ECE 111 - Homework #10

ECE 343 Signals & Systems- Due Due 11am, Tuesday, November 1st

1) A filter has the following transfer function

$$Y = \left(\frac{2s+50}{s^2+4s+20}\right)X$$

1a) What is the differential equation relating X and Y?

Cross multiply

$$(s^2 + 4s + 20)Y = (2s + 5)X$$

note: sY means *the derivative of y*

$$y'' + 4y' + 20y = 2x' + 5x$$

or

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 20y = 2\frac{dx}{dt} + 5x$$

1b) Find y(t) assuming $x(t) = 5$

Express in phasor form

$$s = 0$$

$$X = 5$$

Evaluate at s = 0

$$Y = \left(\frac{2s+50}{s^2+4s+20}\right)X = \left(\frac{2s+50}{s^2+4s+20}\right)_{s=0} \cdot (5)$$

$$Y = 12.5$$

meaning

$$y(t) = 12.5$$

1c) Find y(t) assuming $x(t) = 5\sin(6t)$

Express in phasor form

$$s = j6$$

$$X = 0 - j5 \qquad \textit{real = cosine, \quad -imag = sine}$$

$$Y = \left(\frac{2s+50}{s^2+4s+20}\right) X = \left(\frac{2s+50}{s^2+4s+20}\right)_{s=j6} \cdot (0 - j5)$$

$$Y = -8.3654 + j3.0769$$

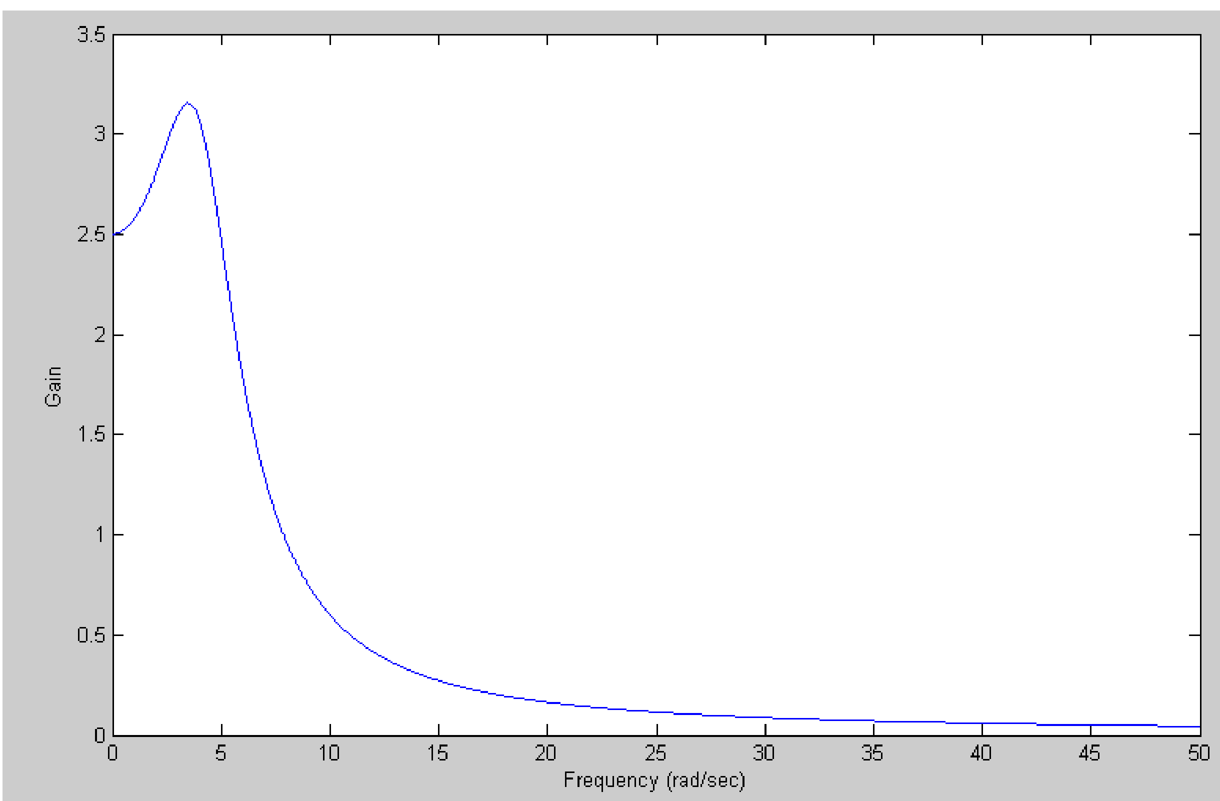
meaning

$$y(t) = -8.3654\cos(6t) - 3.0769\sin(6t)$$

2) Plot the gain vs. frequency for this filter from 0 to 50 rad/sec.

