ECE 111 - Homework #1

Week #1: Algebra. Due 8am Tuesday, January 18th
Please submit as a Word or pdf file and email to Jacob_Glower@yahoo.com with header ECE 111 HW#1

functions *poly* and *roots*:

1) Use MATLAB, find the roots the the following polynomials:

a) $x^3 + 9x^2 + 26x + 24 = 0$

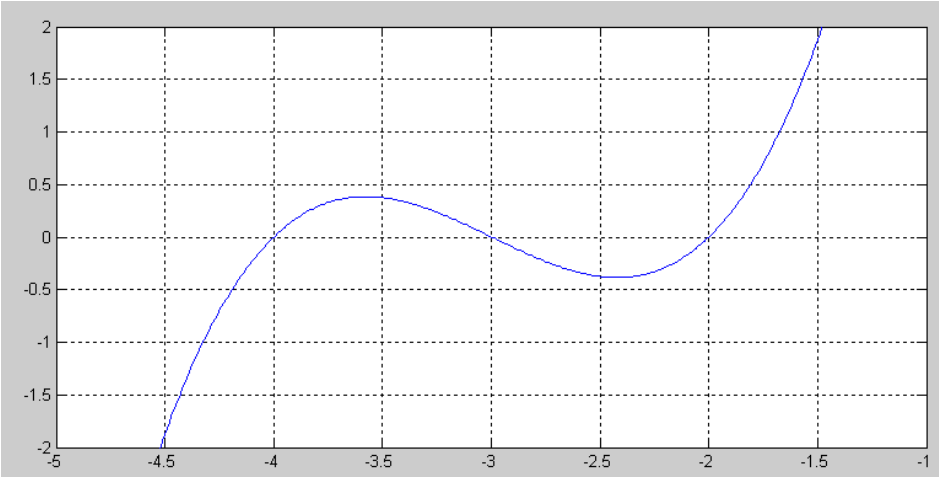
```
>> roots([1,9,26,24])

   -4.0000
   -3.0000
   -2.0000
```

You can also find these by plotting

$$y = x^3 + 9x^2 + 26x + 24$$

and finding the zero crossings. This has three real roots, meaning it crosses the X axis three times:

```
>> x = [-5:0.01:-1]';
>> y = x.^3 + 9*x.^2 + 26*x + 24;
>> plot(x,y)
>> grid
>> ylim([-2,2])
```



The zero crossings are the roots

b) $x^4 + 15x^3 + 80x^2 + 180x + 144 = 0$

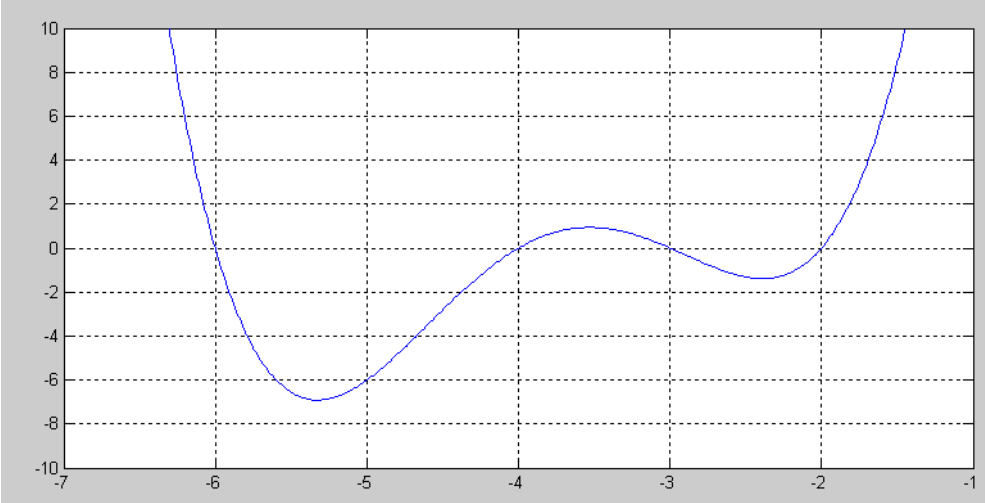
```
>> roots([1,15,80,180,144])

   -6.0000
   -4.0000
   -3.0000
   -2.0000
```

