ECE 111 - Homework #11

Week #11 - ECE 343 Signals- Due Tuesday, November 8th

Problem 1-5) Let x(t) be a function which is periodic in 2π

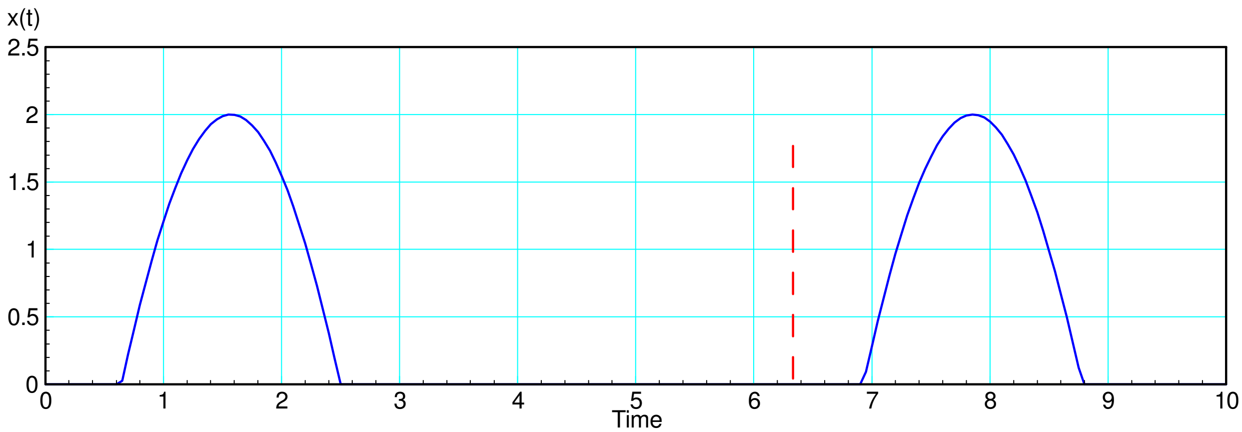
$$x(t) = x(t + 2\pi)$$

Over the interval $(0, 2\pi)$ x(t) is

$$x(t) = \max(0, 5\sin(t) - 3)$$

or in Matlab:

```
t = [0:0.001:2*pi]';
x = max(0, 5*sin(t) - 3);
plot(t,x)
```



x(t) Note that x(t) repeats repeats every 2π seconds

Curve Fitting with a power series:

1) Using least squares, approximate x(t) over the interval (0, 2π) as

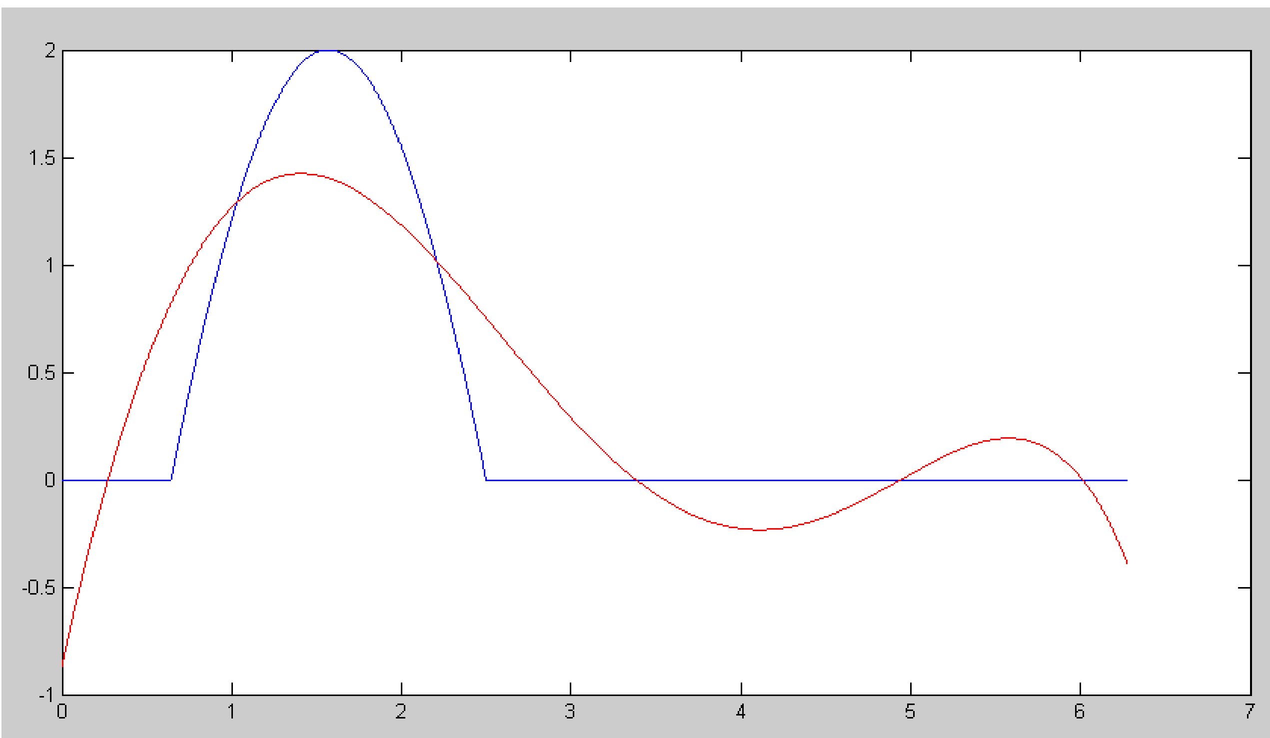
$$x(t) \approx a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

