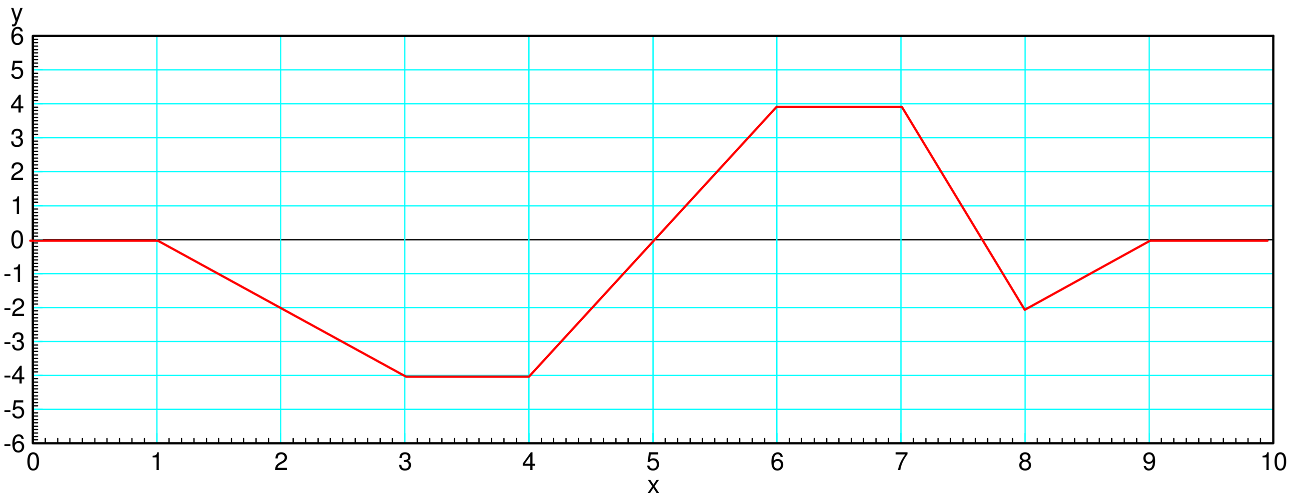
ECE 111 - Homework #6:

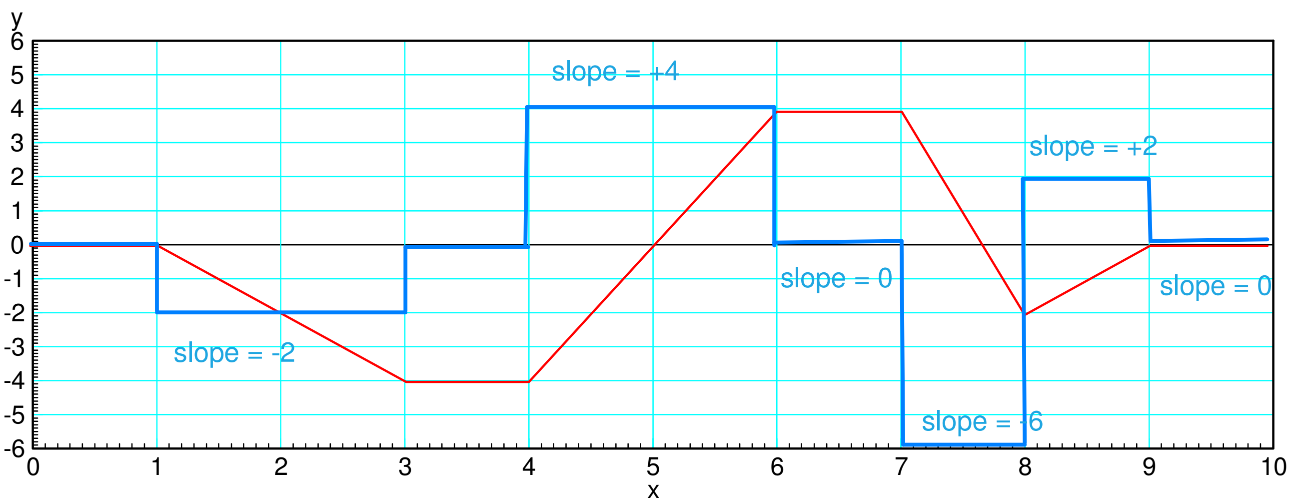
Math 165: Differentiation
Due Tuesday, October 3rd
Please submit via email, via hard copy, or on BlackBoard

1) Sketch the derivative of the following funciton

If this is the balance of your checking account, how much money are you adding (positive) or withdrawing (negative) for the balance to be as shown?



