ECE 111 - Homework #3

Week #3: Linear Algebra.   Due 8am Tuesday, February 1st
Please submit as a Word or pdf file and email to Jacob_Glower@yahoo.com with header ECE 111 HW#3

1) Solve for {x, y}

$$x + 5y = 10$$

$$x - 5y = 15$$

Step 1: Express in matrix form

$$\begin{bmatrix} 1 & 5 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

Step 2: Input into Matlab and solve

```
>> B = [1,5 ; 1,-5]

     1     5
     1    -5

>> A = [10;15]

    10
    15


>> inv(B)*A

x   12.5000
y   -0.5000
```

2) Solve for {x, y, z}

$$x + y = 5$$

$$y + 2z = 10$$

$$x + 2y + 3z = 15$$

Step 1: Put into matrix form

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}$$

Step 2: Input into matlab and solve

```
>> B = [1,1,0 ; 0,1,2 ; 1,2,3]

     1     1     0
     0     1     2
     1     2     3

>> A = [5;10;15]

     5
    10
    15

>> inv(B)*A

x    -5
y    10
z     0

>>
```

3) Solve for {a, b, c, d}

$$a - b = 0$$

$$a + b + 2c = 5$$

$$a + 2b + 3c + 4d = 10$$

$$c + d = 0$$

Step 1: Place in matrix form

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 10 \\ 0 \end{bmatrix}$$

Put into Matlab and solve

```
>> B = [1,-1,0,0 ; 1,1,2,0 ; 1,2,3,4 ; 0,0,1,1]

     1    -1     0     0
     1     1     2     0
     1     2     3     4
     0     0     1     1

>> A = [0;5;10;0]

     0
     5
    10
     0

>> inv(B)*A

a    3.1250
b    3.1250
c   -0.6250
d    0.6250

>>
```

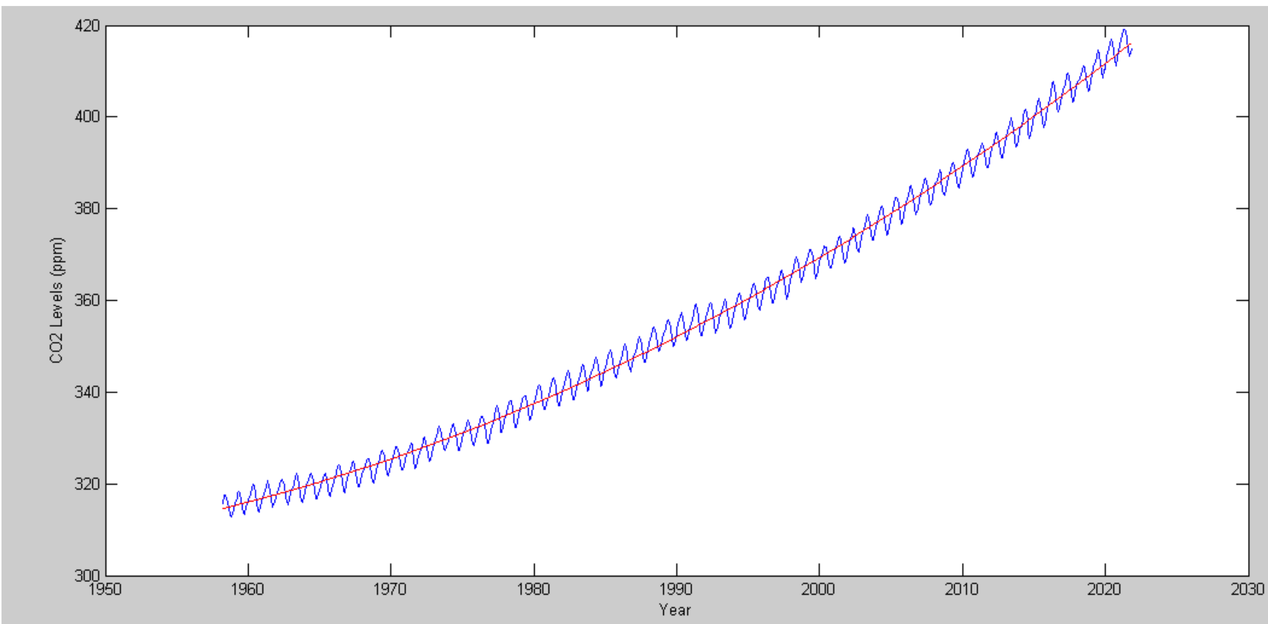
Problem 4-5: CO2 Levels. The CO2 levels measured at Mauna Loa observatory for the past 52 years are:

Problem 4) Determine a parabolic curve fit for this data in the form of

$$CO_2 \approx ay^2 + by + c$$

where 'y' is the year.

```
>> DATA = [  <paste data>  ];
>> year = DATA(:,3);
>> CO2 = DATA(:,4);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*CO2

a    1.307904363187570e-002
b   -5.045969126850365e+001
c    4.897257338516259e+004

>> plot(year,CO2,'b',year,B*A,'r');
>> xlabel('Year');
>> ylabel('CO2 Levels (ppm)');
```



Problem 5) From this data, when do you predict that we will hit

- 400ppm?
- 2000 ppm of CO2? (the same as what was observed during the Permian extinction)

```
>> roots(A - [0;0;400])

    2.014930866195835e+003        Predict to reach 400ppm in 2014
    1.843125784485060e+003

>> roots(A - [0;0;2000])

    2.289184333419512e+003        Predict to reach 2000ppm in 2289
    1.568872317261384e+003
```

Problem 6-7) Sea Ice: The area covered by sea ice is recored by the National Snow and Ice Data Center:

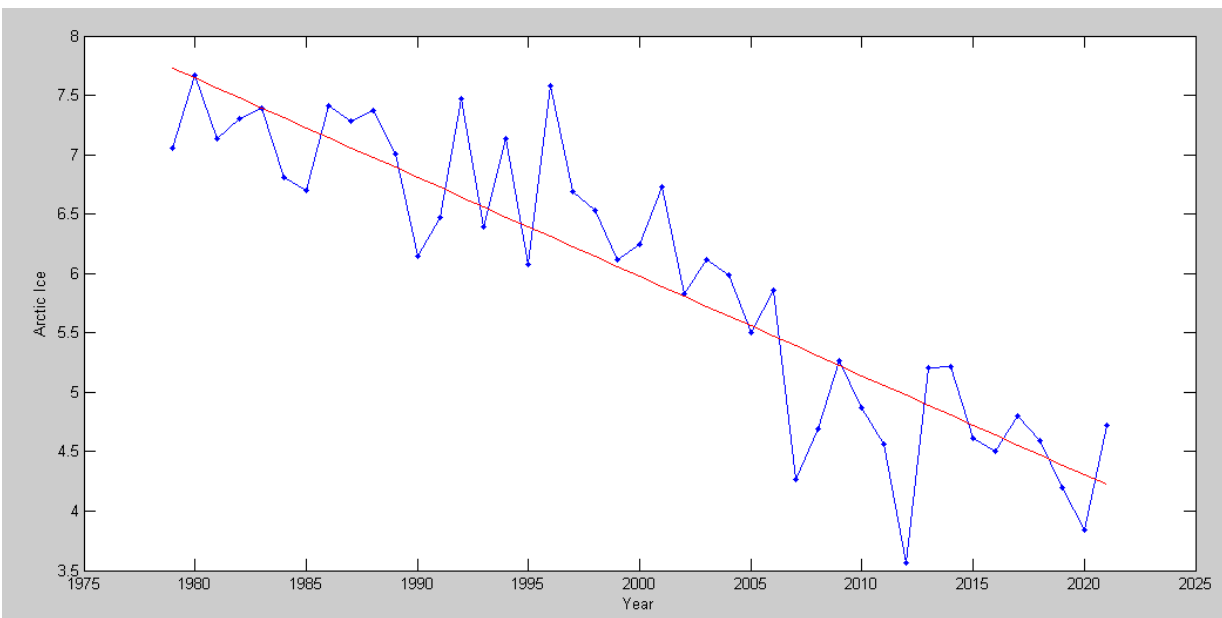
6) Approximate this data from the years 1979 - 2021 with a line

$$Area \approx ay + b$$

```
>> DATA = [ <paste> ];
>> year = DATA(:,1);
>> ICE = DATA(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*ICE

a   -8.345620658429354e-002
b    1.728875992150993e+002

>> plot(year,ICE,'b.-',year,B*A,'r');
>> xlabel('Year');
>> ylabel('Arctic Ice')
>> roots(A)

    2.071596664778635e+003

>>
```

From this curve fit, when do you expect the Arctic to be ice free? (First time in 5 million years)

Using a linear curve fit, the date suggests that the Arctic will be ice free in the year 2071