You can also plot

$$y = x^4 + 15x^3 + 80x^2 + 180x + 144$$

and find the zero crossings. This has four real roots, meaning it crosses the X axis for times:

```
>> x = [-7:0.01:-1]';
>> y = x.^4 + 15*x.^3 + 80*x.^2 + 180*x + 144;
>> plot(x,y)
>> grid
>> ylim([-10,10])
```

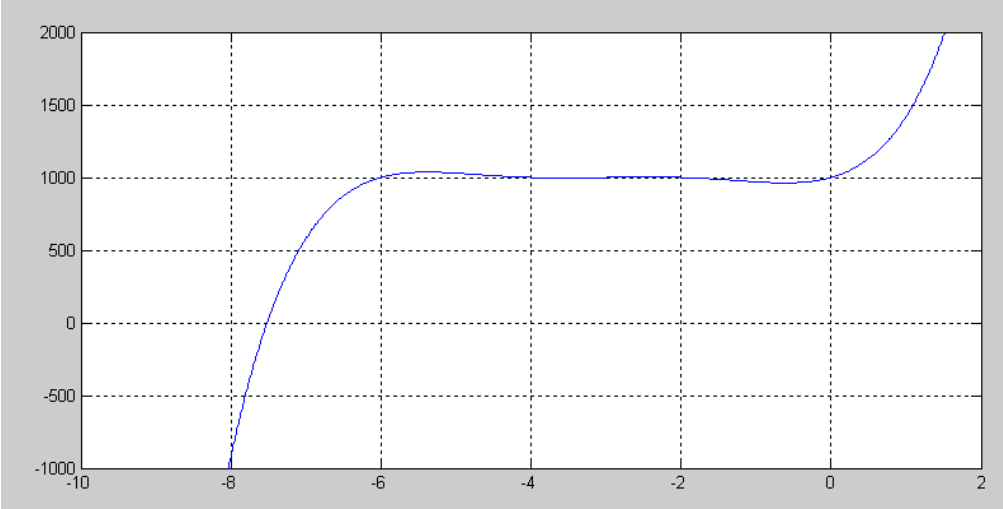


c) $x^5 + 15x^4 + 80x^3 + 180x^2 + 144x + 1000 = 0$

```
>> roots([1,15,80,180,144,1000])

 -7.5174
 -4.3579 + 3.3293i
 -4.3579 - 3.3293i
  0.6166 + 2.0107i
  0.6166 - 2.0107i
```

This only has one real root, meaning it only crosses the X axis one time

```
>> x = [-10:0.01:10]';
>> y = x.^5 + 15*x.^4 + 80*x.^3 + 180*x.^2 + 144*x + 1000;
>> plot(x,y)
>> grid
>> ylim([-1000,2000])
```



2) Use Matlab to multiply our the following polynomials.

a) $(x+5)(x+6)(x+7)(x+8)=0$

```
>> poly([-5,-6,-7,-8])

ans =
           1          26         251        1066        1680
```

meaning this is the same as

$$x^4+26x^3+251x^2+1066x+1680=0$$

b) $(x-1)(x+1)(x-3)(x+3)(x+4)(x+5)=0$

```
>> poly([1,-1,3,-1,-4,-5])

ans =

     1     7    -2   -74   -59    67    60
```

