ECE 111 - Homework #3

Week #3: Linear Algebra. Due 11am Tuesday, September 13th

N equations & N unknowns

1) Solve for {x, y}

$$20x + 24y = 29$$

$$5x + 15y = 11$$

Solution: Place in matrix form

$$\begin{bmatrix} 20 & 24 \\ 5 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 29 \\ 11 \end{bmatrix}$$

Solve using Matlab

```
>> A = [20,24 ; 5,15]

    20    24
     5    15

>> B = [29;11]

    29
    11

>> inv(A)*B

x    0.9500
y    0.4167
```

2) Solve for {x, y, z}

$$4x + 17y - 18z = 1$$

$$3x + 2y - 14z = 19$$

$$-16x + 12y = 6$$

Place in matrix form

$$\begin{bmatrix} 4 & 17 & -18 \\ 3 & 2 & -14 \\ -16 & 12 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 19 \\ 6 \end{bmatrix}$$

Solve using Matlab

```
>> A = [4,17,-18 ; 3,2,-14 ; -16,12,0]

     4    17   -18
     3     2   -14
   -16    12     0

>> B = [1;19;6]

     1
    19
     6

>> inv(A)*B
```

x -1.5811
y -1.6081
z -1.9257

3) Solve for {a, b, c, d}

$$13a + 13b + 8c = 2$$

$$12b + 5c + 10d = 20$$

$$7a + 9b + d = 16$$

$$a + 2b + 3c + 4d = 0$$

Place in matrix form

$$\begin{bmatrix} 13 & 13 & 8 & 0 \\ 0 & 12 & 5 & 10 \\ 7 & 9 & 0 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 20 \\ 16 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> A = [13,13,8,0 ; 0,12,5,10 ; 7,9,0,1 ; 1,2,3,4]

    13    13     8     0
     0    12     5    10
     7     9     0     1
     1     2     3     4

>> B = [2;20;16;0]

     2
    20
    16
     0

>> inv(A)*B

a   -0.2929
b    1.9058
c   -2.3709
d    0.8985
```

Global CO2 Levels

Problem 4: CO2 Levels. The CO2 levels measured at Mauna Loa observatory for the past 52 years are:

Determine a parabolic curve fit for this data in the form of

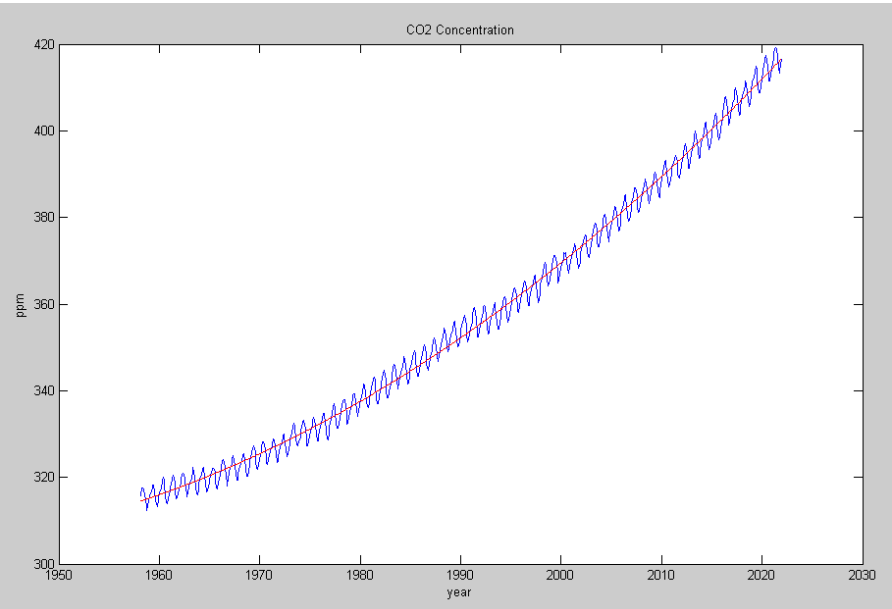
$$CO_2 \approx ay^2 + by + c$$

where 'y' is the year. From this data, when do you predict that we will hit

- 400ppm?
- 2000 ppm of CO2? (the same as what was observed during the Permian extinction)

In Matlab