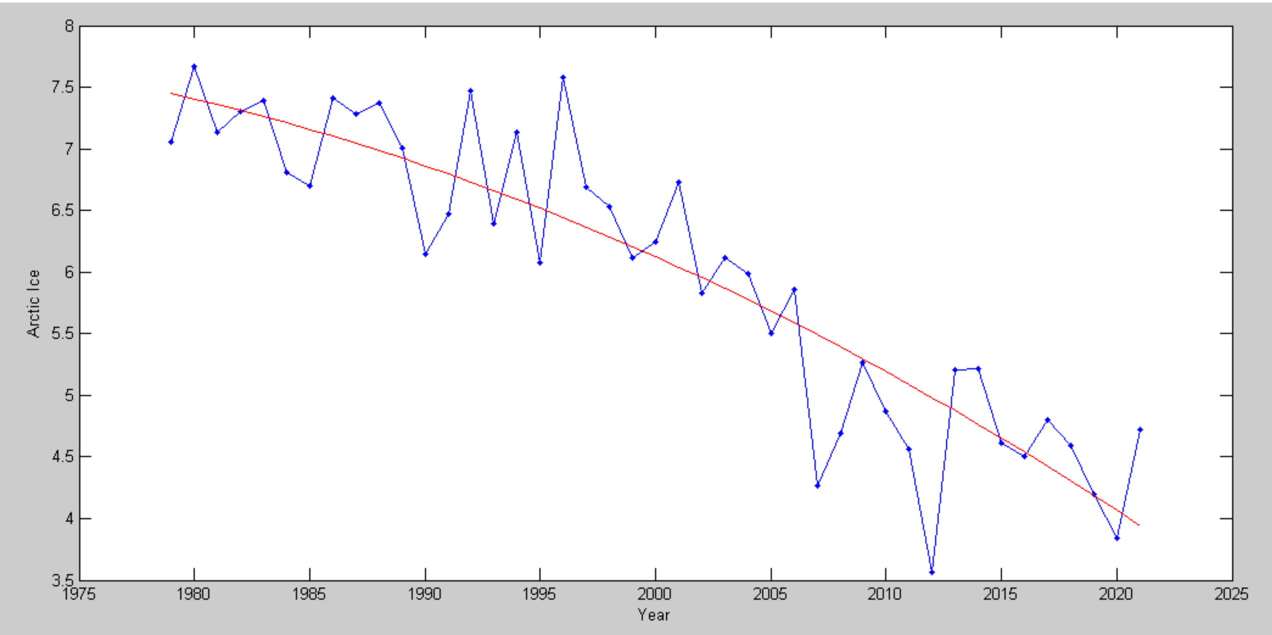
7) Approximate this data with a parabolic curve fit:

$$Area \approx ay^2 + by + c$$

From this curve fit, when do you expect the Arctic to be ice free?

```
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*ICE

a   -9.765118348656295e-004
b    3.822591133382362e+000
c   -3.733009356841957e+003

>> plot(year,ICE,'b.-',year,B*A,'r');
>> xlabel('Year');
>> ylabel('Arctic Ice')
>> roots(A)

    2.047262182294541e+003
    1.867274228734579e+003
```

Using a parabolic curve fit, the data suggests that the Arctic will be ice free in the year 2047 (25 years from now)



Problem 8-9: World Temperatures. NASA Goddard has been keep records since 1880 (139 years of data).

