ECE 111 - Homework #12

Week #12: ECE 341 Random Processes. Due November 23rd

Please submit as a Word or pdf file to BlackBoard or email to Jacob_Glower@yahoo.com with header ECE 111 HW#12

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Chi-Squared Tests

Problem 1: The following Matlab code generates 90 random die rolls for a six sided die

```
RESULT = zeros(1,6);
for i=1:90
   D6 = ceil( 6*rand );
   RESULT(D6) = RESULT(D6) + 1;
   end
RESULT
```

Determine whether this is a fair or loaded die using a Chi-Squared test.

Running the code (results vary - it's random)

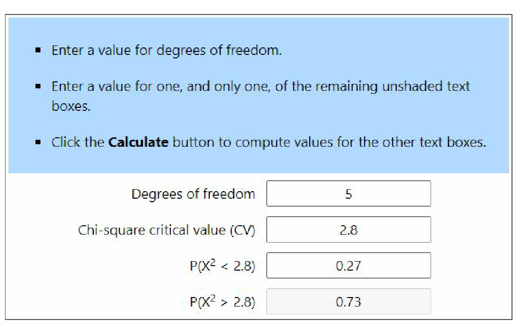
```
RESULT =

     13     14     13     13     20     17
```

Put this into a table:

die roll	p	np	N	chi-squared
1	1/6	15	13	0.27
2	1/6	15	14	0.07
3	1/6	15	13	0.27
4	1/6	15	13	0.27
5	1/6	15	20	1.67
6	1/6	15	17	0.27
			Total	2.8

From StatTrek, a chi-squared total of 2.8 with 5 degrees of freedom corresponds to a probability of 0.27

There is a 27% chance that the die is loaded (no conclusion)



Problem 2: The following Matlab code generates 90 rolls of a loaded six-sided die (12% of the time, you roll a 6):

```
RESULT = zeros(1,6);
for i=1:90
   if(rand < 0.12)
      D6 = 6;
   else
      D6 = ceil( 6*rand );
      end
   RESULT(D6) = RESULT(D6) + 1;
   end
RESULT
```

Running the code gives the following results (vary with each trial)

```
RESULT =     13     14     16     13     12     22
```

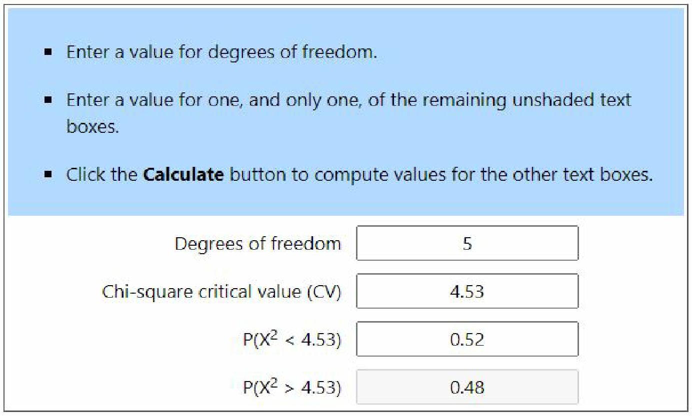
Putting this into a table:

die roll	p	np	N	chi-squared
1	1/6	15	13	0.27
2	1/6	15	14	0.07
3	1/6	15	16	0.07
4	1/6	15	13	0.27
5	1/6	15	12	0.6
6	1/6	15	22	3.27
			Total	4.53

From StatTrek, a chi-squared critical value of 4.53 with 5 degrees of freedom corresponds to a probability of 0.52

There is a 52% chance the die is loaded (no conclusion)

note: It is very hard to tell if a die is loaded only 12% of the time with only 90 rolls.



Am I Psychic?

Problem #3: Shuffle a deck of 52 playing cards and place it face down on a table.

- Predict the suit of the top card then reveal it. If correct, place the card in one pile (correct). If incorrect, place it in another pile.
- Repeat for all 52 cards.