The slope is the change in y / change in x



Numerical Differentiation:

2) Use numerical methods to determine the derivative of y:

$$y = \left(\frac{\cos(x)}{x^2+0.5}\right)$$

$$z = \frac{d}{dx}(y)$$

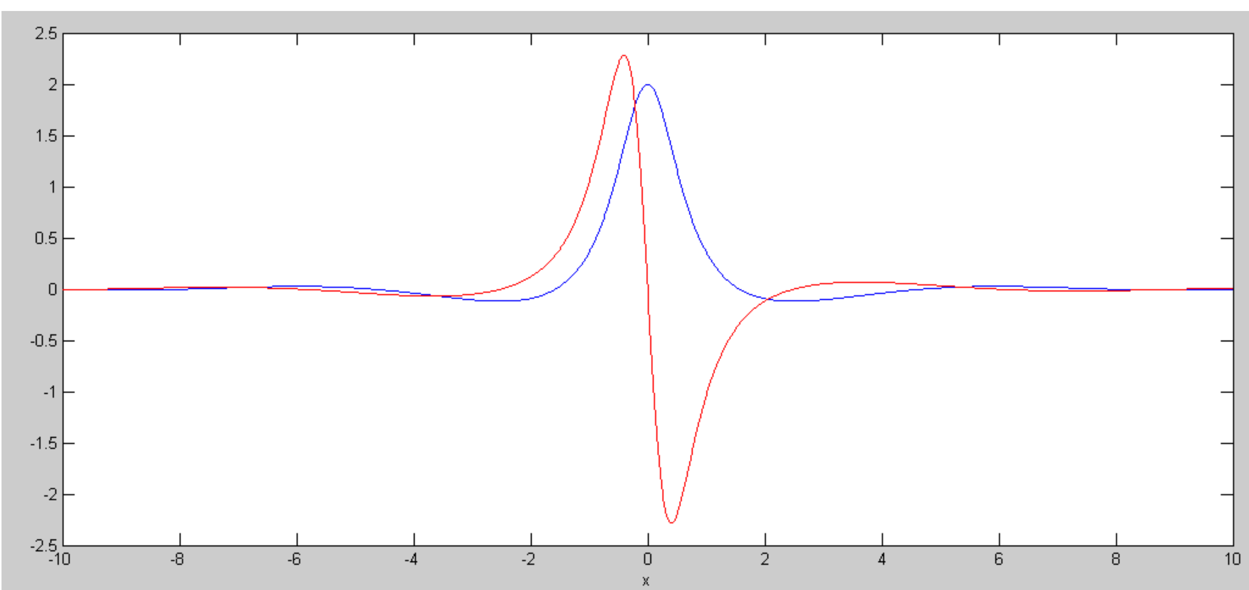
for -10 < x < 10. (a plot is sufficient).

Start with the a function, Derivative.m

```
% ECE 111 Lecture #6
% Compute the derivative of y(x)

function [dy] = Derivative(x, y)

   npt = length(x);

   dy = 0*x;

   for i=2:npt-1
      dy(i) = ( y(i+1)-y(i-1) ) / (x(i+1)-x(i-1));
      end

   dy(1) = (y(2)-y(1))/(x(2)-x(1));
   dy(npt) = (y(npt)-y(npt-1))/(x(npt)-x(npt-1));
   end
```

Plot y(x) and y'(x)

```
>> x = [-10:0.01:10]';
>> y = cos(x) ./ ( x.^2 + 0.5);
>> dy = Derivative(x,y);
>> plot(x,y,'b',x,dy,'r');
>> xlabel('x');
```



y(x) (bue) and y'(x) (red)

