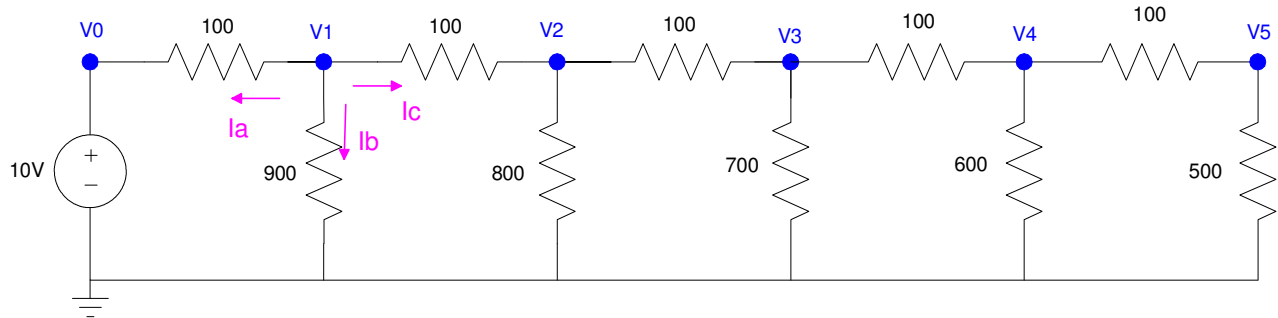


ECE 111 - Homework #6

Week #6: EE 206 Circuits I - - Due 8am Tuesday, February 22nd
Please submit as a Word or pdf file and email to Jacob_Glower@yahoo.com with header ECE 111 HW#6

1) Use Voltage Nodes write N equations for N unknowns for the following circuit.



Sum the current from each node to zero. At Node V1

$$I_a + I_b + I_c = 0$$

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{900}\right) + \left(\frac{V_1 - V_2}{100}\right) = 0$$

Repeating for all five nodes:

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{900}\right) + \left(\frac{V_1 - V_2}{100}\right) = 0$$

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{800}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$$

$$\left(\frac{V_3 - V_2}{100}\right) + \left(\frac{V_3}{700}\right) + \left(\frac{V_3 - V_4}{100}\right) = 0$$

$$\left(\frac{V_4 - V_3}{100}\right) + \left(\frac{V_4}{600}\right) + \left(\frac{V_4 - V_5}{100}\right) = 0$$

$$\left(\frac{V_5 - V_4}{100}\right) + \left(\frac{V_5}{500}\right) = 0$$

Note that at node V1 (2nd equation)

- All of the V1 terms are positive
- All of the other terms are negative

At node V2 (3rd equation)

- All of the V2 terms are positive
- All of the other terms are negative

You'll usually (always?) see this pattern.

2) Solve for the node voltages in Matlab.

Group terms

$$V_0 = 10$$

$$\left(\frac{-1}{100}\right)V_0 + \left(\frac{1}{100} + \frac{1}{900} + \frac{1}{100}\right)V_1 + \left(\frac{-1}{100}\right)V_2 = 0$$

$$\left(\frac{-1}{100}\right)V_1 + \left(\frac{1}{100} + \frac{1}{800} + \frac{1}{100}\right)V_2 + \left(\frac{-1}{100}\right)V_3 = 0$$

$$\left(\frac{-1}{100}\right)V_2 + \left(\frac{1}{100} + \frac{1}{700} + \frac{1}{100}\right)V_3 + \left(\frac{-1}{100}\right)V_4 = 0$$

$$\left(\frac{-1}{100}\right)V_3 + \left(\frac{1}{100} + \frac{1}{600} + \frac{1}{100}\right)V_4 + \left(\frac{-1}{100}\right)V_5 = 0$$

$$\left(\frac{-1}{100}\right)V_4 + \left(\frac{1}{100} + \frac{1}{500}\right)V_5 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \left(\frac{-1}{100}\right) & \left(\frac{2}{100} + \frac{1}{900}\right) & \left(\frac{-1}{100}\right) & 0 & 0 & 0 \\ 0 & \left(\frac{-1}{100}\right) & \left(\frac{2}{100} + \frac{1}{800}\right) & \left(\frac{-1}{100}\right) & 0 & 0 \\ 0 & 0 & \left(\frac{-1}{100}\right) & \left(\frac{2}{100} + \frac{1}{700}\right) & \left(\frac{-1}{100}\right) & 0 \\ 0 & 0 & 0 & \left(\frac{-1}{100}\right) & \left(\frac{2}{100} + \frac{1}{600}\right) & \left(\frac{-1}{100}\right) \\ 0 & 0 & 0 & 0 & \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{500}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> a1 = [1,0,0,0,0,0];
>> a2 = [-1/100,2/100+1/900,-1/100,0,0,0];
>> a3 = [0,-1/100,2/100+1/800,-1/100,0,0];
>> a4 = [0,0,-1/100,2/100+1/700,-1/100,0];
>> a5 = [0,0,0,-1/100,2/100+1/600,-1/100];
>> a6 = [0,0,0,0,-1/100,1/100+1/500];
>> A = [a1;a2;a3;a4;a5;a6]
```