Use a chi-squared test to test the hypothesis that you're just guessing (probability of being correct is 25%)

Flipping 52 cards resulted in

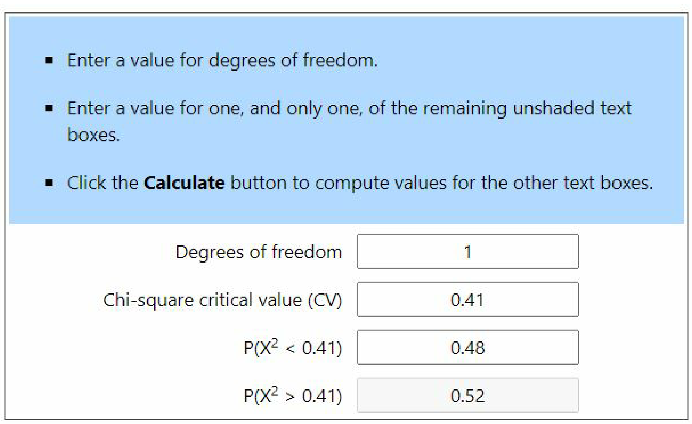
- 11 times correct
- 41 times incorrect

Checking if this is random guessing (25% chance of getting it right)

prediction	p	np	N	chi-squared
correct	1/4	13	11	0.31
incorrect	3/4	39	41	0.1
			Total	0.41

From StatTrek, a chi-squared critical value of 0.41 with 1 degree of freedom gives a probability of 0.48

There is a 48% chance that I'm not guessing (no conclusion)



Normal Approximation

The mean and standard deviation for a fair 6-sided die and 8-sided die are:

$$\mu_{d6} = 3.5 \qquad \mu_{d8} = 4.5$$

$$\sigma_{d6} = 1.7078 \qquad \sigma_{d8} = 2.291$$

Problem 4: Let Y be the sum of rolling five 6-sided dice (5d6) plus five 8-sided dice (5d8).

Y = 5d6 + 5d8

a) What is the mean and standard deviation of Y?