meaning this is the same as

$$x^6+7x^5-2x^4-74x^3-59x^2+67x+60=0$$

Graphing in Matlab

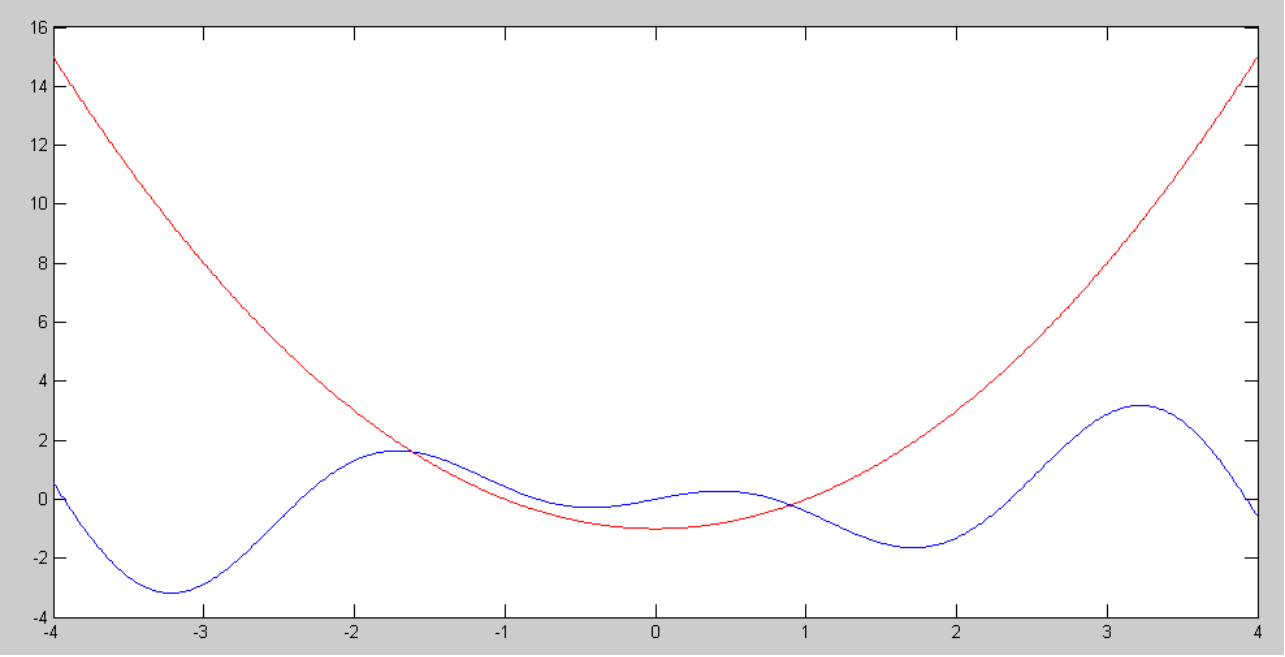
3) Plot the two functions in Matlab and determine all solutions in the range of -4 < x < +4

$$y = x \cdot \cos(2x)$$

$$y = x^2 - 1$$

There are two solutions (where the curves intersect)

- (-1.615, 1.61)
- (0.8961, -0.197)



4) Plot the two functions in Matlab and determine all solutions in the range of -4 < x < +4