```
>> DATA = [ <paste date from Bison Academy> ];
>> year = DATA(:,3);
>> CO2 = DATA(:,4);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*CO2

  1.3206e-002
 -5.0958e+001
  4.9462e+004

>> plot(year,CO2,'b',year,B*A,'r')
>> xlabel('year');
>> ylabel('ppm');
>> title('CO2 Concentration')

>> roots(A - [0;0;400])

  2014.8          reached 400ppm in 2014
  1844.0

>> roots(A - [0;0;2000])

  2287.8          at this rate, we'll reach 2000ppm in 2287 (265 years from now)
  1571.0
```

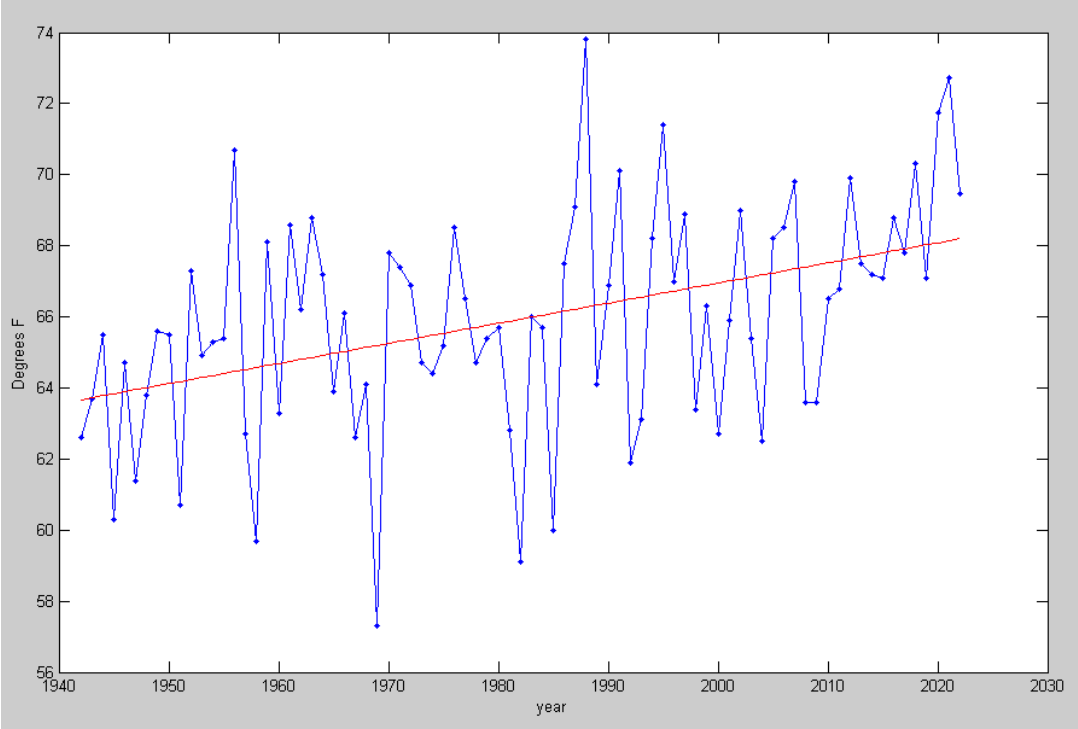


Fargo Temperatures

5) The average temperature in June for Fargo, ND is as follows:

5a) Determine a curve fit of the form of $T = ay + b$

```
>> year = DATA(:,1);
>> June = DATA(:,7);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*June

a   5.6725e-002
b  -4.6496e+001

>> plot(year,June,'b',year,B*A,'r')
>> xlabel('year');
>> ylabel('Degrees F');
```



5b) How much has Fargo warmed up over the past 80 years?