Plot x(t) along with it's approximation.

```
>> t = [0:0.001:2*pi]';
x = max(0, 5*sin(t) - 3);
plot(t,x)
>> B = [t.^0,t,t.^2,t.^3,t.^4,t.^5];
>> A = inv(B'*B)*B'*x

a0 -8.6820e-001
a1  3.6333e+000
a2 -1.6422e+000
a3  1.0495e-001
a4  4.2435e-002
a5 -5.0751e-003

>> plot(t,x,'b',t,B*A,'r')
```



x(t) (blue) and curve fit (red)

Note:

- It's not a great curve-fit: more terms would help
- The results doesn't help in finding y(t). x(t) isn't in the form of sinusoids.

Curve Fitting using a Fourier Series

2) Using least squares, approximate x(t) over the interval (0, 2π) as

$$x(t) = a_0 + a_1\cos(t) + b_1\sin(t) + a_2\cos(2t) + b_2\sin(2t) + a_3\cos(3t) + b_3\sin(3t)$$

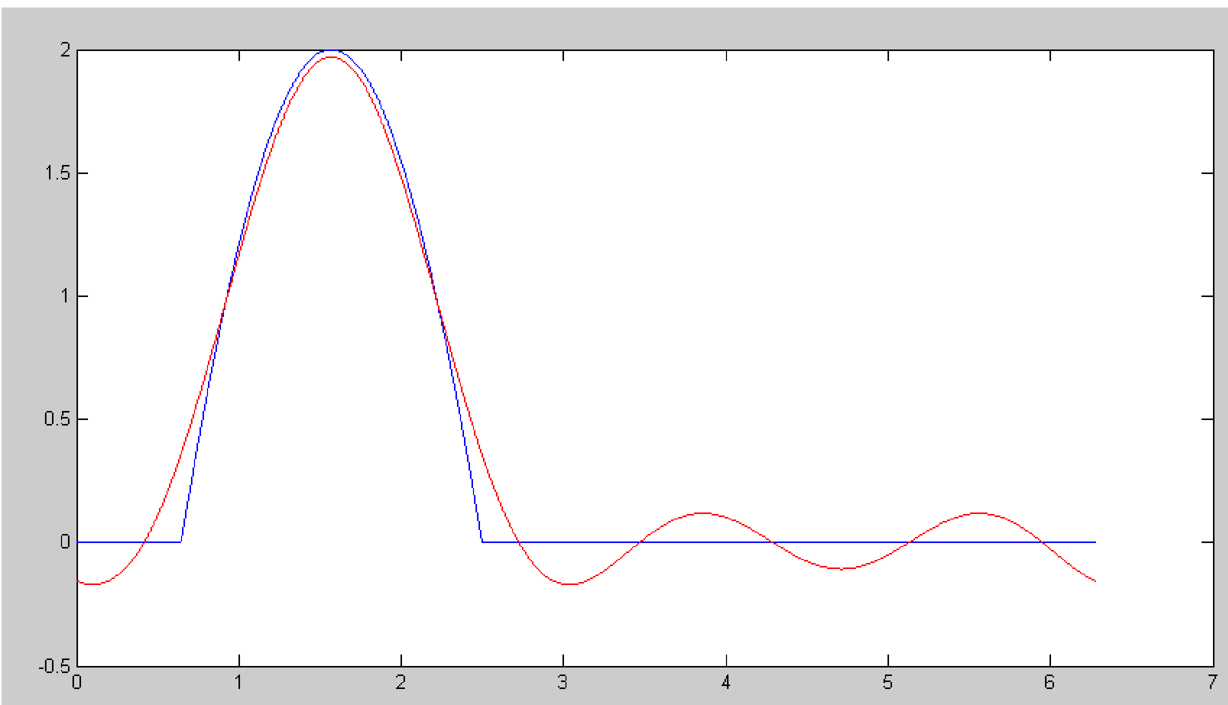
Plot x(t) along with it's approximation.

```
>> B = [t.^0,cos(t),sin(t),cos(2*t),sin(2*t),cos(3*t),sin(3*t)];
>> A = inv(B'*B)*B'*x

A =

a0   3.8776e-001
a1   4.0370e-005
b1   7.1189e-001
a2  -5.4321e-001
b2   9.4496e-008
a3   4.0247e-005
b3  -3.2595e-001

>> plot(t,x,'b',t,B*A,'r')
>>
```



