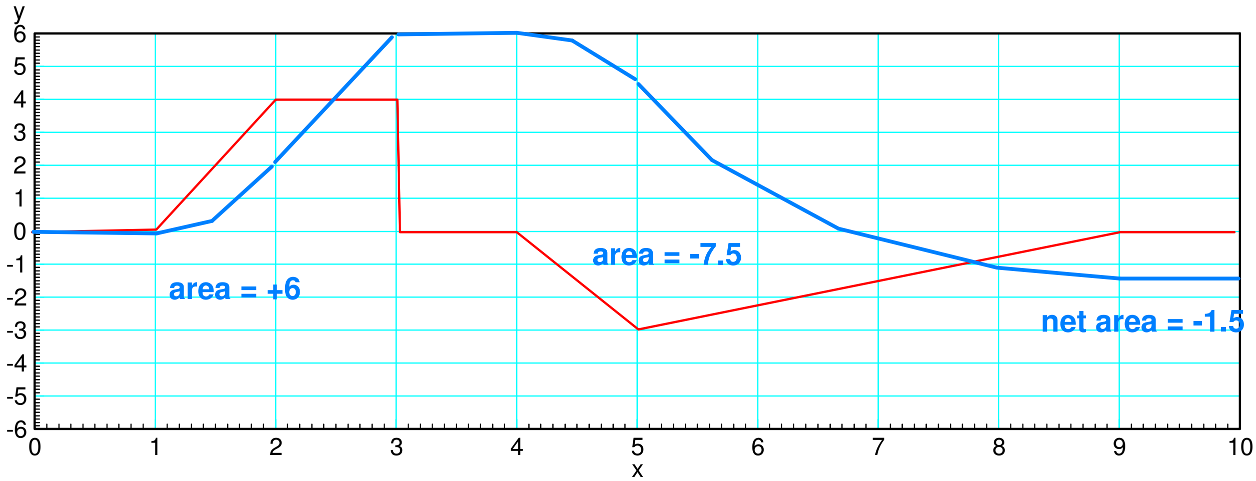
ECE 111 - Homework #7

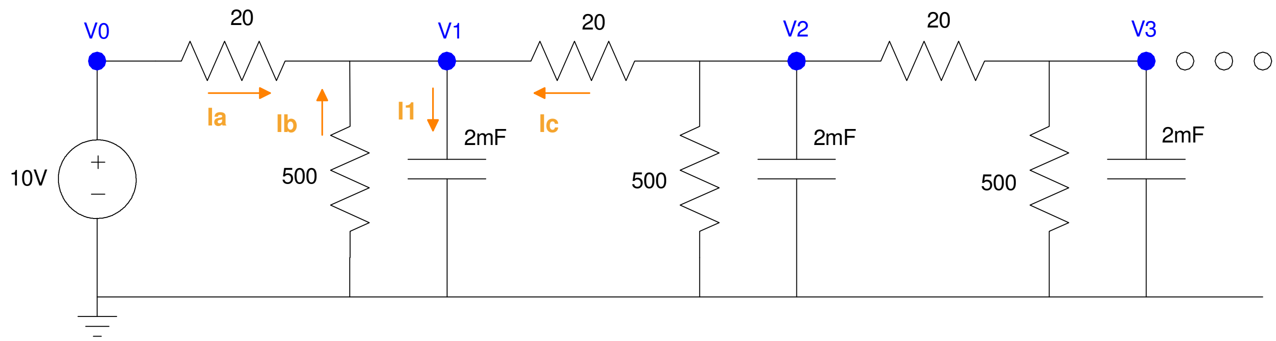
Week #7: ECE 311 Circuits II - - Due 11am Tuesday, October 4th

1) Assume the current flowing through a one Farad capacitor is shown below. Sketch the voltage. Assume V(0) = 0. The voltage is the integral of the current (capacitors are integrators)

$$V = \frac{1}{C}\int I \cdot dt$$



Problem 2-5: Assume a 10-stage RC filter (V0 .. V10)



