## ECE 111: Homework 15

Week \#15 - ECE 331 Energy Conversion. Due Deember 14th
Please submit as a Word or pdf file to BlackBoard or email to Jacob_Glower@yahoo.com with header ECE 111 HW\#15 www.BisonAcademy.com

1) Determine the circuit model for a $13.2 \mathrm{~V}: 240 \mathrm{~V}$ transformer is tested with the following test results:


Transformer Model

|  | V | Power | pf |
| :---: | :---: | :---: | :---: |
| Open-Circuit Test | $\mathrm{V} 1=9.6 \mathrm{kV}$ | 85 W | 0.06 |
| Short-Circuit Test | $\mathrm{V} 2=24 \mathrm{~V}$ | 15 W | 0.97 |

Open Circut Test:

$$
\begin{aligned}
& P=V \cdot I \cdot p f \\
& I=\left(\frac{P}{V \cdot p f}\right)=\left(\frac{85 W}{9600 V \cdot 0.06}\right)=0.1476 A \\
& Z=\left(\frac{V}{A}\right) \angle \arccos (p f)=\left(\frac{9600 V}{0.1476 A}\right) \angle 86.56^{0} \\
& Z=65054 \angle 85.56^{0}=3903.2+j 64,937
\end{aligned}
$$

This is the series model (resistors in series add). The parallel model uses the inverse

$$
\begin{aligned}
& \frac{1}{Z}=9.2231 \cdot 10^{-7}-j 1.5344 \cdot 10^{-5} \\
& R_{c}=\left(\frac{1}{9.2231 \cdot 10^{-7}}\right)=1.0842 M \Omega \\
& X_{c}=\left(\frac{1}{1.5344 \cdot 10^{-5}}\right)=65.172 \mathrm{k} \Omega
\end{aligned}
$$

In Matlab

```
>> V = 9600;
>> P = 85;
>> pf = 0.06;
>> I = P / (V*pf)
I = 0.1476
>> Z = (V/I) * exp(j*acos(pf))
```

```
Z = 3.9032e+003 +6.4937e+004i
>> 1/Z
ans = 9.2231e-007-1.5344e-005i
>> Rc = 1/ real(1/Z)
Rc = 1.0842e+006
>> Xc = 1 / abs(imag(1/Z))
Xc =
    6.5172e+004
```

Line Model:

$$
\begin{aligned}
& I=\left(\frac{P}{V \cdot p f}\right)=\left(\frac{15 W}{24 V \cdot 0.97}\right)=0.6443 A \\
& Z=\left(\frac{V}{A}\right) \angle \arccos (p f)=\left(\frac{24 V}{0.6443 A}\right) \angle 14.07^{0} \\
& Z=36.1306+j 9.0552
\end{aligned}
$$

This is the series model $\left(R_{L}+j X_{L}\right.$

$$
\begin{aligned}
& R_{L}=36.1306 \Omega \\
& j X_{L}=j 9.0552 \Omega
\end{aligned}
$$

In Matlab

```
>> P = 15;
>> V = 24;
>> pf = 0.97;
>> I = P / (V*pf)
I = 0.6443
>> Z = (V/I) * exp(j*acos(pf))
Z = 36.1306 + 9.0552i
>>
```

2) Convert the voltages and impeances to the 120 V node (right side)


Voltages convert as the turn ratio:

$$
128 k V \Rightarrow(128 k V)\left(\frac{9.6 k V}{128 k V}\right)\left(\frac{120 V}{9.6 k V}\right)=120 \mathrm{~V}
$$

Impedances convert as the turn ratio squared

$$
R_{01} \Rightarrow 214\left(\frac{120 V}{128 k V}\right)^{2}=0.000188 \Omega
$$

In matlab

```
>> R01 = 214 * (120/128000)^2
R01 = 1.8809e-004
>> X1 = 300000 * (120/128000)^2
x1 = 0.2637
>> R1 = 1.5e6 * (120/128000)^2
R1 = 1.3184
>> R12 = 1.07 * (120/9600)^2
R12 = 1.6719e-004
>> X2 = 3e3 * (120/9600)^2
x2 = 0.4688
>> R2 = 9e3 * (120/9600)^2
R2 = 1.4063
>> R23 = 0.01704
R23 = 0.0170
>> R3 = 1.44
R3 = 1.4400
```