```
>> a = A(1);
>> a*80

   4.5380
```

The average temperature in Fargo has risen 4.53 degrees F over the past 80 years

5c) What will the average temperature in Fargo be in June in the year 2050?

```
>> a = A(1);
>> b = A(2);
>> F = a*2050 + b

F =   69.791          average temperature in Fargo in the year 2050 (est)
```

Problem 6-7) Sea Ice: The area covered by sea ice is recored by the National Snow and Ice Data Center:

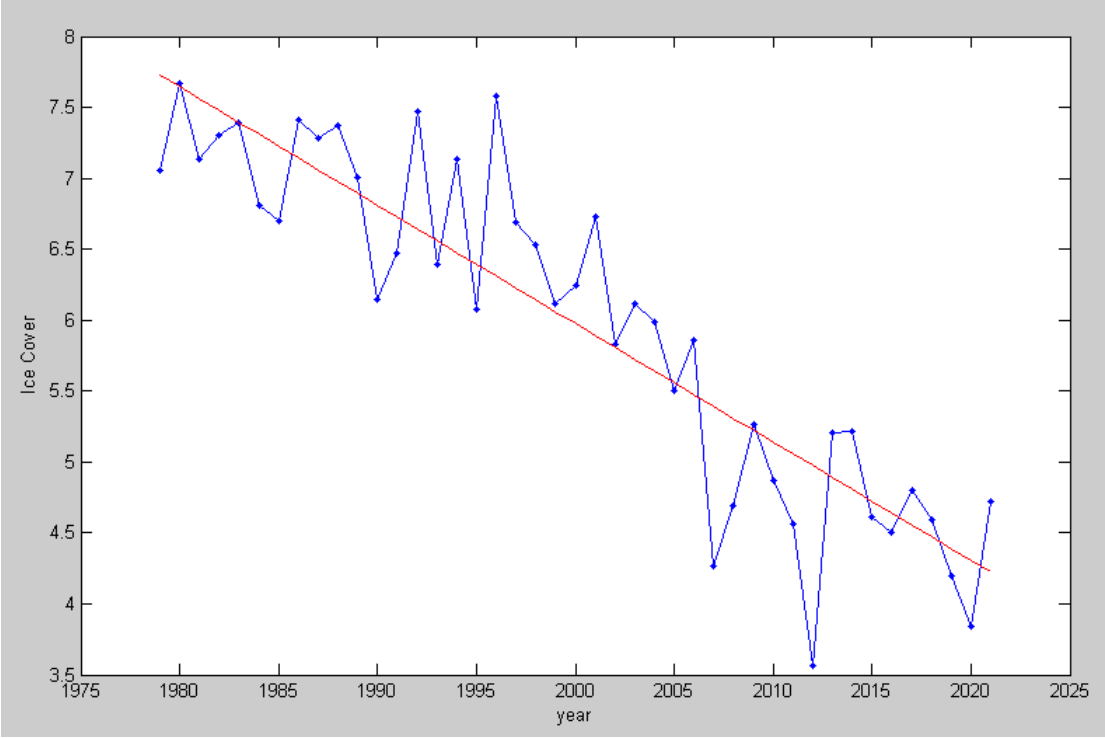
6) Approximate this data from the years 1979 - 2022 with a line

$$Area \approx ay + b$$

From this curve fit, when do you expect the Arctic to be ice free? (First time in 5 million years)