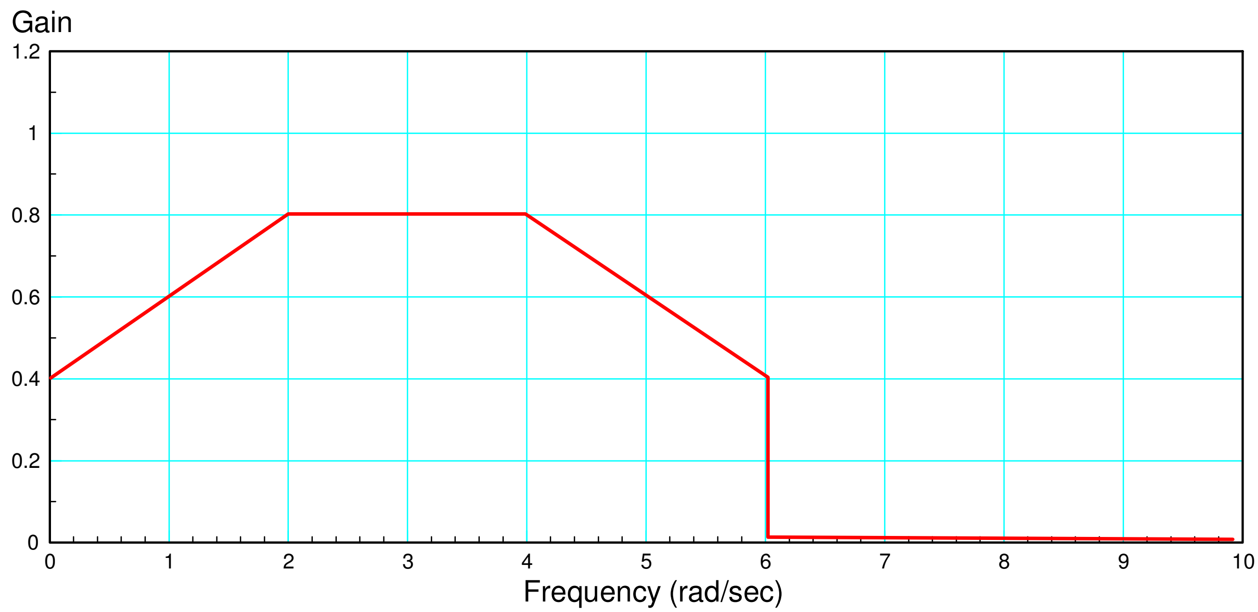
$$Y = \left(\frac{2s+50}{s^2+4s+20}\right) X$$

```
>> w = [0:0.01:50]';
>> s = j*w;
>> G = (2*s + 50) ./ (s.^2 + 4*s + 20);
>> plot(w,abs(G))
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
```



Problem 3-5) Design a filter of the following form so that the gain matches the graph below:

$$G(s) = \left(\frac{a}{\left(s^2+bs+c\right)\left(s^2+ds+e\right)\left(s^2+fs+g\right)} \right)$$



3) Write an m-file, cost.m, which

- Is passed an array, z, with each element representing (a, b, c, d, e, f, g)
- Computes the gain, G(s) for this value of (a, b, c, d, e, f, g)
- Computes the difference between the gain, G, and the target (above), and
- Returns the sum-squared error in the gain

```
function [ J ] = costF( z )
 a = z(1);
 b = z(2);
 c = z(3);
 d = z(4);
 e = z(5);
 f = z(6);
 g = z(7);

 w = [0:0.1:10]';
 s = j*w;
 Gideal = (0.2*w+0.4).*(w<=2) + 0.8*(w>2).*(w<=4) + (1.6 - 0.2*w).*(w>4).*(w<6);
 G = a ./ ( (s.^2 + b*s + c).*(s.^2 + d*s + e) .* (s.^2 + f*s + g));
 e = abs(Gideal) - abs(G);
 J = sum(e .^ 2);
 plot(w,abs(Gideal),'r',w,abs(G),'b');
 ylim([0,1.2]);
 pause(0.01);
end
```