Problem 2) Write the dynamics for this system as a set of ten coupled differential equations:

Start with node V1:

$$I_1 = C\frac{dV_1}{dt} = \sum(\text{current to node} V_1)$$

$$I_1 = C\frac{dV_1}{dt} = I_a + I_b + I_c$$

$$0.002\frac{dV_1}{dt} = \left(\frac{V_0 - V_1}{20}\right) + \left(\frac{0 - V_1}{500}\right) + \left(\frac{V_2 - V_1}{20}\right)$$

$$\frac{dV_1}{dt} = 25V_0 - 51V_1 + 25V_2$$

Node V2..V9 are the same (just shift the indicies).

$$\frac{dV_2}{dt} = 25V_1 - 51V_2 + 25V_3$$

$$\vdots$$

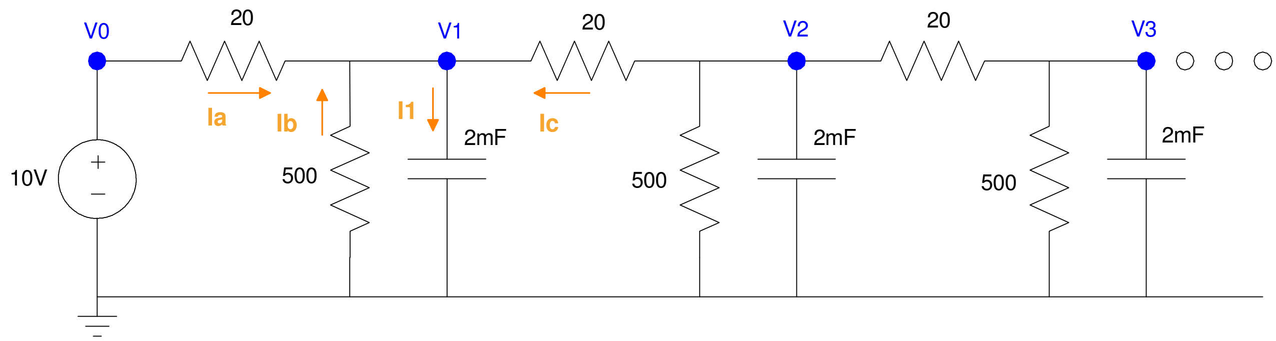
$$\frac{dV_9}{dt} = 25V_8 - 51V_9 + 25V_{10}$$

V10 is slightly different due to having only one neighbor

$$0.002\frac{dV_{10}}{dt} = \left(\frac{V_9 - V_{10}}{20}\right) + \left(\frac{0 - V_{10}}{500}\right)$$

$$\frac{dV_{10}}{dt} = 25V_9 - 26V_{10}$$

Forced Response for a 10-Node RC Filter (heat.m):