The means add

$$\mu = 5 \cdot 3.5 + 5 \cdot 4.5$$

$$\mu = 40$$

The variance also adds

$$\sigma^2 = 5 \cdot 1.7078^2 + 5 \cdot 2.291^2$$

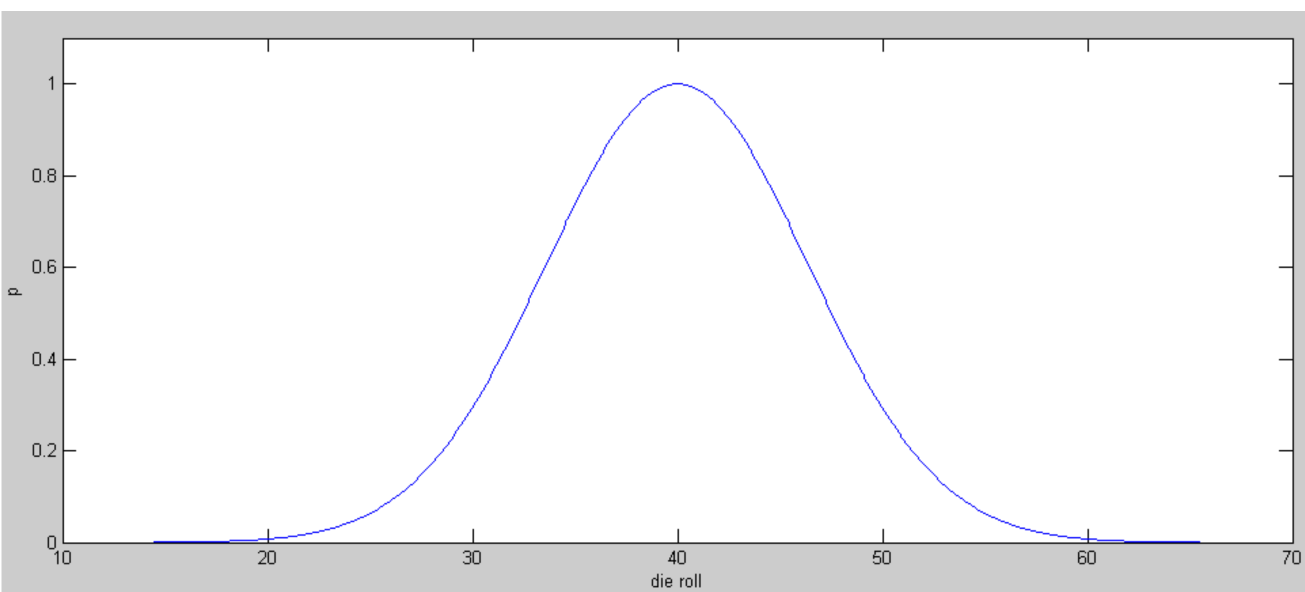
$$\sigma^2 = 40.8263$$

The standard deviation is then

$$\sigma = \sqrt{\sigma^2} = 6.3895$$

The pdf then looks like the following:

```
>> s1 = [-4:0.01:4]';
>> p = exp(-s1.^2 / 2);
>> plot(s1*6.3895+40,p)
```

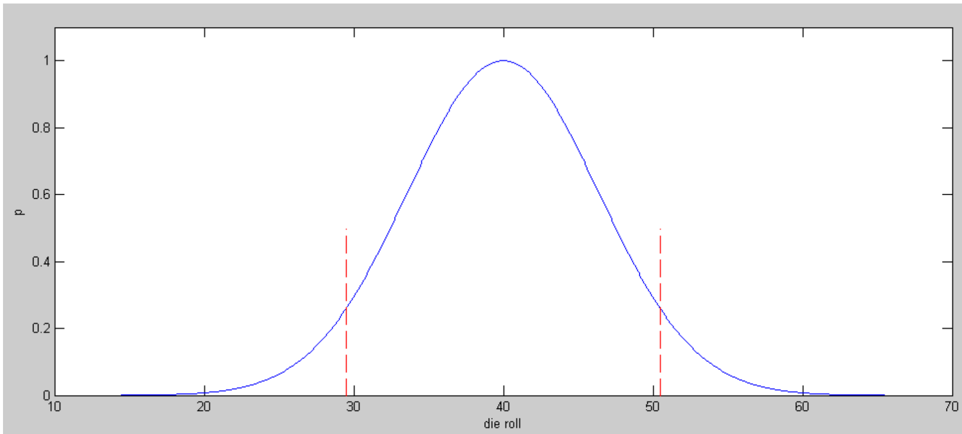


b) Using a normal approximation, what is the 90% confidence interval for Y?

5% tails corresponds to a z-score of 1.645. The 90% confidence interval is then

$$\mu - 1.645\sigma < sum < \mu + 1.645\sigma \qquad p = 0.9$$

$$29.489 < sum < 50.511$$

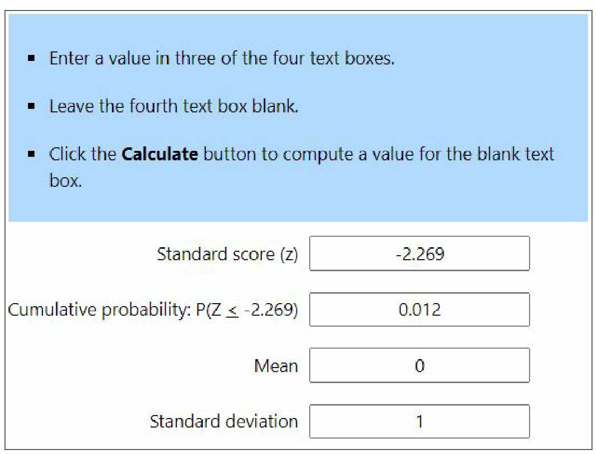


c) Using a normal approximation, what is the probability that the sum the dice will be more than 54.5?