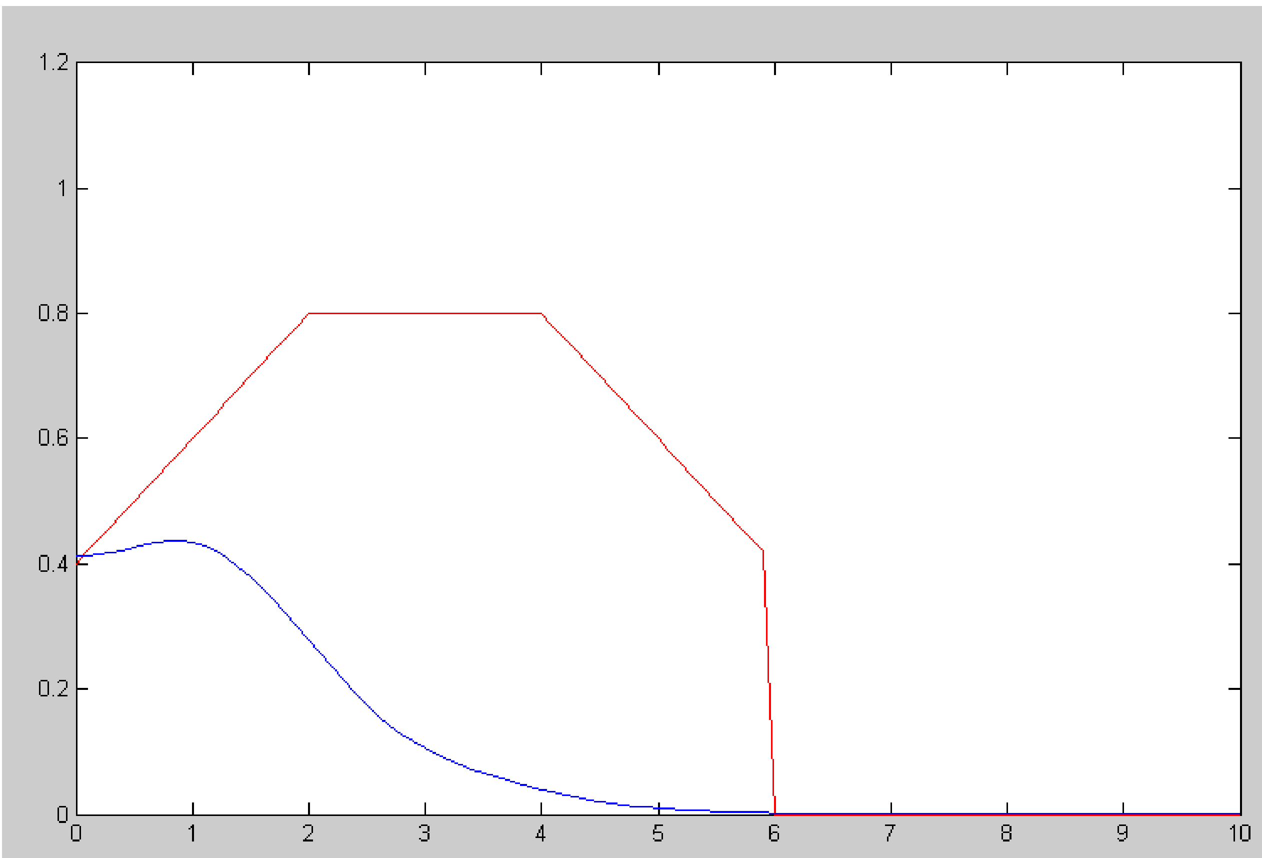
4) Use your m-file to determine how 'good' the following filter is:

$$G(s) = \left(\frac{a}{(s^2+bs+c)(s^2+ds+e)(s^2+fs+g)} \right) = \left(\frac{70}{(s^2+2s+2)(s^2+2s+5)(s^2+2s+17)} \right)$$

```
>> costF([70,2,2,2,5,2,17])

ans =

   17.5094
```



5) Use fminsearch() to find the 'best' filter of the form

$$G(s) = \left(\frac{a}{(s^2+bs+c)(s^2+ds+e)(s^2+fs+g)} \right)$$

5a) Give the resulting (a, b, c, d, e, f, g)

```
>> [Z,e] = fminsearch('costF',[70,2,2,2,5,2,17])

Z =

  1.0e+003 *

    2.0605    0.0036    0.0094    0.0033    0.0149    0.0011    0.0286

e =    0.3540

>> [Z,e] = fminsearch('costF',Z)

Z =

  1.0e+003 *

    1.0951    0.0020    0.0047    0.0017    0.0165    0.0008    0.0304


e =    0.2086
```

5b) Give the resulting filter, and

```
>> format short e
>> Z'

ans =

a  1.0951e+003
b  1.9547e+000
c  4.7101e+000
d  1.6712e+000
e  1.6491e+001
f  8.4123e-001
g  3.0376e+001
```

$$G(s) = \left(\frac{1095.1}{(s^2+1.9547s+4.7101)(s^2+1.6712s+16.491)(s^2+0.84123s+30.3076)} \right.$$

```
>> roots([1,Z(2),Z(3)])

 -9.7736e-001 +1.9377e+000i
 -9.7736e-001 -1.9377e+000i

>> roots([1,Z(4),Z(5)])

 -8.3558e-001 +3.9741e+000i
 -8.3558e-001 -3.9741e+000i

>> roots([1,Z(6),Z(7)])

 -4.2061e-001 +5.4954e+000i
 -4.2061e-001 -5.4954e+000i

>>
```

5c) Plot the 'optimal' filter's gain vs. frequency