Note:

- This is a better approximation
- More terms would improve the result
- The result *is* useful: x(t) is now in terms of sine waves

Superposition

3) Assume X and Y are related by

$$Y = \left(\frac{0.5}{s^2+s+0.5} \right) X$$

3a) Determine x(t) in terms of its Fourier Transform out to 3 rad/sec

```
>> a0 = mean(x)

a0 =  3.8769e-001

>> a1 = 2*mean(x .* cos(t))

a1 =  8.4944e-008

>> b1 = 2*mean(x .* sin(t))

b1 =  7.1180e-001

>> a2 = 2*mean(x .* cos(2*t))

a2 = -5.4318e-001

>> b2 = 2*mean(x .* sin(2*t))

b2 =  1.0195e-007

>> a3 = 2*mean(x .* cos(3*t))

a3 = -3.7419e-008

>> b3 = 2*mean(x .* sin(3*t))

b3 = -3.2591e-001
```

Note the the results are the same as we got using least squares

Fourier Transform is nothing more than a least-squares curve fit with the basis consisting of sinusoids

	Least Squres	Fourier Coeff
a0	0.38776	0.38769
a1	0.00004	0
b1	0.71189	0.71180
a2	-0.54321	-0.54318
b2	0	0
a3	0.00004	0
b3	-0.32595	-32591

3b) Plot x(t) and its Fourier approximation taken out to 3 rad/sec

It's the same result as problem #2 - the graph will also be the same

4) Determine the gain of this filter at each frequency present in problem #2 (i.e. 0, 1, 2, 3 rad/sec)

- *note: You should get a complex number for the gain at each frequency*

5a) Determine the phasor representation for Y(jw) at each frequency.

- *note: You should get a complex number for Y - the phasor representation for y(t) at 0, 1, 2, and 3 rad/sec*

Use superpositon to find Y(jw) at each frequency

DC

```
>> s = 0;
>> G0 = 0.5 / (s^2 + s + 0.5)

G0 =     1

>> x0 = a0

x0 =  3.8769e-001

>> y0 = G0 * x0

y0 =  3.8769e-001
```

meaning

$$y_0 = 0.38769$$

1 rads/sec

```
>> s = j;
>> x1 = a1 - j*b1

x1 =  8.4944e-008 -7.1180e-001i

>> G1 = 0.5 / (s^2 + s + 0.5)

G1 = -2.0000e-001 -4.0000e-001i

>> y1 = G1 * x1

y1 = -2.8472e-001 +1.4236e-001i
```

meaning

$$y_1 = -0.28472\cos(t) - 0.14236\sin(t)$$

2 rad/sec

```
>> s = j*2;
>> x2 = a2 - j*b2

x2 = -5.4318e-001 -1.0195e-007i

>> G2 = 0.5 / (s^2 + s + 0.5)

G2 = -1.0769e-001 -6.1538e-002i

>> y2 = G2 * x2

y2 =  5.8496e-002 +3.3426e-002i
```

meaning

$$y_2 = 0.058496\cos(2t) - 0.033426\sin(2t)$$

3 rad/sec

```
>> x3 = a3 - j*b3

x3 = -3.7419e-008 +3.2591e-001i

>> s = j*3;
>> G3 = 0.5 / (s^2 + s + 0.5)

G3 = -5.2308e-002 -1.8462e-002i

>> y3 = G3 * x3

y3 =  6.0167e-003 -1.7047e-002i
```

meaning

$$y_3(t) = 0.0060167\cos(3t) + 0.017047\sin(3t)$$

5b) From this, determine y(t)