```
>> year = DATA(:,1);
>> ICE = DATA(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*ICE

a -8.3456e-002
b  1.7289e+002

>> plot(year,ICE,'b.-',year,B*A,'r')
>> xlabel('year');
>> ylabel('Ice Cover');
>> roots(A)
```

2071.6 when the Arctic will be ice free assuming constant melt rate



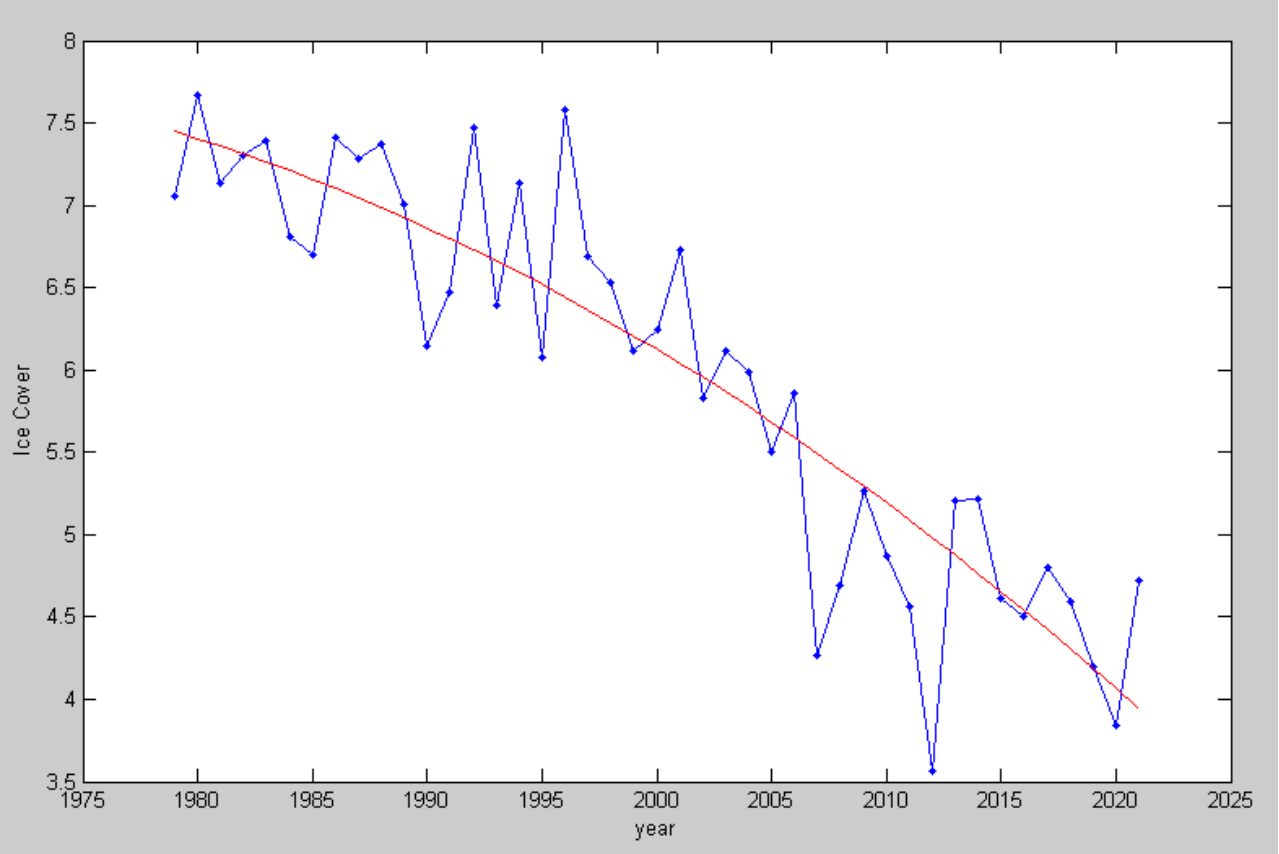
7) Approximate this data with a parabolic curve fit:

$$Area \approx ay^2 + by + c$$

From this curve fit, when do you expect the Arctic to be ice free?