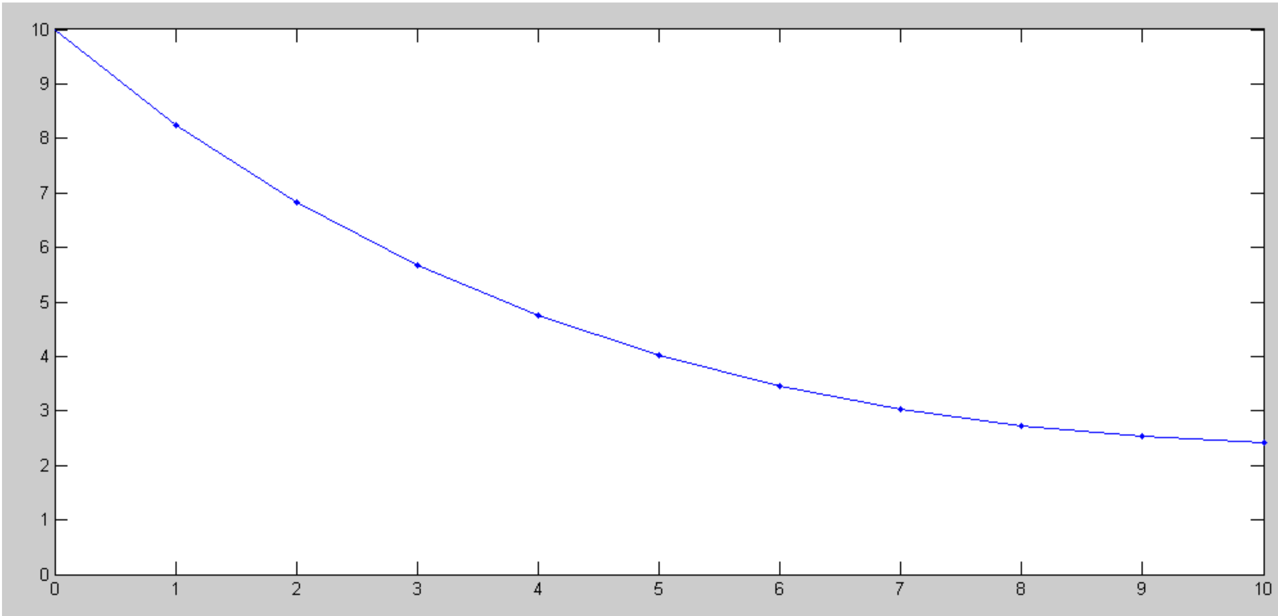


Problem 3) Using Matlab, solve these ten differential equations for 0 < t < 5 s assuming

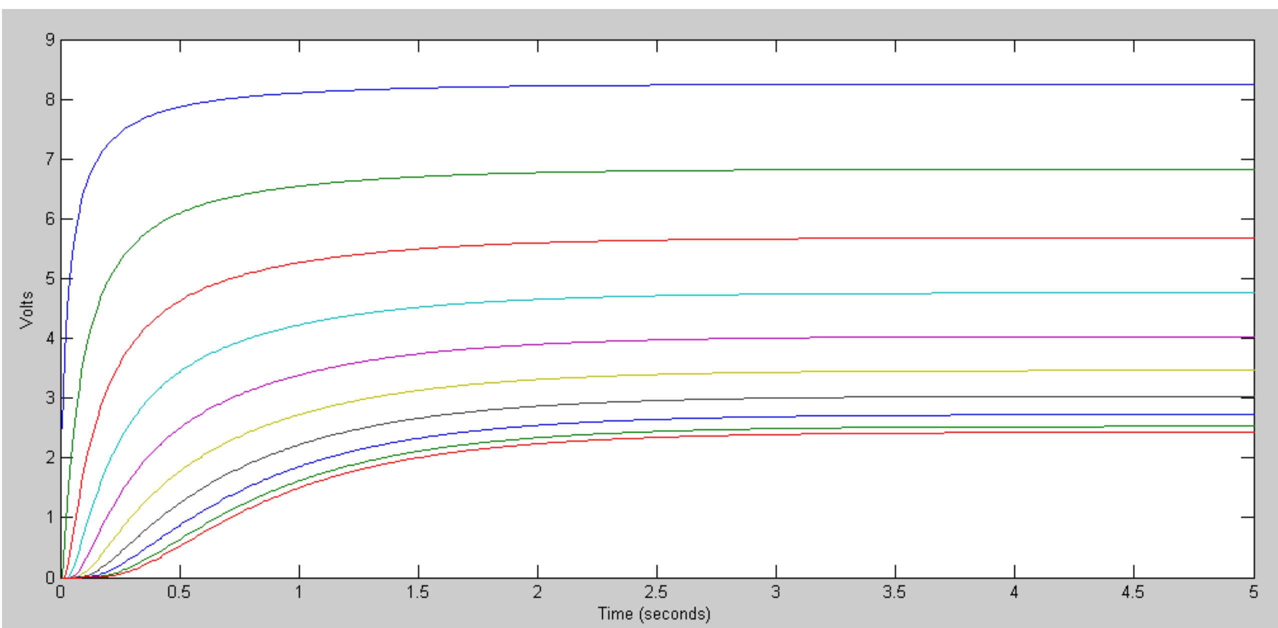
- The initial voltages are zero, and
- V0 = 10V.