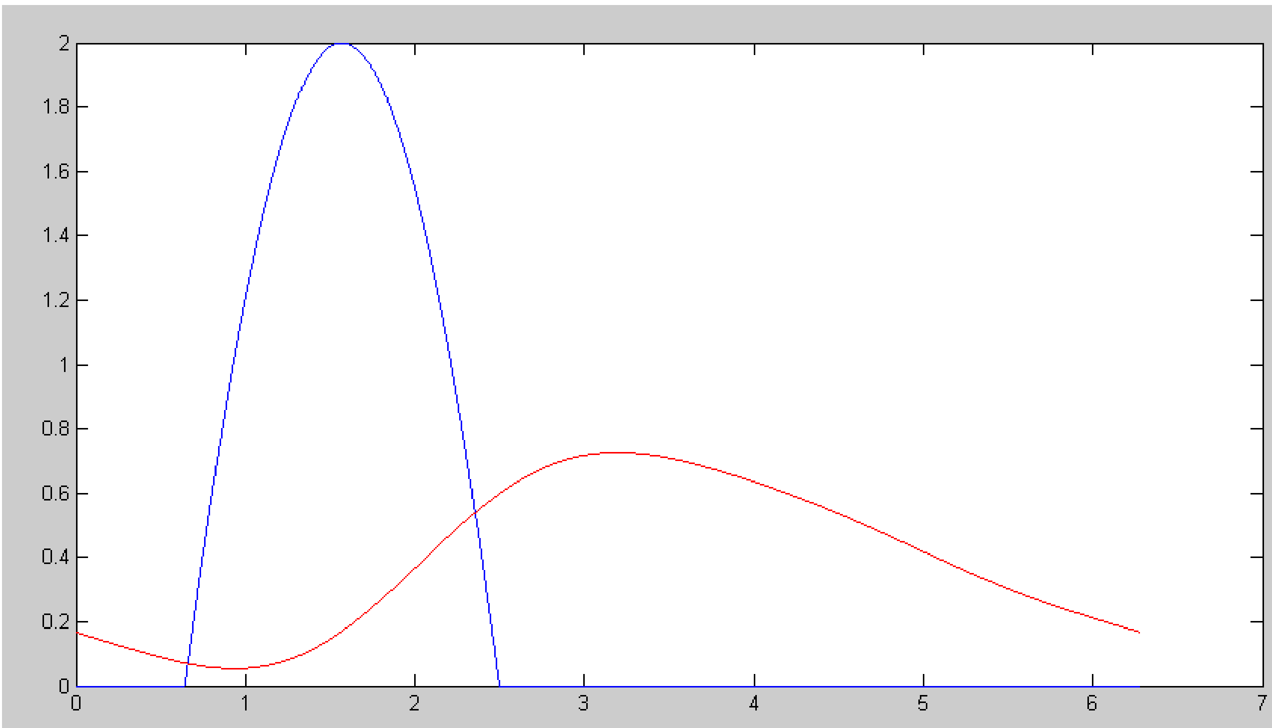
The total answer is the sum of all of the harmonics

$$y(t) = y_0 + y_1 + y_2 + y_3$$

$$\begin{aligned} y(t) = & \quad 0.38769 \\ & -0.28472\cos(t) - 0.14236\sin(t) \\ & +0.058496\cos(2t) - 0.033426\sin(2t) \\ & +0.0060167\cos(3t) + 0.017047\sin(3t) \end{aligned}$$

6) Plot x(t) and y(t).

```
>> y = y0;
>> y = y + real(y1)*cos(t) - imag(y1)*sin(t);
>> y = y + real(y2)*cos(2*t) - imag(y2)*sin(2*t);
>> y = y + real(y3)*cos(3*t) - imag(y3)*sin(3*t);
>> plot(t,x,'b',t,y,'r');
```



x(t) (blue) and y(t) (red)

Note:

This isn't exact.

- y(t) contains harmonics out to infinity.
- We only went out to 3 rad/sec

It's close

- X(jw) gets smaller as frequency goes up
- G(jw) gets smaller as frequenc goes up
- Y(jw) = G(jw)*X(jw) goes to zero fairly quickly
- Ignoring the higher-frequency term doesn't affect y(t) very much