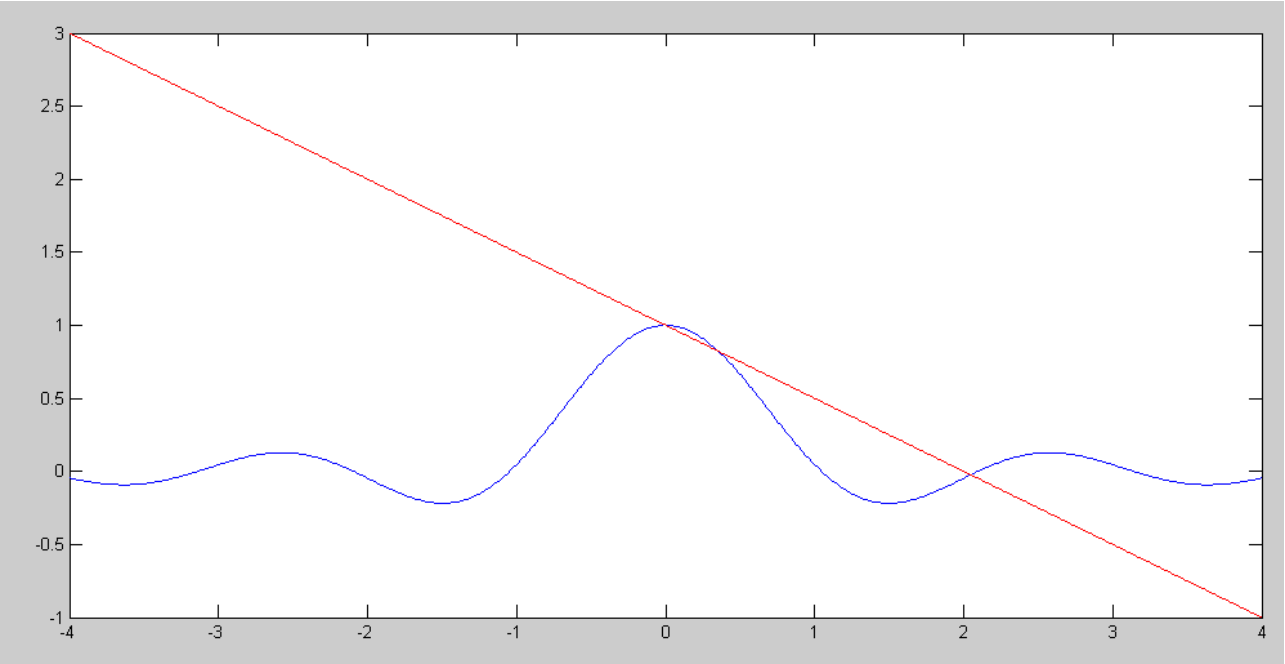
$$y = \left(\frac{\sin(3x)}{3x}\right)$$

$$y = 1 - \frac{x}{2}$$

```
>> x = [-4:0.01:4]';
>> y1 = sin(3*x) ./ (3*x);
>> y2 = 1 - x/2;
>> plot(x,y1,'b',x,y2,'r')
```

From the graph, there are three solutions (where the two curves intersect)

- (0, 1)
- (0.325, 0.824)
- (2.046, -0.023)



Monte-Carlo Simulations:

Two teams, A and B, are playing a game. Team A has a 70% chance of winning any given game.

5) For Loops: Suppose the two teams play a 9-game match. The match winner is whoever has 5 wins or more. Determine the probability that team A will win the match.

hint: use a for-loop (for i=1:9) and count how many times team A wins during the 9-game match).

Code:

```
% Best of 9 series
% Team A has a 70% chance of winning a given game

tic
WINS = 0;

for i=1:1e5
    A = 0;
    for j=1:9
        if(rand < 0.70) A = A + 1;
        end
    end
    if(A >= 5)
        WINS = WINS + 1;
    end
end

WINS/1e5

toc

ans =

    0.9001

Elapsed time is 0.112847 seconds.
```

Team A has a 90.01% chance of winning the match

6) While Loops: Suppose the two teams play until one team is up by 5 games. Determine the probability that team A will win the match.

hint: use a while-loop and keep looping until one team is up by 5 games.

```
% Win by 5 series
% Team A has a 70% chance of winning a given game

tic
WINS = 0;

for i=1:1e5
    A = 0;
    while(abs(A) < 5)
        if(rand < 0.7)
            A = A + 1;
        else
            A = A - 1;
        end
    end
    if(A >=5)
        WINS = WINS + 1;
    end
end

WINS/1e5

toc

ans =

    0.9856

Elapsed time is 0.094450 seconds.
```

Team A has a 98.56% chance of winning the match

7) Gauss' Dilema: Play the following game 100 times. (i.e. use Matlab and a for loop along with a while loop)

- It costs $25 to play. The pot starts at $1.
- Flip a coin. If you get a heads, the pot doubles. If you get a tails, the game is over and you collect the money in the pot.
- Keep flipping until you get a tails.

How much money do you expect to win (or lose) each time you play this game?

