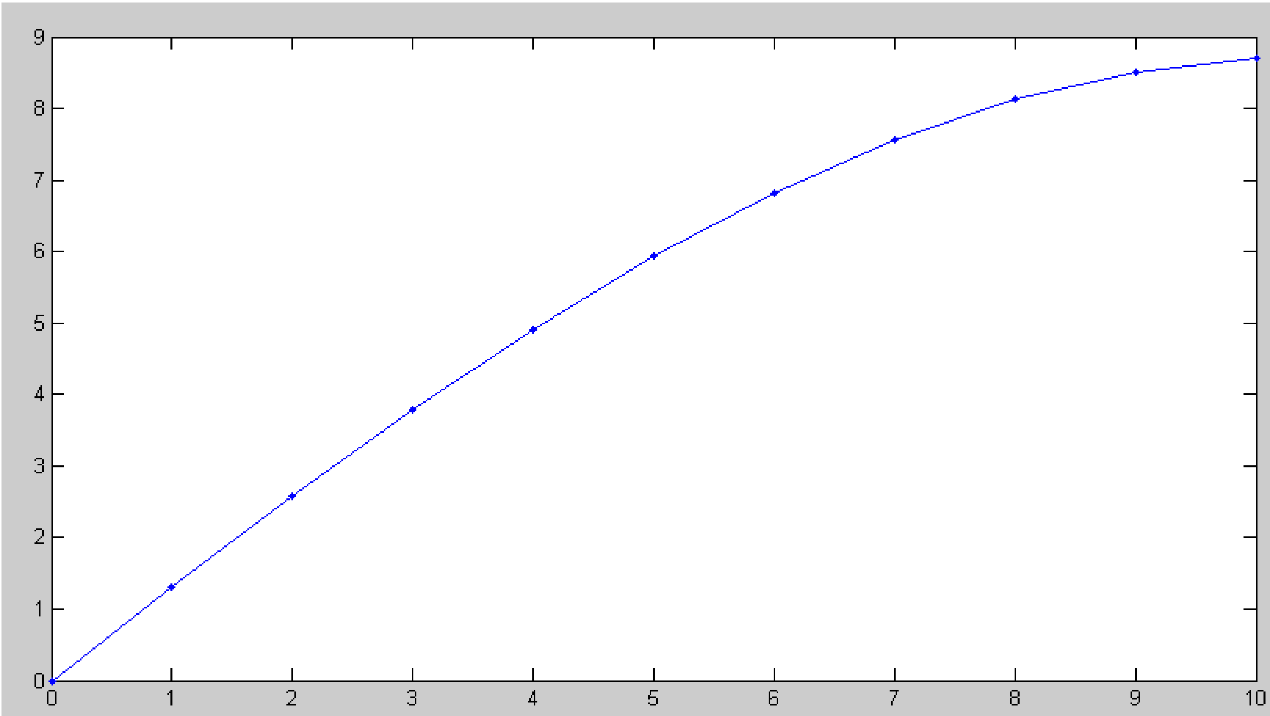
Slow Mode (blue): Make the initial conidtion proportional to the slow eigenvector and the transient decays as exp(-0.2656t)



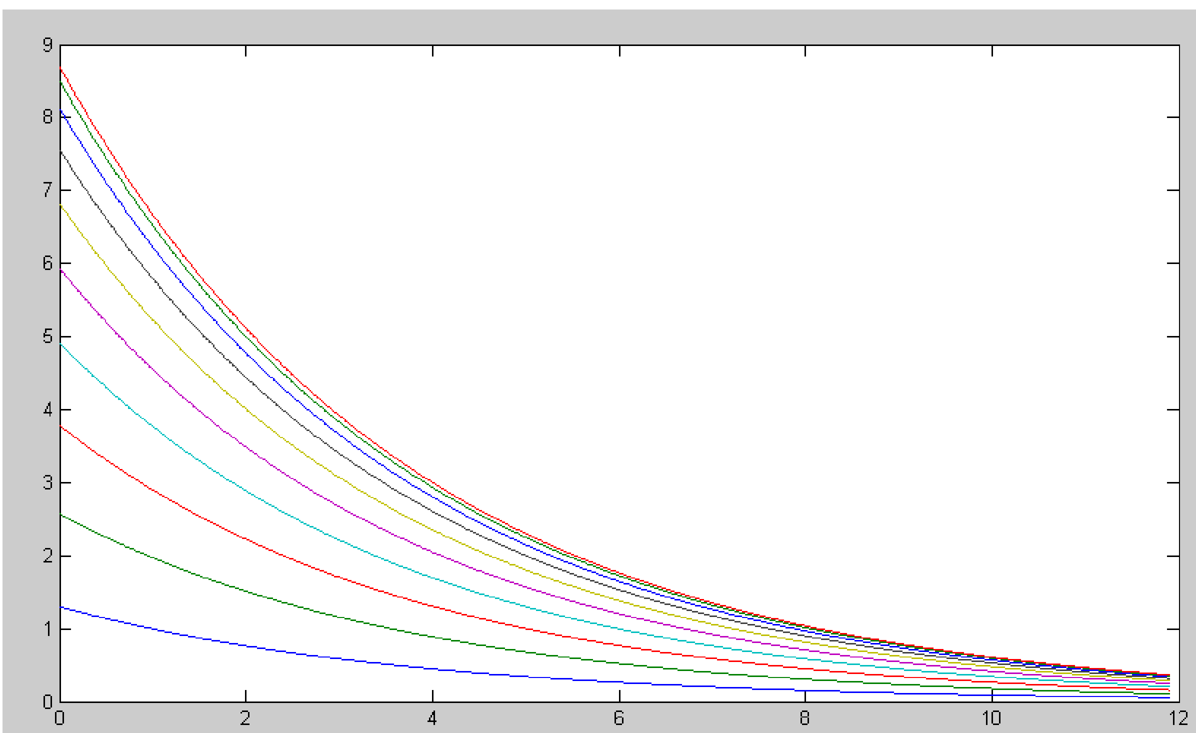
Fast Mode (Initial Condition)