Code:

```
% ECE 111 Homework #7
% 10-stage RC filter

V = zeros(10,1);
dV = zeros(10,1);
V0 = 10;
dt = 0.01;
t = 0;
y = [];
while(t < 5)
 dV(1) = 25*V0 - 51*V(1) + 25*V(2);
 dV(2) = 25*V(1) - 51*V(2) + 25*V(3);
 dV(3) = 25*V(2) - 51*V(3) + 25*V(4);
 dV(4) = 25*V(3) - 51*V(4) + 25*V(5);
 dV(5) = 25*V(4) - 51*V(5) + 25*V(6);
 dV(6) = 25*V(5) - 51*V(6) + 25*V(7);
 dV(7) = 25*V(6) - 51*V(7) + 25*V(8);
 dV(8) = 25*V(7) - 51*V(8) + 25*V(9);
 dV(9) = 25*V(8) - 51*V(9) + 25*V(10);
 dV(10) = 25*V(9) - 26*V(10);
 V = V + dV*dt;
 t = t + dt;
 plot([0:10], [V0;V], '.-');
 ylim([0,10]);
 pause(0.01);

 y = [y ; V'];
end
pause(3)

t = [1:length(y)]' * dt;
plot(t,y);
xlim([0,5]);
xlabel('Time (seconds)');
ylabel('Volts');
```



