## ECE 111: Homework 15

Week \#15 - ECE 331 Energy Conversion. Due 11am, Tuesday December 6th

1) Determine the circuit model for a $9.6 \mathrm{kV}: 240 \mathrm{~V}$ transformer is tested with the following test results:


Transformer Model

|  | V | Power | pf |
| :---: | :---: | :---: | :---: |
| Open-Circuit Test | $\mathrm{V} 1=13.2 \mathrm{kV}$ | 75 W | 0.02 |
| Short-Circuit Test | $\mathrm{V} 2=24 \mathrm{~V}$ | 12 W | 0.98 |

## Open Circuit Test

The current is from

$$
\begin{aligned}
& P=V \cdot I \cdot p f \\
& 75 W=13.2 k V \cdot I \cdot 0.02
\end{aligned}
$$

$$
I=0.2841 A
$$

The impedance in polar form is

$$
\begin{aligned}
& Z=\left(\frac{V}{I}\right) \angle \arccos (p f) \\
& Z=\left(\frac{13,200 V}{0.2841 A}\right) \angle \arccos (0.02) \\
& Z=46,464 \angle 88.854^{0}
\end{aligned}
$$

or in rectangular form

$$
Z=929.2865+j 46,454.7 \Omega
$$

This is the series model. To get the parallel model, take the inverse

$$
\frac{1}{Z}=\frac{1}{R}+\frac{1}{j X}=4.3044 e-7-j 2.1518 e-5
$$

The real part is $1 / \mathrm{R}$, the complex part is $1 / \mathrm{jX}$

$$
\begin{aligned}
& R_{c}=2.323 \mathrm{M} \Omega \\
& j X_{c}=j 46.47 \mathrm{k} \Omega
\end{aligned}
$$

## Short Circuit Test

Again

$$
\begin{aligned}
& P=V \cdot I \cdot p f \\
& 12 W=24 V \cdot I \cdot 0.98 \\
& I=0.5102 A
\end{aligned}
$$

The impedance is

$$
\begin{aligned}
& Z=\left(\frac{V}{I}\right) \angle \arccos (p f) \\
& Z=\left(\frac{24 V}{0.5102 A}\right) \angle \arccos (0.98) \\
& Z=47.04 \angle 11.4783^{\circ} \Omega
\end{aligned}
$$

or in rectalgular form

$$
Z=46.0992+j 9.3608
$$

This is the series model

$$
\begin{aligned}
& R_{L}=46.0992 \Omega \\
& j X_{L}=j 9.3608 \Omega
\end{aligned}
$$

For the utility grid on the back of the page....
2) Convert the voltages and impeances to the 120 V node (right side)

Impedances:

- Go through the transformer as the turns-ratio squared


## Votlages:

- Go through the transformer as the turns-ratio

$$
128 \mathrm{kV} \gg 120 \mathrm{~V}
$$

In Matlab

```
v0 = 128000 * (120/128000)
R01 = 428 * (120/128000)^2
R1 = 3.3e6 * (120/128000)^2
X1 = j*1e6 * (120/128000)^2
R12 = 2.14 * (120/13200)^2
x2 = j*10e3 * (120/13200)^2
R2 = 43e3 * (120/13200)^2
R23 = 0.0214
R3 = 1.44
```


## Matlab Results

```
V0 = 120
R01 = 3.7617e-004
R1 = 2.9004
X1 = 0 + 0.8789i
R12 = 1.7686e-004
X2 = 0 + 0.8264i
R2 = 3.5537
R23 = 0.0214
R3 = 1.4400
```


3) Determine the voltages at each node


Writing the voltage node equations

$$
\begin{aligned}
& V_{0}=120 \\
& \left(\frac{V_{1}-V_{0}}{R_{01}}\right)+\left(\frac{V_{1}}{R_{1}}\right)+\left(\frac{V_{1}}{j X_{1}}\right)+\left(\frac{V_{1}-V_{2}}{R_{12}}\right)=0 \\
& \left(\frac{V_{2}-V_{1}}{R_{12}}\right)+\left(\frac{V_{2}}{R_{2}}\right)+\left(\frac{V_{2}}{j X_{2}}\right)+\left(\frac{V_{2}-V_{3}}{R_{23}}\right)=0 \\
& \left(\frac{V_{3}-V_{2}}{R_{23}}\right)+\left(\frac{V_{3}}{R_{3}}\right)=0
\end{aligned}
$$

Place in matrix form

$$
\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
\left(\frac{-1}{R_{01}}\right) \\
\left(\begin{array}{cc}
\left(\frac{1}{R_{01}}+\frac{1}{R_{1}}+\frac{1}{j X_{1}}+\frac{1}{R_{12}}\right) & \left(\frac{-1}{R_{12}}\right)
\end{array}\right. \\
\left(\frac{-1}{R_{12}}\right) & \left(\frac{1}{R_{12}}+\frac{1}{R_{2}}+\frac{1}{j X_{2}}+\frac{1}{R_{23}}\right) & \left(\frac{-1}{R_{23}}\right) \\
\left(\frac{-1}{R_{23}}\right) & \left(\frac{1}{R_{23}}+\frac{1}{R_{3}}\right)
\end{array}\right]\left[\begin{array}{c}
V_{0} \\
V_{1} \\
V_{2} \\
V_{3}
\end{array}\right]=\left[\begin{array}{c}
120 \\
0 \\
0 \\
0
\end{array}\right]
$$

Solve in Matlab

```
\(\mathrm{aO}=[1,0,0,0] ;\)
\(a 1=[-1 / R 01,1 / R 01+1 / R 1+1 / X 1+1 / R 12,-1 / R 12,0] ;\)
\(a 2=[0,-1 / R 12,1 / R 12+1 / R 2+1 / X 2+1 / R 23,-1 / R 23] ;\)
a3 \(=[0,0,-1 / R 23,1 / R 23+1 / R 3]\);
\(A=[a 0 ; a 1 ; a 2 ; a 3]\)
\(B=[120 ; 0 ; 0 ; 0]\)
\(\mathrm{V}=\operatorname{inv}(\mathrm{A}) * \mathrm{~B}\)
```

vo $120.00-0.00 i$
V1 $119.94+0.11 i$
V2 $119.92+0.13 i$
v3 $118.16+0.13 i$
abs (V)
V0 120.0000
V1 119.9408
V2 119.9203
V3 118.1643

4) Determine the efficiency of this system

- Ignoring the core losses
- Assumes a large number of customers share these losses
- Including the core losses
- Assumes a single customer

In Matlab

```
V1 = V(1);
V2 = V(2);
V3 = V (3);
P01 = ( abs(V0 - V1) )^2 / R01
P1 = abs(V1)^2 / R1
P12 = ( abs(V1 - V2) )^2 / R12
P2 = abs(V2)^2 / R2
P23 = ( abs(V2 - V3) )^2 / R23
P3 = abs(V3)^2 / R3
eff1 = P3 / (P3 + P1 + P2 + P01 + P12 + P23)
eff2 = P3 / (P3 + P01 + P12 + P23)
```


## Result

```
P01 = 39.1127
P1 = 4960.0
P12 = 6.0956
P2 = 4046.7
P23 = 144.0991
P3 = 9696.4
eff1 = 0.5132 Efficiency including all losses
eff2 = 0.9809 Efficiency ignoring core losses
```




