## ECE 111 - Homework \#4:

## Week \#4: Math 165 Calculus - Due 11am Tuesday, September 20th

1) Sketch the derivative of the following funciton

If this is the balance of your checking account, how much money are you adding (positive) or withdrawing (negative) for the balance to be as shown?

2) Sketch the integral of the following funciton


If this is how much money you are depositing (positive) or withdrawing (negative) from your checking account, what is the balance at each instance?

## Numerical Differentiation:

```
function [ dy ] = derivative( x, y )
dy = 0*y;
n = length(y);
for i=2:n-1
    dy(i) = ( y(i+1) - y(i-1) ) / (x(i+1) - x(i-1));
end
dy(1) = (y(2) - y(1))/(x(2) - x(1));
dy(n) = (y(n) - y(n-1))/(x(n) - x(n-1));
end
```

3) Use numerical methods to determine $y(x)$

$$
d y=\frac{d}{d x}\left(\frac{\cos (2 x)}{x^{2}+1}\right)
$$

for $-10<x<10$. ( a plot of $y(t)$ is sufficient ).

```
>> x = [-10:0.01:10]';
>> y = cos(2*x) ./ (x.^2 + 1);
>> dy = derivative(x, y);
>> plot(x,y,'b',x,dy,'r')
>> xlabel('x')
>>
```


$\mathrm{y}(\mathrm{x})$ (blue) and dy/dx (red)
4) Use numerical methods to determine $y(x)$

$$
d y=\frac{d}{d x}(\sin (x)+0.2 \cos (10 x))
$$

for $-10<x<10$. ( a plot of $y(t)$ is sufficient ).

```
>> dx = 0.01;
>> xlabel('x')
>> y = sin(x) + 0.2*\operatorname{cos(10*x);}
>> dy = derivative(y, dx);
>> plot(x,y,'b',x,dy,'r')
>> xlabel('x')
```


note: Differentiation amplifies the high-frequency term.
This is a problem since noise tends to be high-frequency. Differentiation amplifies noise.

## Numerical Integration

function [y ] = Integrate( $x, d y$ )

```
    npt = length(x);
    y = 0*dy;
    for i=2:npt
    y(i) = y(i-1) + 0.5*(dy(i) + dy(i-1)) * (x(i) - x(i-1));
    end
```

end
5) Use numerical methods to determine $y(x)$

$$
y=\int\left(\frac{\cos (2 x)}{x^{2}+1}\right) d x
$$

for $-10<x<10$. (a plot of $y(x)$ is sufficient ). Assume $y(-10)=0$.

```
>> x = [-10:0.01:10]';
>> dy = cos(2*x) ./ (x.^2 + 1);
>> y = Integrate(x,dy);
>> plot(x,y,'b',x,dy,'r')
>> xlabel('x')
>>
```


dy(x) (red) and its integral (blue)
6) Use numerical methods to determine $y(x)$

$$
y=\int(\sin (x)+0.2 \cos (10 x)) \cdot d x
$$

for $-10<x<10$. ( a plot of $y(x)$ is sufficient ). Assume $y(-10)=0$.

```
>> x = [-10:0.01:10]';
>> dy = sin(x) + 0.2* cos(10*x);
>> y = Integrate(x,dy);
>> plot(x,y,'b',x,dy,'r')
>> xlabel('x')
>>
```


$\mathrm{dy}(\mathrm{x})$ (red) and its integral (blue)

Note: Integration attenuates high-frequency terms.
This is good: integration helps to remove noise.

## Wind Energy

7) Load the 4-weeks worth of wind-speed data from NDAWN. (close to your home town if you're from North Dakota). Plot this in MATLAB as wind speed vs hour.
https://ndawn.ndsu.nodak.edu/
Pick Garrison, ND for June 22 - August 18, 2022

8) Write a function in Matlab where you pass the wind speed at 180 m (about 2.2 x the wind speed at the ground) and it returns the power generated by a Vestas V90-2MW

| Wind Speed <br> $(\mathrm{m} / \mathrm{s})$ | $0 . .3$ | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | $13+$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| kW | 0 | 93 | 211 | 391 | 601 | 884 | 1,247 | 1,594 | 1,861 | 1,993 | 2,000 |

https://en.wind-turbine-models.com/turbines/16-vestas-v90
8a) Determine a function in Matlab to approximate this curve.

```
function [kW] = PowerCurve( Wind )
x = [3,4,5,6,7,8,9,10,11,12,13]';
y = [0,93,211,391,601,884,1247,1594,1861,1993,2000]';
B = [x.^3, x.^2, x, x.^0];
A = inv(B'*B)*B'*y;
kW = 0*Wind;
for i=1:length(Wind)
    if(Wind(i) < 3)
        kW(i) = 0;
    elseif(Wind(i) > 13)
        kW(i) = 2000;
    else
        kW(i) = [Wind(i)^3, Wind(i)^2, Wind(i), 1]*A;
    end
end
plot(x,y,'r',Wind,kW,'r.')
end
```

Check: From the command window

```
>> x = [0:0.1:15]';
>> PowerCurve(x);
>> xlabel('Wind Speed (m/s)');
>> ylabel('kW');
```



8b) Use this function to compute how much power a Vestas V90-2MW wind turbine would produce from the wind data your found in problem 3.

```
>> kW = PowerCurve(WIND*2.2);
>> plot(hr/24,kW)
>> xlabel('Time (days)');
>> ylabel('Power (kW)');
```


9) It takes 1.78 pounds of North Dakota lignite coal to produce 1 kWh of electricity. How many pounds of coal does this wind turbine offset over 4 weeks?

```
>> kWh = sum(kW)
kWh = 3.8666e+005
>> Pounds = kWh * 1.78
Pounds = 6.8825e+005
```


## Every 4 weeks, this wind turbine offsets $\mathbf{6 8 8 , 2 5 0}$ pounds of coal.

10. Assume

- This wind turbine costs $\$ 2.6$ million to build $(\$ 1300 / \mathrm{kW})$, and - You get $\$ 0.11 / \mathrm{kWh}$ for the energy you produce.

How long will it take for this wind turbine to pay for itself?

```
>> Dollars = kWh * 0.11
Dollars = 4.2532e+004
>> Dollars_per_Year = Dollars * 52/4
Dollars_per_Year = 5.5292e+005
>> Years = 2.6e6 / Dollars_per_Year
Years = 4.7023
```

It would take about 4.7 years for this wind turbine to pay for itself.

- From a financial standpoint, this is a good investment (payback time is less than 7 years)


Vestas V90-2MW (https://en.wind-turbine-models.com/turbines/16-vestas-v90)