After 100 games, I'm down $1951. It isn't a great game to play....

```
% Gauss' Dilema

tic
BANK = 0;

for i=1:1e2
    POT = 1;
    while(rand < 0.5)
        POT = POT * 2;
        end
    BANK = BANK + POT - 25;
end

BANK

toc

BANK =

       -1951

Elapsed time is 0.000535 seconds.
```

Dice:

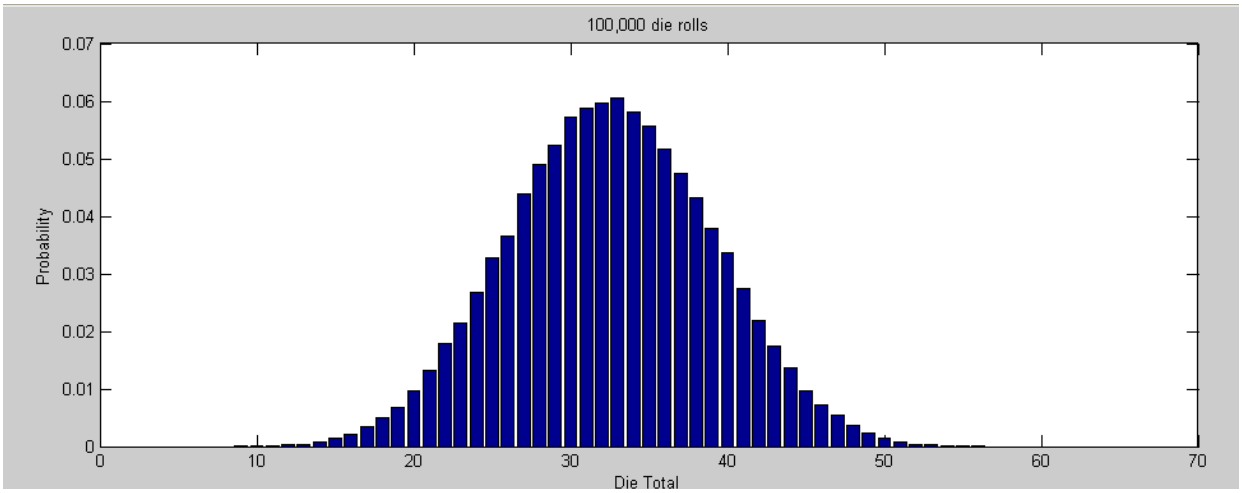
8a) Determine the probability distribution for the following:

- Roll three 6-sided dice and four 10-sided dice.
- The total is the sum of all of the dice.

  Y = 3d6 + 4d10

Code:

```
X = zeros(60,1);
for i=1:1e5
   D = sum( ceil( 6*rand(1,3) ) ) + sum( ceil( 10*rand(1,4)));
   X(D) = X(D) + 1;
   end
bar(X / 1e5)
xlabel('Die Total');
ylabel('Probability');
title('100,000 die rolls')
```



8b) What is the probability of the total being 30?

```
>> X(30) / 1e5

ans =    0.0572
```

There is a 5.72% chance of the total being 30

8c) What is the probability of the total being 30 or more?

```
>> sum(X(30:60))/1e5

ans =    0.6760
```

There is a 67.6% chance that the total will be 30 or more

9) Two people are playing a dice game:

- Player A rolls seven dice and takes the total (3d6 + 4d10)
- Player B rolls two 100-sided dice and takes the lower of the two numbers.
- Whoever has the highest score wins.

Determine the probability that

- A wins
- There is a tie, and
- B wins

- A wins 52.21% of the time
- It's a tie 1.40% of the time
- B wins 46.38% of the time

Code:

```
W = 0;
T = 0;
L = 0;

for i=1:1e5
   A = sum( ceil( 6*rand(1,3) ) ) + sum( ceil( 10*rand(1,4)));
   B = min( ceil(100*rand(1,2) ) );
   if (A > B)
       W = W + 1;
   elseif (A == B)
       T = T + 1;
   else
       L = L + 1;
   end
end

[W,T,L] / 1e5

ans =

    0.5221    0.0140    0.4638
```