```
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*ICE
Warning: Matrix is close to singular or badly
scaled.
         Results may be inaccurate. RCOND =
         7.386767e-023.

a -9.7651e-004
b  3.8226e+000
c -3.7330e+003

>> plot(year,ICE,'b.-',year,B*A,'r')
>> xlabel('year');
>> ylabel('Ice Cover');
>> roots(A)
```

2047.3 **when the Arctic will be ice free assuming increasing melt rate**
1867.3

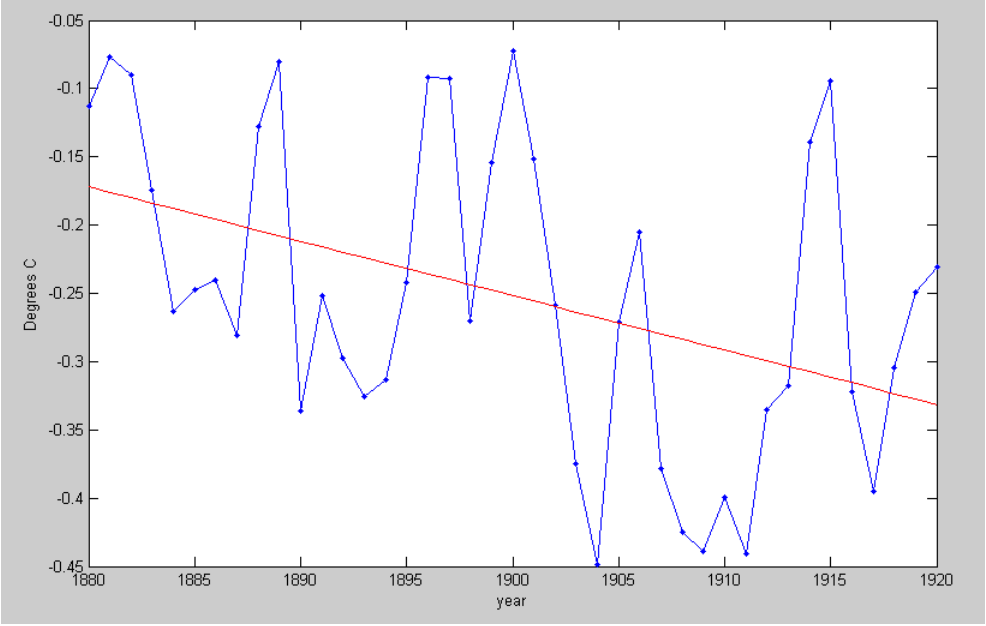


Problem 8-9: World Temperatures. NASA Goddard has been keep records since 1880 (139 years of data).

8) Determine a least-squares curve fit for this data from the year 1880 - 1920 in the form of

$$\delta T = ay + b$$

```
>> year = DATA(:,1);
>> B = [year, year.^0];
>> dT = DATA(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*dT

a -3.9853e-003
b  7.3204e+000

>> plot(year,dT,'b.-',year,B*A,'r')
>> xlabel('year');
>> ylabel('Degrees C');
```



Based upon this data, what *should* the temperature deviation be in the year 2022?