Fast Mode (transient response)



Slow Mode: Initial Condition



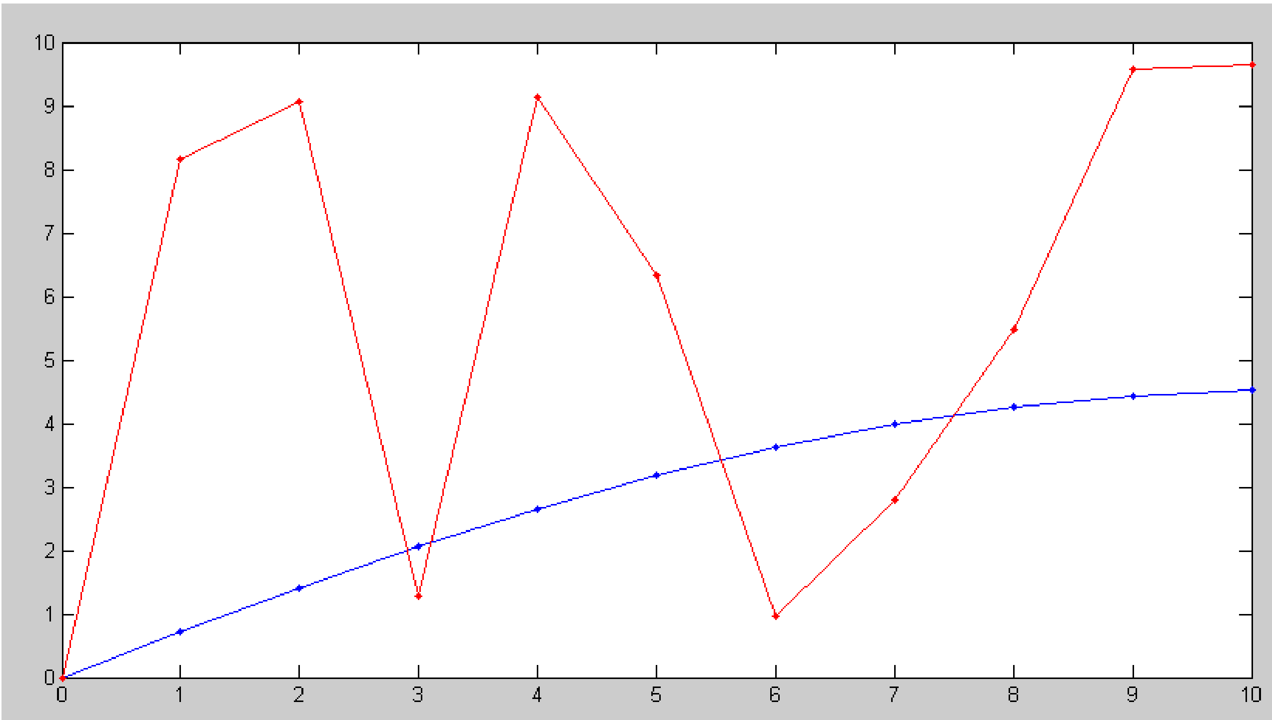
Slow Mode: Transient Response

Problem 6) Assume Vin = 0V. Pick random voltages for V1 .. V10 in the range of (0V, 10V):

```
V = 10 * rand(10,1)
```

Plot the votlages at t = 1. Which eigenvector does it look like?

- At 2 seconds (blue below), the voltage looks like the slow eigenvector



Random initial condition (red) and the voltages at 2.00 seconds (blue)