3) Determine the voltages at each node

Write the voltage node equations

$$
\begin{aligned}
& V_{0}=120 \\
& \left(\frac{V_{1}-V_{0}}{R_{01}}\right)+\left(\frac{V_{1}}{R_{1}}\right)+\left(\frac{V_{1}}{j X_{1}}\right)+\left(\frac{V_{1}-V_{2}}{R_{12}}\right)=0 \\
& \left(\frac{V_{2}-V_{1}}{R_{12}}\right)+\left(\frac{V_{2}}{R_{2}}\right)+\left(\frac{V_{2}}{j X_{2}}\right)+\left(\frac{V_{2}-V_{3}}{R_{23}}\right)=0 \\
& \left(\frac{V_{3}-V_{2}}{R_{23}}\right)+\left(\frac{V_{3}}{R_{3}}\right)=0
\end{aligned}
$$

## Group terms

$$
\begin{aligned}
& V_{0}=120 \\
& -\left(\frac{1}{R_{01}}\right) V_{0}+\left(\frac{1}{R_{01}}+\frac{1}{R_{1}}+\frac{1}{j X_{1}}+\frac{1}{R_{12}}\right) V_{1}-\left(\frac{1}{R_{12}}\right) V_{2}=0 \\
& -\left(\frac{1}{R_{12}}\right) V_{1}+\left(\frac{1}{R_{12}}+\frac{1}{R_{2}}+\frac{1}{j X_{2}}+\frac{1}{R_{23}}\right) V_{2}-\left(\frac{1}{R_{23}}\right) V_{3}=0 \\
& -\left(\frac{1}{R_{23}}\right) V_{2}+\left(\frac{1}{R_{23}}+\frac{1}{R_{3}}\right) V_{3}=0
\end{aligned}
$$

Place in matrix form

$$
\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
\left(\frac{-1}{R_{01}}\right) & \left(\frac{1}{R_{01}}+\frac{1}{R_{1}}+\frac{1}{j X_{1}}+\frac{1}{R_{12}}\right) & \left(\frac{-1}{R_{12}}\right) & 0 \\
0 & \left(\frac{-1}{R_{12}}\right) & \left(\frac{1}{R_{12}}+\frac{1}{R_{2}}+\frac{1}{j X_{2}}+\frac{1}{R_{23}}\right) & \left(\frac{-1}{R_{23}}\right) \\
0 & 0 & \left(\frac{-1}{R_{23}}\right) & \left(\frac{1}{R_{23}}+\frac{1}{R_{3}}\right)
\end{array}\right]\left[\begin{array}{c}
V_{0} \\
V_{1} \\
V_{2} \\
V_{3}
\end{array}\right]=\left[\begin{array}{c}
120 \\
0 \\
0 \\
0
\end{array}\right]
$$

Solve (using Matlab)

```
>>a1 = [1,0,0,0];
>> a2 = [-1/R01,1/R01+1/R1+1/(j*X1)+1/R12,-1/R12,0];
>> a3 = [0,-1/R12,1/R12+1/R2+1/ (j*X2) +1/R23,-1/R23];
>> a4= [0,0,-1/R23,1/R23+1/R3];
>> A = [a1;a2;a3;a4]
>> A = [a1;a2;a3;a4]
A =
    1.0e+004 *
        0.0001 0 0 0
    -0.5317 1.1299-0.0004i -0.5981 0
        0 -0.5981 
>> B = [120;0;0;0]
    120
        0
        0
        0
>> V = inv(A)*B
    1.0e+002 *
        1.2000 + 0.0000i
        1.1995 + 0.0013i
        1.1992 + 0.0018i
        1.1852 + 0.0017i
>> abs(V)
V0 120.0000
V1 119.9513
V2 119.9232
V3 118.5208
```

The customer's voltage drops to 118.52 V
4) Determine the efficiency of this system a) Ignoring the core losses, and b) Including the core losses

```
V0 = V(1);
V1 = V(2);
V2 = V(3);
V3 = V(4);
P01 = (abs(V1-V0))^2 / R01
P12 = (abs(V1-V2))^2 / R12
P23 = (abs(V2-V3))^2 / R23
P1 = (abs(V1))^2 / R1
P2 = (abs(V2))^2 / R2
P3 = (abs(V3))^2 / R3
eff1 = P3 / (P3 + P01 + P12 + P23)
eff2 = P3 / (P3 + P01 + P12 + P23 + P1 + P2)
```

results in

```
P01 = 107.5859
P12 = 15.6382
P23 = 115.4339
P1 = 1.0914e+004
P2 = 1.0227e+004
P3 = 9.7550e+003
```



The efficiency when ignoring the core losses is $97.6 \%$

- $98.8 \%$ of the power transmitted gets to the customer as long as you're serving a city (so the core losses are spread out over a large number of customers)
effi $=0.9761$

The efficiency when including the core losses is only $31.3 \%$

- If there is only one customer, only $31 \%$ of the energy transmitted gets to the customer
- It's not efficient or economic to build a power grid to serve a single customer eff2 $=0.3133$