The z-score is

$$z = \left(\frac{54.5-\mu}{\sigma}\right) = \left(\frac{54.5-40}{6.3895}\right) = 2.269$$

From StatTrek, a z-score of 2.269 corresponds to a probability of 1.2%



Problem 5: Check your answer using a Monte-Carlo simulation in Matlab with 100,000 rolls:

```
N = 0;
for i=1:1e5
   Y = sum( ceil( 6*rand(5,1) ) ) + sum( ceil( 8*rand(5,1) ) );
   if(Y > 54.5)
      N = N + 1;
      end
   end
N / 1e5
```

```
ans =

    0.0106
```

A Monte-Carlo run gives a probability of 1.06% chance of rolling more than 54.5

A normal approximation gives a probability of 1.2%

t-Tests

Problem 6: Using Matlab, cast five level-7 fireballs (the sum of seven 6-sided dice, or 7d6)

```
damage = [];
for i=1:5
  x = sum( ceil( 6*rand(7,1) ) );
  damage = [damage ; x];
  end
```

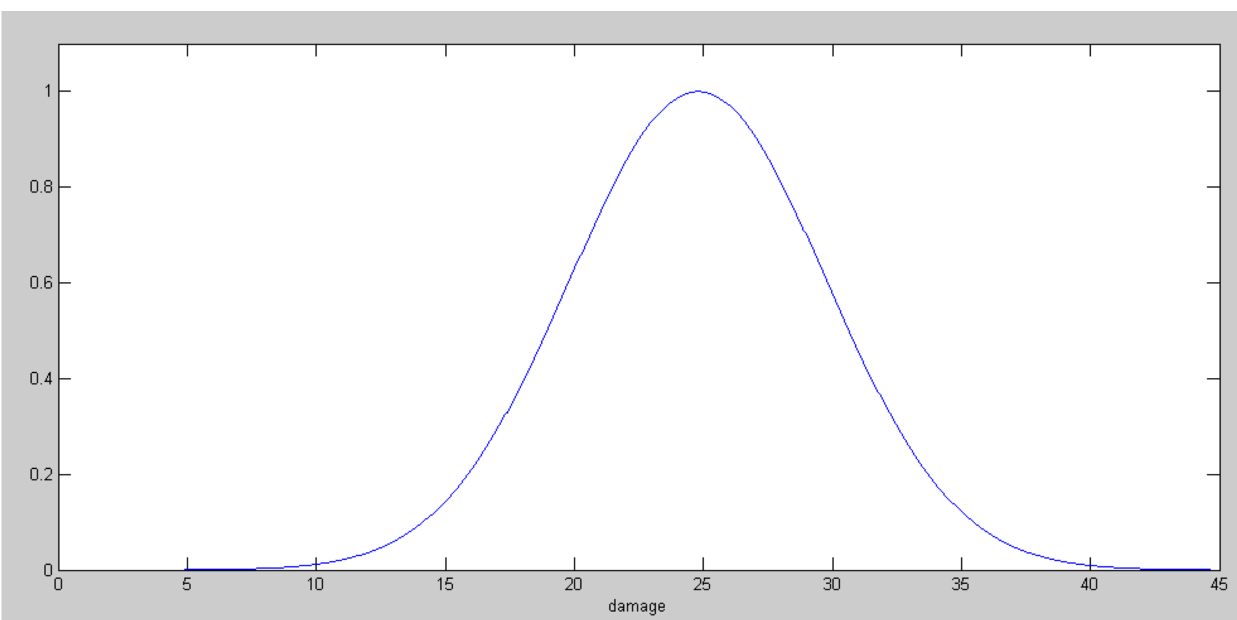
From this, determine the mean and standard deviation of your data set.

Running the program gives

```
damage =

    28
    31
    23
    18
    24
```

Find the mean and standard deviation

```
>> x = mean(damage)

x =   24.8000

>> s = std(damage)

s =    4.9699

>> s1 = [-4:0.01:4]';
>> p = exp(-s1.^2 / 2);
>> plot(s1*4.9699+24.8,p)
```



pdf for a level-7 fireball

Problem 7: Use a t-test to determine

- The 90% confidence interval for a level 7 fireball.
- The probabillity of doing 35 damage or more with a level-7 fireball