```
    1.0000         0         0         0         0         0
   -0.0100    0.0211   -0.0100         0         0         0
         0   -0.0100    0.0213   -0.0100         0         0
         0         0   -0.0100    0.0214   -0.0100         0
         0         0         0   -0.0100    0.0217   -0.0100
         0         0         0         0   -0.0100    0.0120
```

```
>> B = [10;0;0;0;0;0]
```

B =













10
0
0
0
0
0

>> V = inv(A) * B

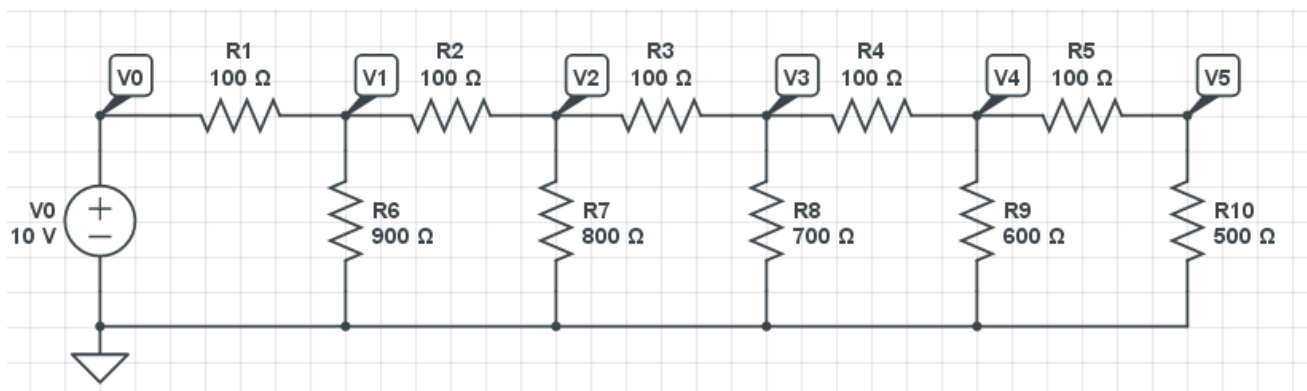
V0 10.0000
V1 7.1408
V2 5.0750
V3 3.6436
V4 2.7327
V5 2.2772

3) Check your answers in CircuitLab

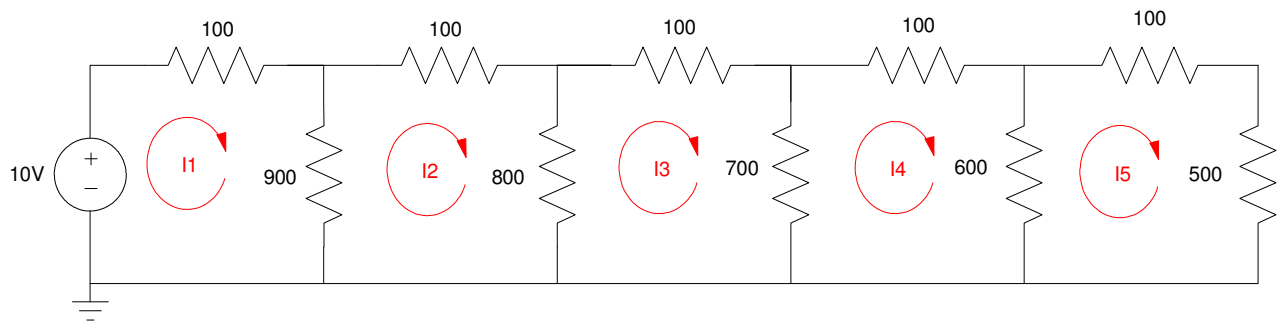
▼ DC

V(V0)	10.00 V		
V(V1)	7.141 V		
V(V2)	5.075 V		
V(V3)	3.644 V		
V(V4)	2.733 V		
V(V5)	2.277 V		

[+ Add Expression](#)



4) Use Current Loops to write N equations for N unknowns for the following circuit.



The voltages around each loop must sum to zero

$$-10 + 100I_1 + 900(I_1 - I_2) = 0$$

$$900(I_2 - I_1) + 100I_2 + 800(I_2 - I_3) = 0$$

$$800(I_3 - I_2) + 100I_3 + 700(I_3 - I_4) = 0$$

$$700(I_4 - I_3) + 100I_4 + 600(I_4 - I_5) = 0$$

$$600(I_5 - I_4) + 100I_5 + 500(I_5) = 0$$

Note that around loop I1 (1st equation)

- All of the I1 terms are positive
- All of the other terms are negative

Around loop I2 (2nd equation)

- All of the I2 terms are positive
- All of the other terms are negative

If you keep the direction of each loop the same (clockwise in this case), you'll usually (always?) get this pattern.

5) Solve for the currents in Matlab

Group terms

$$1000I_1 - 900I_2 = 10$$

$$-900I_1 + 1800I_2 - 800I_3 = 0$$

$$-800I_2 + 1600I_3 - 700I_4 = 0$$

$$-700I_3 + 1400I_4 - 600I_5 = 0$$

$$-600I_4 + 1200I_5 = 0$$

Place in matrix form

$$\begin{bmatrix} 1000 & -900 & 0 & 0 & 0 \\ -900 & 1800 & -800 & 0 & 0 \\ 0 & -800 & 1600 & -700 & 0 \\ 0 & 0 & -700 & 1400 & -600 \\ 0 & 0 & 0 & -600 & 1200 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> A = [1000,-900,0,0,0 ; -900,1800,-800,0,0 ; 0,-800,1600,-700,0 ;  
0,0,-700,1400,-600 ; 0,0,0,-600,1200]
```

```
    1000         -900             0             0             0  
   -900         1800          -800             0             0  
    0         -800         1600          -700             0  
    0             0          -700         1400          -600  
    0             0             0          -600         1200
```