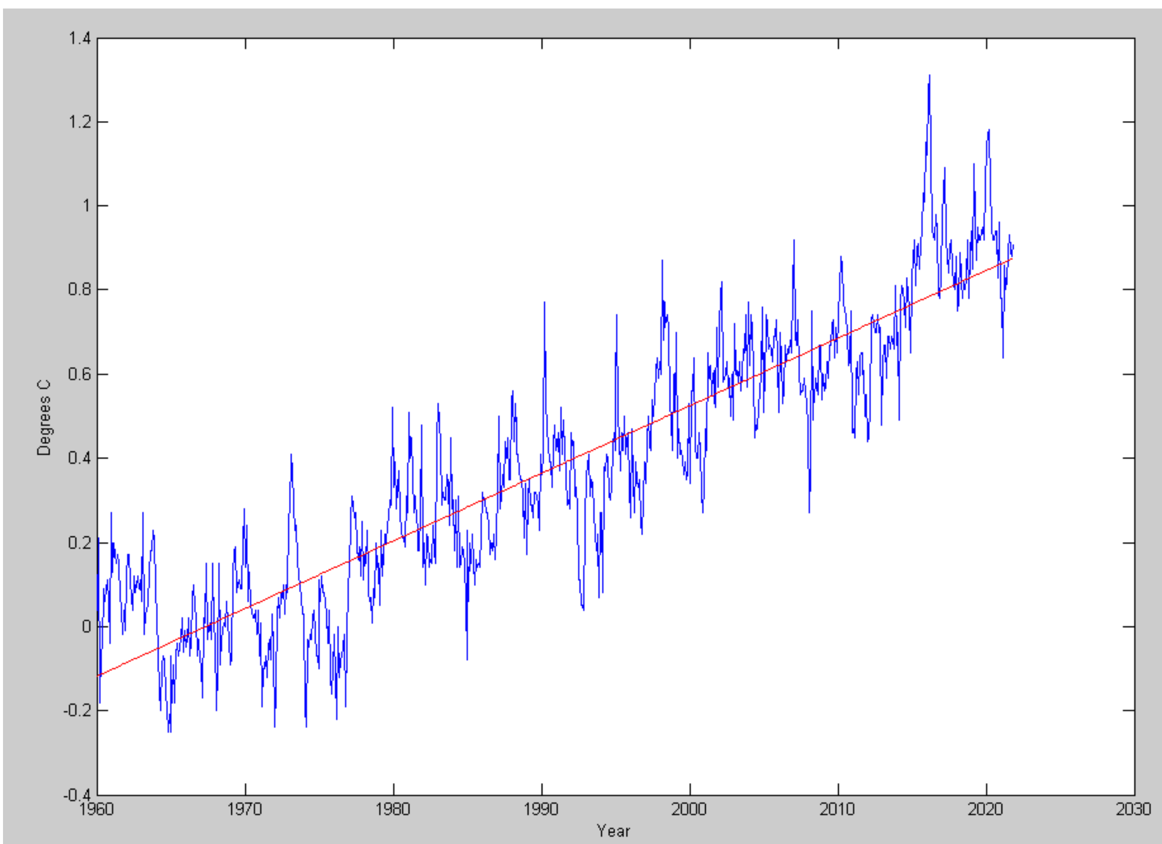
8a) Determine a least-squares curve fit for this data from the year 1960 - 2021 in the form of

$$\delta T = aT + b$$

Based upon this data, predict when we will see a 10 degree temperature increase if nothing changes.

Using a linear curve fit, the data suggests that we'll hit +10 degrees C in the year 2591

```
>> year = DATA(:,1);
>> dT = DATA(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*dT

    0.0161
  -31.6242

>> plot(year,dT,'b',year,B*A,'r');
>> xlabel('Year');
>> ylabel('Degrees C');
>> roots(A - [0;10])

  2589.4   year we hit +10 degrees C
```