From StatTrek, a t-distribution

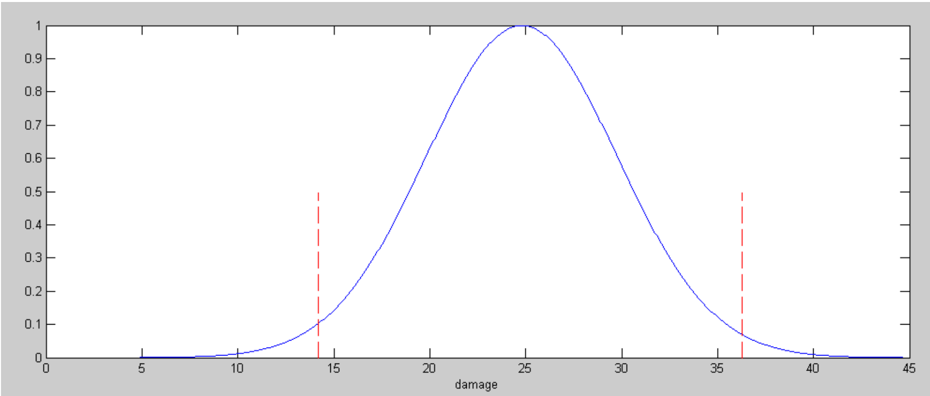
- with 4 degrees of freedom (sample size of 5)
- with 5% tails (90% in the middle)

gives a z--score of 2.132

The 90% confidence interval is then

$$\bar{x} - 2.132s < sum < \bar{x} + 2.132s$$

$$14.2042 < sum < 36.3302$$



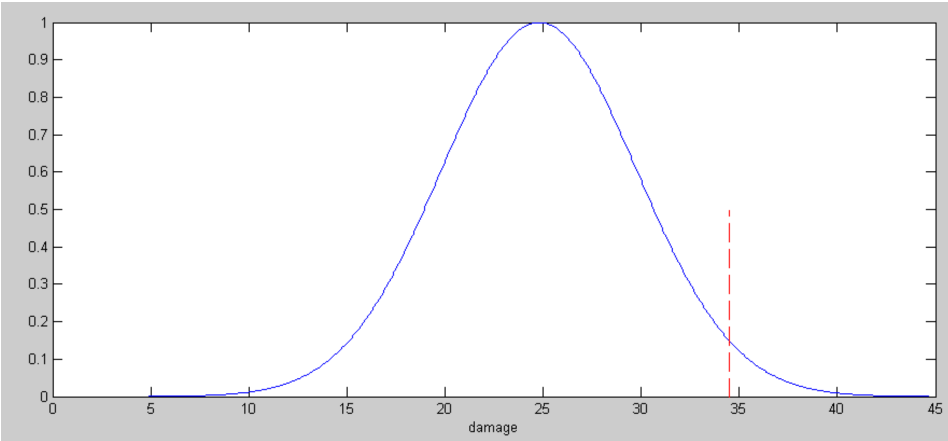
To find the probabiliy of doing 35 damage or more, find the t-score for 34.5

$$t = \left(\frac{34.5 - \bar{x}}{s}\right) = \left(\frac{34.5 - 24.80}{4.9699}\right) = 1.9517$$

Using StatTrek, a t-score of 1.9517 corresponds to a probability of 0.9386

There is a 93.86% chance or dong less than 35 damage with a level-7 fireball

There is a 6.14% chance of doing 35 or more damage with a level-7 fireball



Problem 8) Check your answer using a Monte-Carlo simulation in Matlab by casting 100,000 level-7 fireballs:

```
Nx = 0;
Ny = 0;

for i=1:1e5
   damage = sum( ceil( 6*rand(7,1) ) );
   if( (damage > 14.2)*(damage < 36.3) )
       Nx = Nx + 1;
       end
   if( damage >= 35)
      Ny = Ny + 1;
      end
   end
[Nx,Ny] / 1e5

ans =

    0.9856    0.0122
```

- The actual probability of doing damage in the range of (14.2, 36.3) is 98.56% (vs. 90%)
- The actual probability of doing 35 or more damage is 1.22% (vs. 6.14%)

These are different due to a small sample size (5 fireballs)