```
>> [2022, 1] * A

ans = -0.73792
```

note: At the start of the 1900's, there was worry that another ice age was coming. This could mean that global warming is actually much higher (+0.74F) than we think.

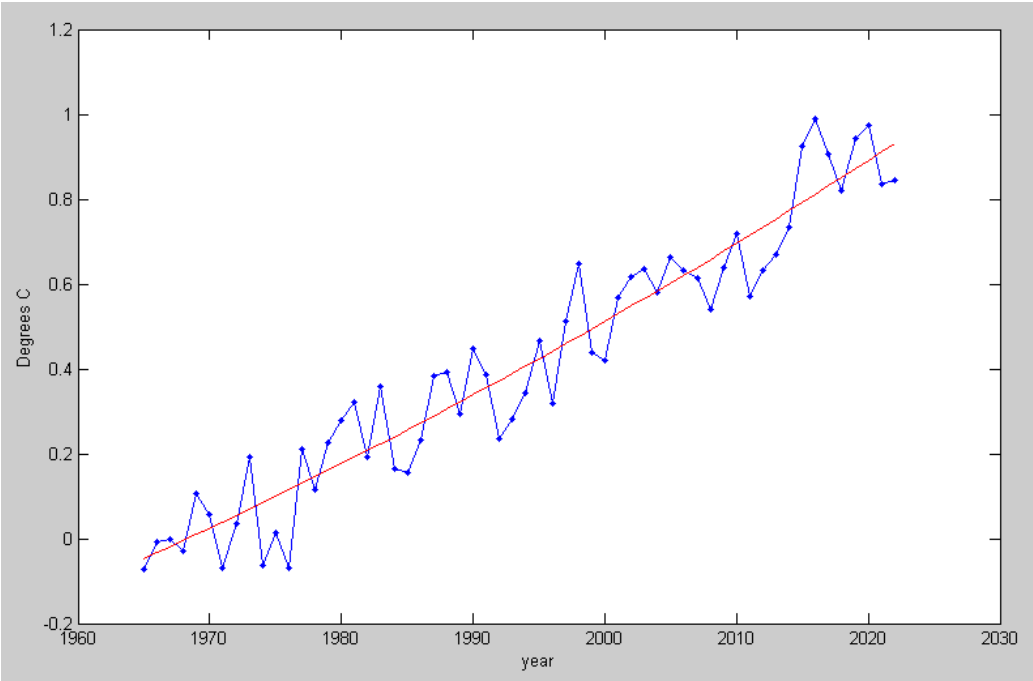
9) Determine a least-squares curve fit for this data from the year 1965 - 2022 in the form of

$$\delta T \approx ay^2 + by + c$$

```
>> year = DATA(:,1);
>> dT = DATA(:,2);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*dT
```

```
a  5.4206e-005
b -1.9895e-001
c  1.8160e+002
```

```
>> plot(year,dT,'b.-',year,B*A,'r')
>> xlabel('year');
>> ylabel('Degrees C');
```



Based upon this data, predict when we will see a 10 degree temperature increase if nothing changes?

```
>> roots(A - [0;0;10])
```

```
2284.8        assuming parabolic curve fit, we'll hit +10C in 2284 (262 years)
1385.5
```

10) What does a temperature rise of 10 degrees mean for the planet?

not graded - too political

The Permian Extinction Event suggests that it's not good: no animals larger than a mouse survived the Permian Extinction - which was triggered by CO2 levels at 2000ppm and a +10 degree C temerature rise.

One Degree: 2025 Summers like 2003 where a heat wave in France caused 10,000 deaths become the norm. Flows of the Po and Rhine river decrease. Crop production drops.

```
-->roots(A - [0;0;1])

  2025.4
  1645.0
```

Two Degrees: 2068. Oceans absorb less CO2 (too hot) and soils start to release CO2. Vacations to the Mediterranean in the summer are just too hot. Crop failures in Africa and Central America cause mass migration. Coastal cities flood. 1/3rd of species face extinction.

```
-->roots(A - [0;0;2])

  2068.9
  1601.5
```

Three Degrees: 2105. Crop failures in China cause the migration of more than 1 billion people. Collapse of equatorial governments.

```
-->roots(A - [0;0;3])

  2105.5
  1564.9
```

Four Degrees: 2137. Spain becomes a desert. Mass migration to Northern latitudes. Rain forests burn up.

```
-->roots(A - [0;0;4])

  2137.7
  1532.6
```

Six Degrees: 2193. Ice caps are gone. Methane hydrates become unstable raising temperatures in a positive-feedback loop. Ocean circulation stops. Hydrogen sulfide producing bacteria flourish poisoning the air. The Ozone layer dissipates leaving the land sterilized with UV radiation. End-Permian-like conditions make life nearly impossible.

```
-->roots(A - [0;0;0;6])

  2193.5
  1476.8
```

Scary? Yes. That's why the rest of the world sees the Paris Climate Accord as being important. That's why the United Nations sees Global Warming as the #1 threat - far greater than terrorism. Far greater than COVID.