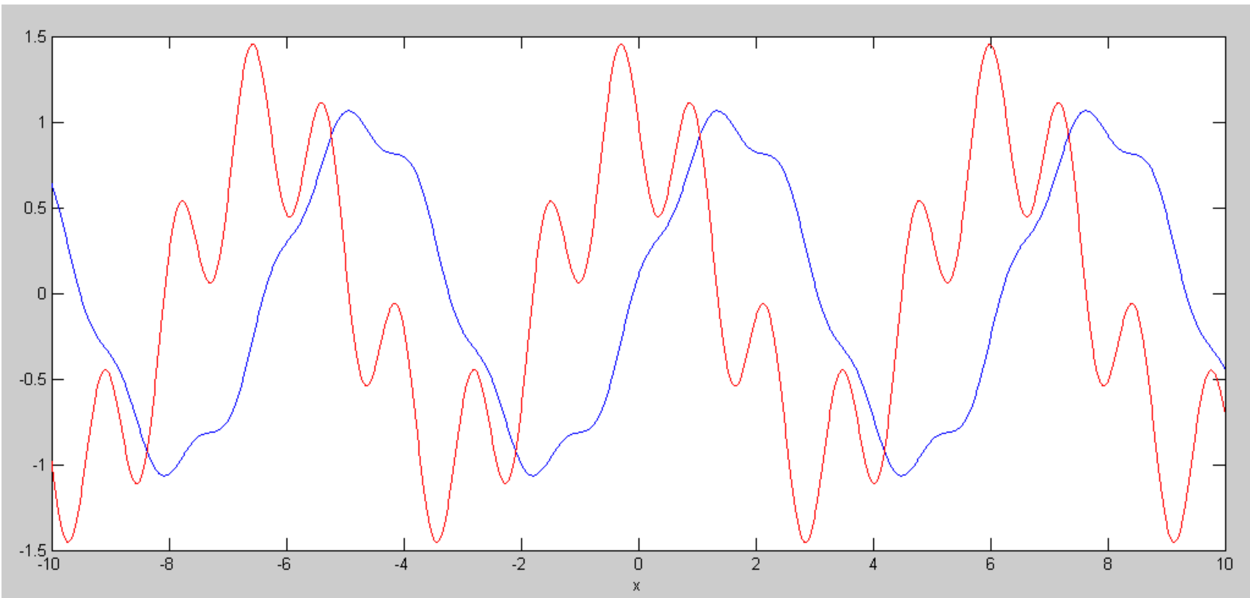
3) Use numerical methods to determine the derivative of y:

$$y = \sin(x) + 0.1\cos(5x)$$

$$z = \frac{d}{dx}(y)$$

for -10 < x < 10. (a plot is sufficient).

```
>> x = [-10:0.01:10]';
>> y = sin(x) + 0.1*cos(5*x);
>> dy = Derivative(x,y);
>> plot(x,y,'b',x,dy,'r');
>> xlabel('x');
>>
```



y(x) (blue) and y'(x) (red)
Note: Differentiation amplifies noise

Path Planning

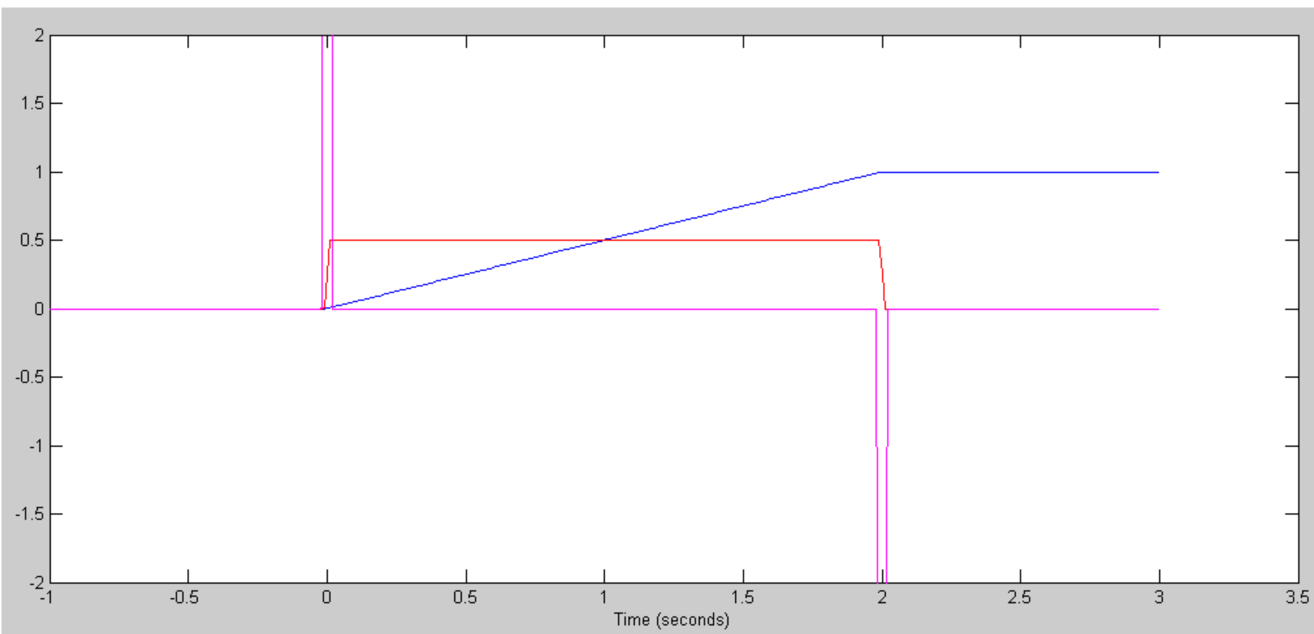
4) Assume a motor's angle is as follows:

$$\theta = \begin{cases} 0 & t < 0 \\ 0.5t & 0 < t < 2 \\ 1 & t > 2 \end{cases}$$

Calculate and plot using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

```
>> t = [-1:0.01:3]' + 1e-6;
>> q = 0.5*t .* (t>0) .* (t<2) + 1 * (t>2);
>> dq = Derivative(t,q);
>> ddq = Derivative(t,dq);
>> plot(t,q,'b',t,dq,'r',t,ddq,'m');
>> ylim([-2,2])
>> xlabel('Time (seconds)');
>>
```



Angle (blue), Velocity (red), and Acceleration (pink)

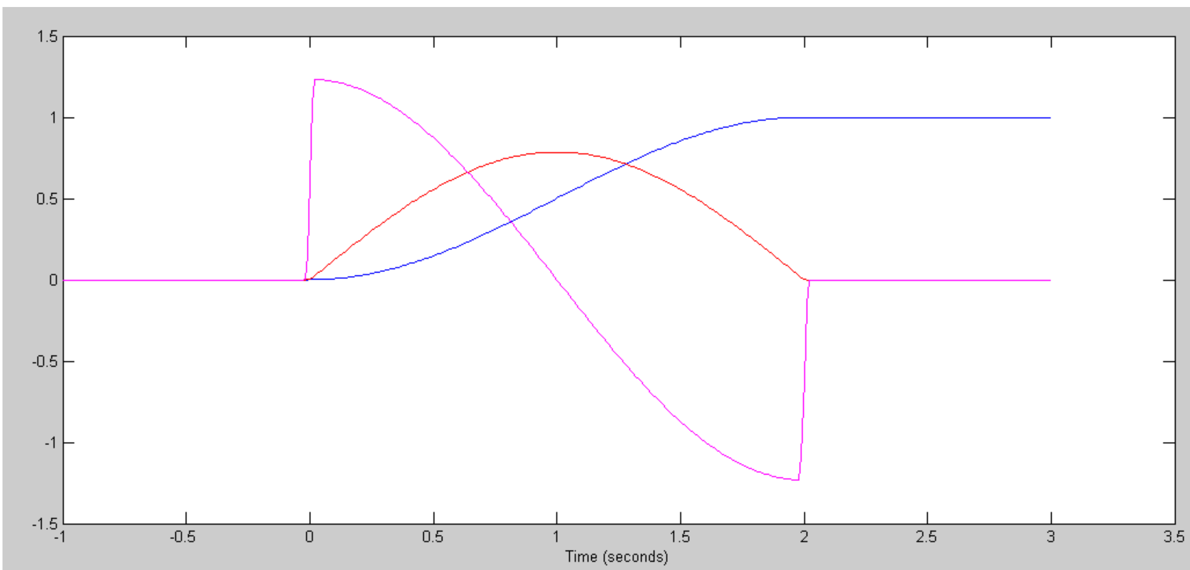
5) Assume a motor's angle is as follows:

$$\theta = \begin{cases} 0 & t < 0 \\ \frac{1}{2} - \frac{1}{2}\cos\left(\frac{1}{2}\pi t\right) & 0 < t < 2 \\ 1 & t > 2 \end{cases}$$