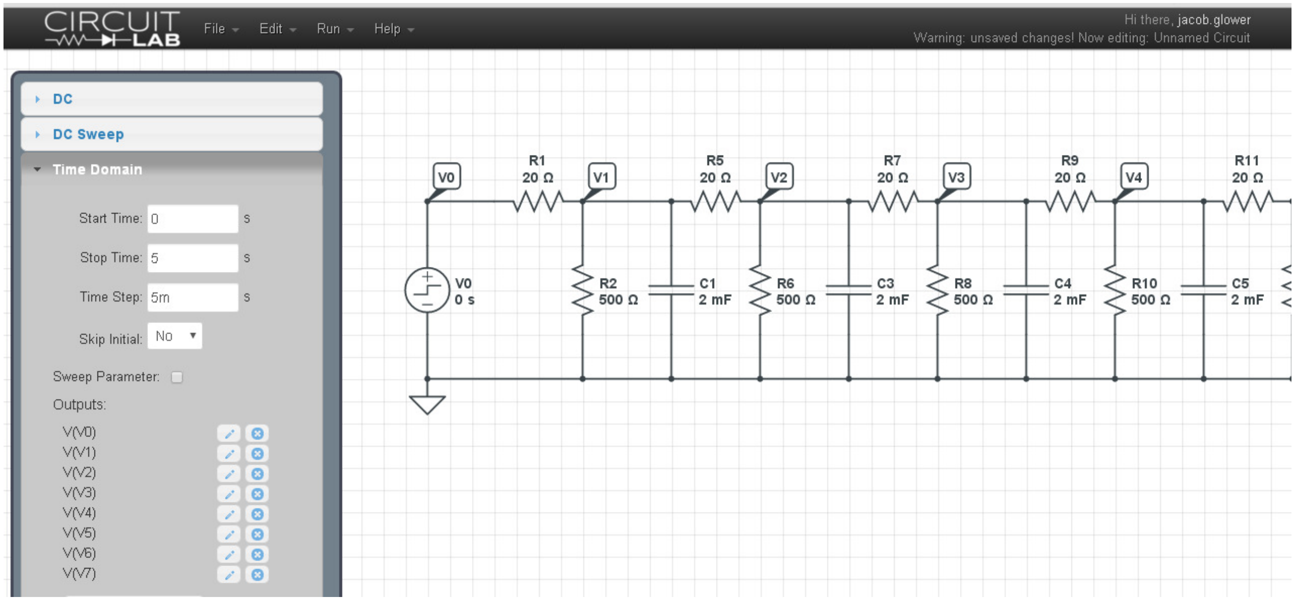
Node vs. Voltage at 5.00 seconds

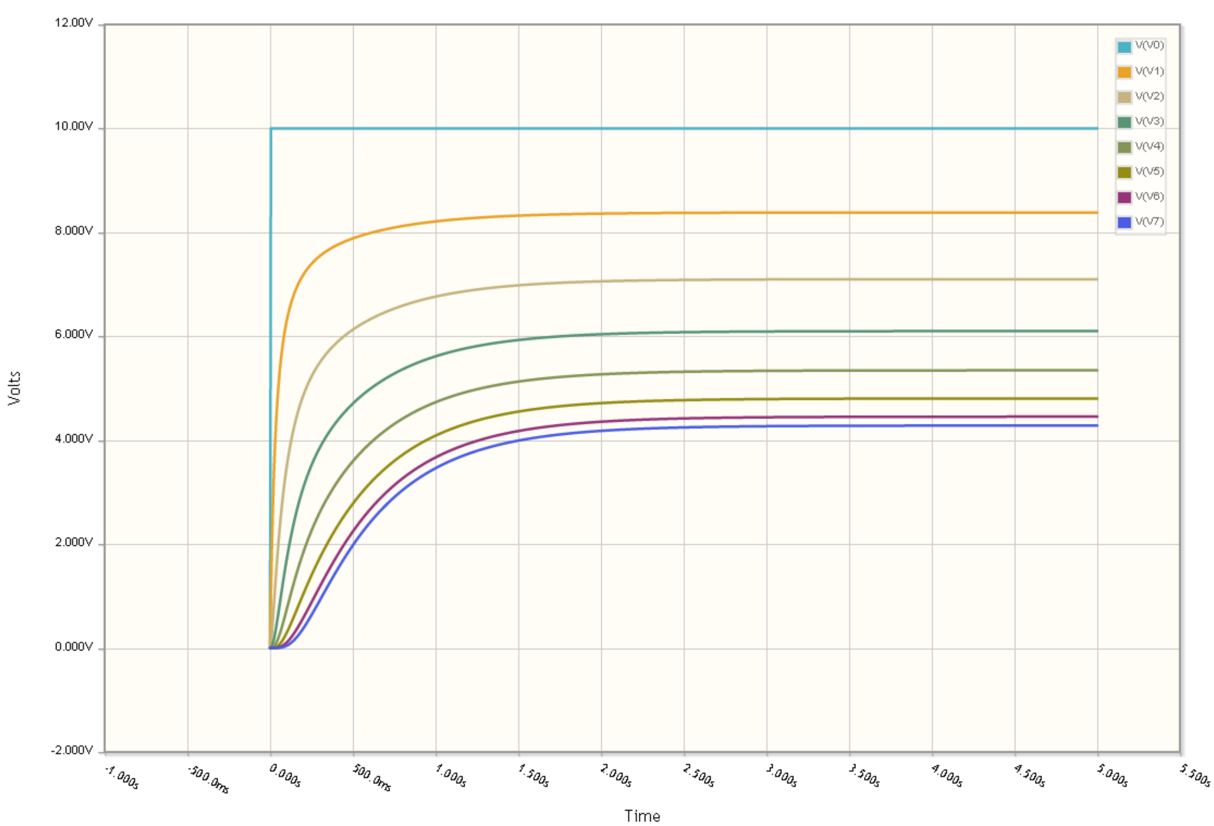


Voltages of each nove vs. time

Problem 4) Using CircuitLab, find the response of this circuit to a 10V step input. *note: It's OK if you only build this circuit to 3 nodes...*

Being someone who doesn't follow directions well, I went to seven nodes. The response matches Matlab





Natural Response

Problem 5) Assume V0 = 0V. Determine the initial conditions of V1..V10 so that

- The maximum voltage is 10V and
- 5a) The voltages go to zero as slow as possible
- 5b) The voltages go to zero as fast as possible.

Simulate the response for these initial conditions in Matlab.