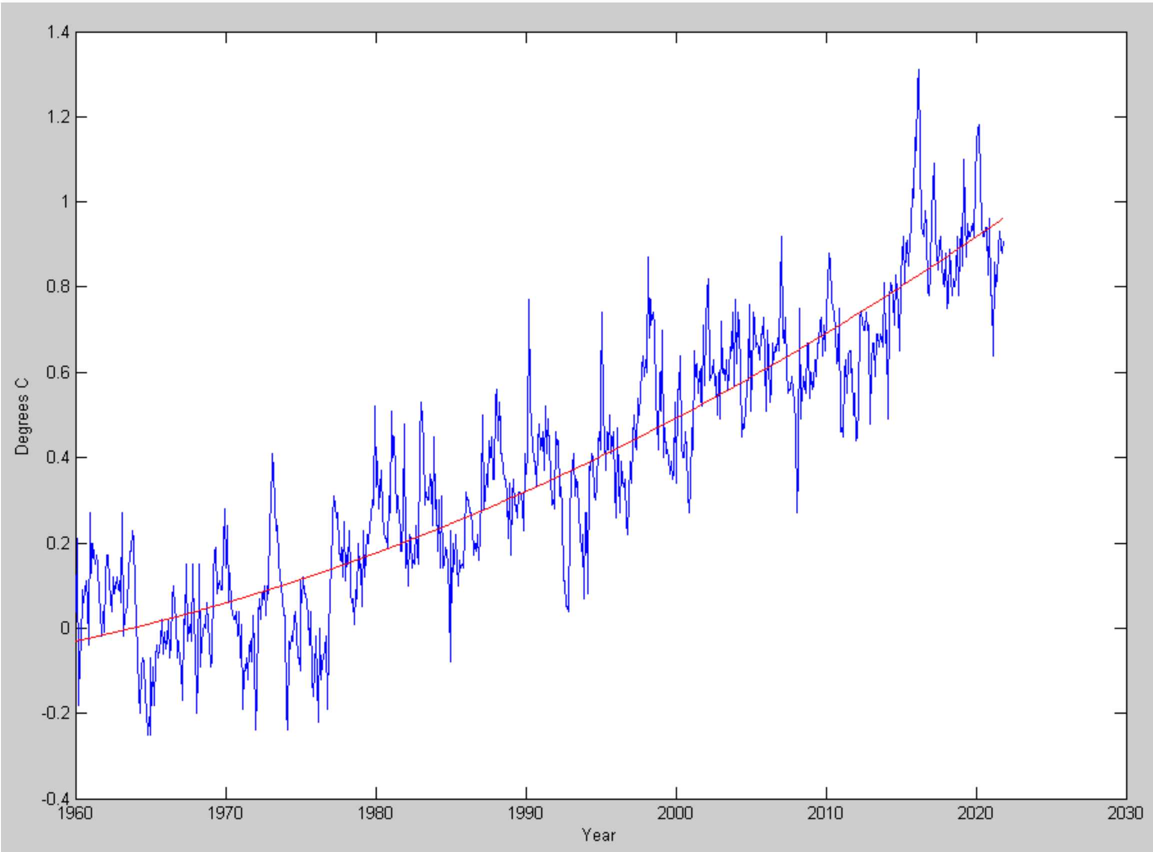


8b) Determine a least-squares cubic curve fit for this data from the year 1960 - 2021 in the form of

$$\delta T \approx ay^2 + by + c$$

Based upon this data, predict when we will see a 10 degree temperature increase if nothing changes.

Using a parabolic curve fit, the data suggests that we'll hit +10 degrees C in the year 2209 (187 years from now)




```
>> year = DATA(:,1);
>> dT = DATA(:,2);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*dT

    0.0001
   -0.5289
  510.8552

>> plot(year,dT,'b',year,B*A,'r');
>> xlabel('Year');
>> ylabel('Degrees C');
>> roots(A - [0;0;10])

    2204.4   year we hit +10 degrees C
    1660.1
```

9) What does a temperature rise of 10 degrees mean for the planet?

not graded - too political

The Permian Extinction Event suggests that it's not good: no animals larger than a mouse survived the Permian Extinction - which was triggered by CO2 levels at 2000ppm and a +10 degree C temerature rise.

One Degree: 2024 Summers like 2003 where a heat wave in France caused 10,000 deaths become the norm. Flows of the Po and Rhine river decrease. Crop production drops.

```
-->roots(A - [0;0;0;1])


     2024.3123
     1813.4666 + 128.13158i
     1813.4666 - 128.13158i
```

Two Degrees: 2056. Oceans absorb less CO2 (too hot) and soils start to release CO2. Vacations to the Mediterranean in the summer are just too hot. Crop failures in Africa and Central America cause mass migration. Coastal cities flood. 1/3rd of species face extinction.

```
-->roots(A - [0;0;0;2])

     2056.7453
     1797.25 + 155.06453i
     1797.25 - 155.06453i  roots(A - [0;0;2])
```

Three Degrees: 2080. Crop failures in China cause the migration of more than 1 billion people. Collapse of equatorial governments.

```
-->roots(A - [0;0;0;3])

     2080.2873
     1785.4791 + 174.84194i
     1785.4791 - 174.84194i
```

Four Degrees: 2099. Spain becomes a desert. Mass migration to Northern latitudes. Rain forests burn up.

```
-->roots(A - [0;0;0;4])

     2099.2884
     1775.9785 + 190.89831i
     1775.9785 - 190.89831i
```

Six Degrees: 2129. Ice caps are gone. Methane hydrates become unstable raising temperatures in a positive-feedback loop. Ocean circulation stops. Hydrogen sulfide producing bacteria flourish poisoning the air. The Ozone layer dissipates leaving the land sterilized with UV radiation. End-Permian-like conditions make life nearly impossible.

```
-->roots(A - [0;0;0;6])

     2129.6492
     1760.7981 + 216.6782i
     1760.7981 - 216.6782i
```

Scary? Yes. That's why the rest of the world sees the Paris Climate Accord as being important. That's why the United Nations sees Global Warming as the #1 threat - far greater than terrorism. Far greater than COVID.