Calculate and plot using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

```
>> t = [-1:0.01:3]' + 1e-6;
>> q = (1 - cos(pi*t/2))/2 .* (t>0) .* (t<2) + 1 * (t>2);
>> dq = Derivative(t,q);
>> ddq = Derivative(t,dq);
>> plot(t,q,'b',t,dq,'r',t,ddq,'m');
>> xlabel('Time (seconds)');
>>
```



Angle (blue), Velocity (red), and Acceleration (pink)

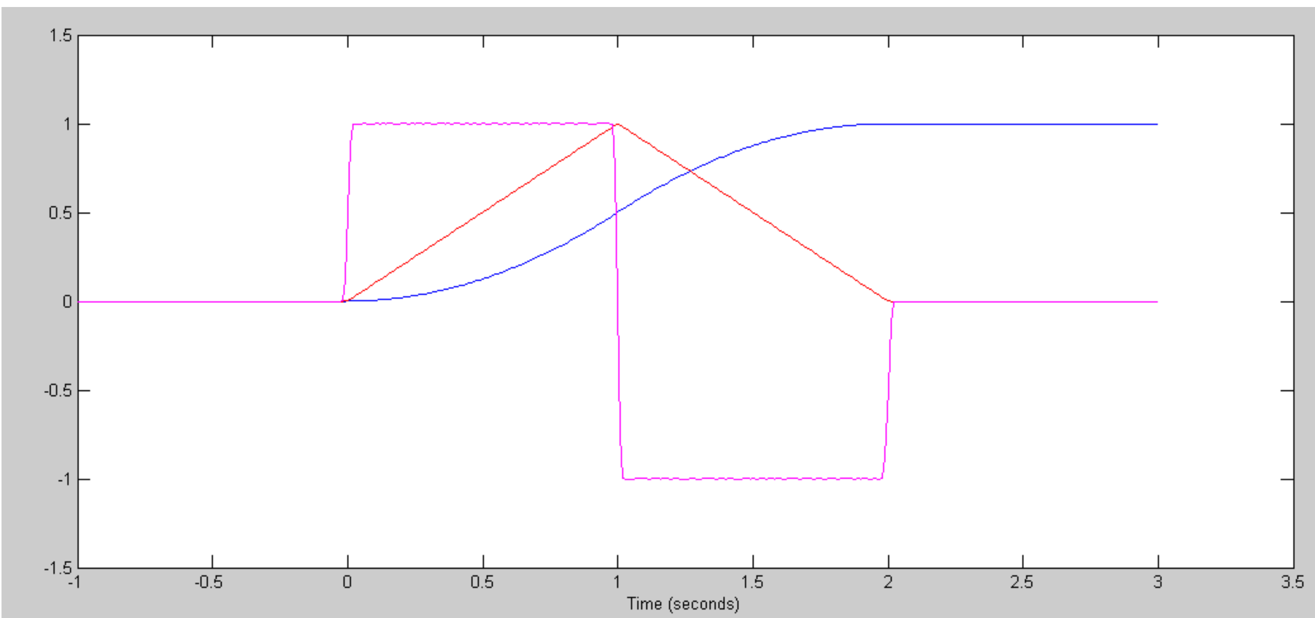
6) Assume a motor's angle is as follows:

$$\theta = \begin{cases} 0 & t < 0 \\ 0.5t^2 & 0 < t < 1 \\ 1 - 0.5(t-2)^2 & 1 < t < 2 \\ 1 & t > 2 \end{cases}$$

Calculate using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

```
>> t = [-1:0.01:3]' + 1e-6;
>> q = t.^2 / 2 .* (t>0) .* (t<1) + (1 - 0.5*(t-2).^2) .* (t>1) .* (t<2) + 1*(t>2);
>> dq = Derivative(t,q);
>> ddq = Derivative(t,dq);
>> plot(t,q,'b',t,dq,'r',t,ddq,'m');
>> xlabel('Time (seconds)');
>>
```



Angle (blue), Velocity (red), and Acceleration (pink)