This is an eigenvector problem.

```
>> A = zeros(10,10);
>> for i=1:9
A(i,i) = -51;
A(i+1,i) = 25;
A(i,i+1) = 25;
end
>> A(10,10) = -26;
>> A

A =

   -51    25     0     0     0     0     0     0     0     0
    25   -51    25     0     0     0     0     0     0     0
     0    25   -51    25     0     0     0     0     0     0
     0     0    25   -51    25     0     0     0     0     0
     0     0     0    25   -51    25     0     0     0     0
     0     0     0     0    25   -51    25     0     0     0
     0     0     0     0     0    25   -51    25     0     0
     0     0     0     0     0     0    25   -51    25     0
     0     0     0     0     0     0     0    25   -51    25
     0     0     0     0     0     0     0     0    25   -26

>> [M,V] = eig(A)

Eigenvectors:

  -0.1286   -0.2459    0.3412    0.4063    0.4352    0.4255    0.3780    0.2969   -0.1894    0.0650
   0.2459    0.4063   -0.4255   -0.2969   -0.0650    0.1894    0.3780    0.4352   -0.3412    0.1286
  -0.3412   -0.4255    0.1894   -0.1894   -0.4255   -0.3412   -0.0000    0.3412   -0.4255    0.1894
   0.4063    0.2969    0.1894    0.4352    0.1286   -0.3412   -0.3780    0.0650   -0.4255    0.2459
  -0.4352   -0.0650   -0.4255   -0.1286    0.4063    0.1894   -0.3780   -0.2459   -0.3412    0.2969
   0.4255   -0.1894    0.3412   -0.3412   -0.1894    0.4255    0.0000   -0.4255   -0.1894    0.3412
  -0.3780    0.3780   -0.0000    0.3780   -0.3780   -0.0000    0.3780   -0.3780   -0.0000    0.3780
   0.2969   -0.4352   -0.3412    0.0650    0.2459   -0.4255    0.3780   -0.1286    0.1894    0.4063
  -0.1894    0.3412    0.4255   -0.4255    0.3412   -0.1894   -0.0000    0.1894    0.3412    0.4255
   0.0650   -0.1286   -0.1894    0.2459   -0.2969    0.3412   -0.3780    0.4063    0.4255    0.4352

>> eig(A)'

Eigenvalues:

 -98.7786  -92.3119  -82.1745  -69.2671  -54.7365  -39.8740  -26.0000  -14.3474   -5.9516   -1.5585
```

The eigenvalues tell you *how* the sytem behaves

The eigenvector tells you *what* behaves that way.

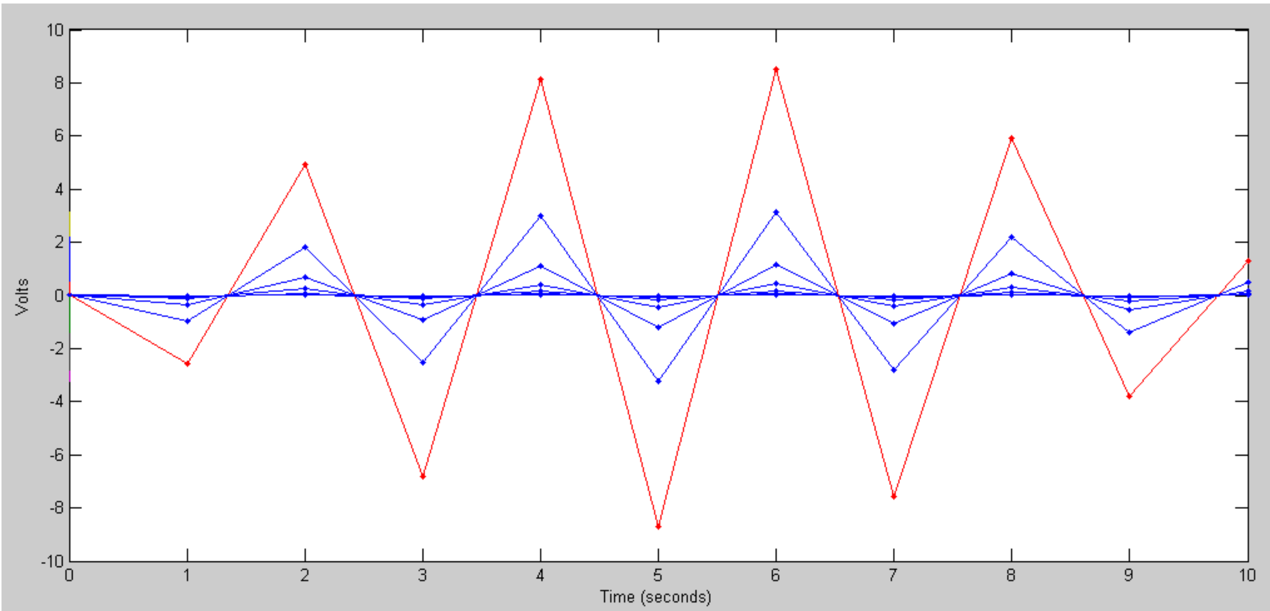
- The fast mode (blue) decays as exp(-98.77t)
- The slow mode (red) decays as exp(-1.55t)