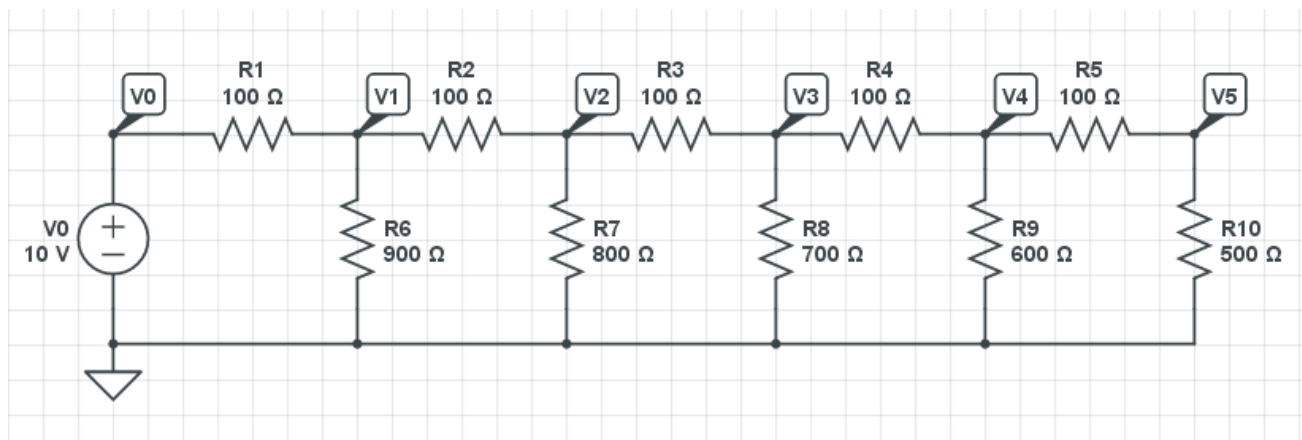
```
>> B = [10;0;0;0;0]
```

```
    10  
     0  
     0  
     0  
     0
```

```
>> I = inv(A)*B
```

```
I1    0.0286  
I2    0.0207  
I3    0.0143  
I4    0.0091  
I5    0.0046
```

6) Check your answers in CircuitLab.



Note: In CircuitLab, you can display the current through each resistor by clicking on the right-side of the resistor. This gives the same answer computed in Matlab

DC	
I(R1.nA)	28.59 mA
I(R2.nA)	20.66 mA
I(R3.nA)	14.31 mA
I(R4.nA)	9.109 mA
I(R5.nA)	4.554 mA
V(V0)	10.00 V
V(V1)	7.141 V
V(V2)	5.075 V
V(V3)	3.644 V
V(V4)	2.733 V
V(V5)	2.277 V

In lab, you normally can't measure current but you can measure voltage. If you know the voltage and the resistances, you can compute the currents

$$I_1 = \left(\frac{V_0 - V_1}{100} \right) = \left(\frac{10.00V - 7.141V}{100\Omega} \right) = 28.59mA$$

$$I_2 = \left(\frac{V_1 - V_2}{100} \right) = \left(\frac{7.141V - 5.075V}{100\Omega} \right) = 20.66mA$$

etc.