Fast Mode: Simulating in Matlab, make the initial condition the fast mode:

```
>> X0 = M(:,1)*20

   -2.5728
    4.9171
   -6.8244
    8.1253
   -8.7043
    8.5099
   -7.5593
    5.9370
   -3.7872
    1.3009
```

Modfying the code:

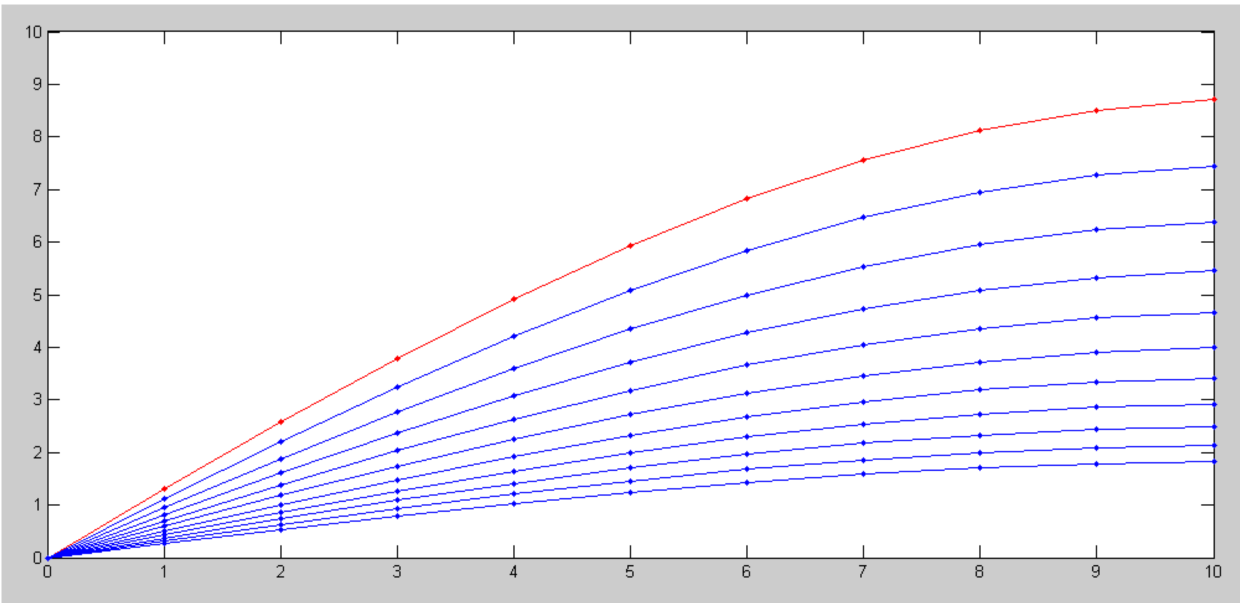
```
% ECE 111 Homework #7
V = M(:,1) * 20;
dV = zeros(10,1);
V0 = 0;
dt = 0.001;
t = 0;
y = [];
while(t < 0.1)
 dV(1) = 25*V0 - 51*V(1) + 25*V(2);
 dV(2) = 25*V(1) - 51*V(2) + 25*V(3);
 dV(3) = 25*V(2) - 51*V(3) + 25*V(4);
```



Fast Mode decays as exp(-98.77t)
The shape (eigenvector) stays the same
The amplitude drops as per the eigenvalue

Slow Mode: Decays as exp(-1.55t)

```
>> X0 = M(:,10) * 20

    1.3009
    2.5728
    3.7872
    4.9171
    5.9370
    6.8244
    7.5593
    8.1253
    8.5099
    8.7043
```



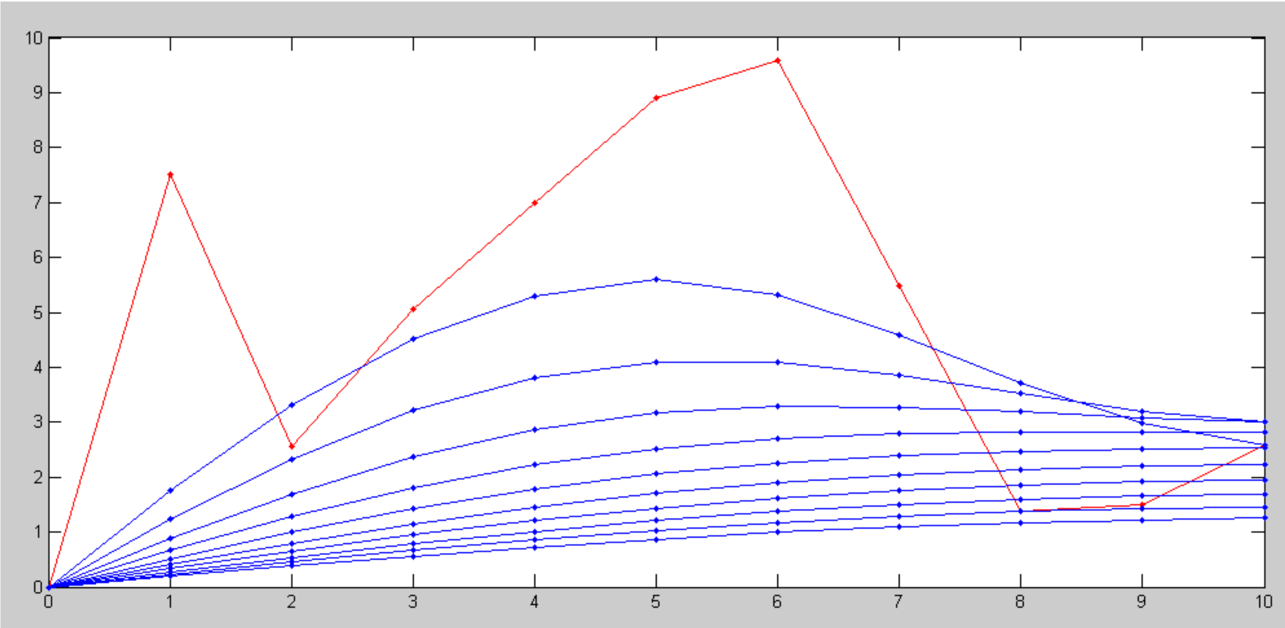
Slow mode: decays as exp(-1.55t)
The shape (eigenvector) stays the same
The amplitude drops as per the eigenvalue

Problem 6) Assume Vin = 0V. Pick random voltages for V1 .. V10 in the range of (0V, 10V):

```
V = 10 * rand(10,1)
```

Plot the votlages at t = 1. Which eigenvector does it look like?

The fast modes die out quickly, leaving the slow mode:



Random initial condition
The fast modes decay quickly, leaving the sloe mode