ECE 111 - Homework #11

Week #11 - ECE 343 Signals

Problem 1-5) Let x(t) be a function which is periodic in 2π

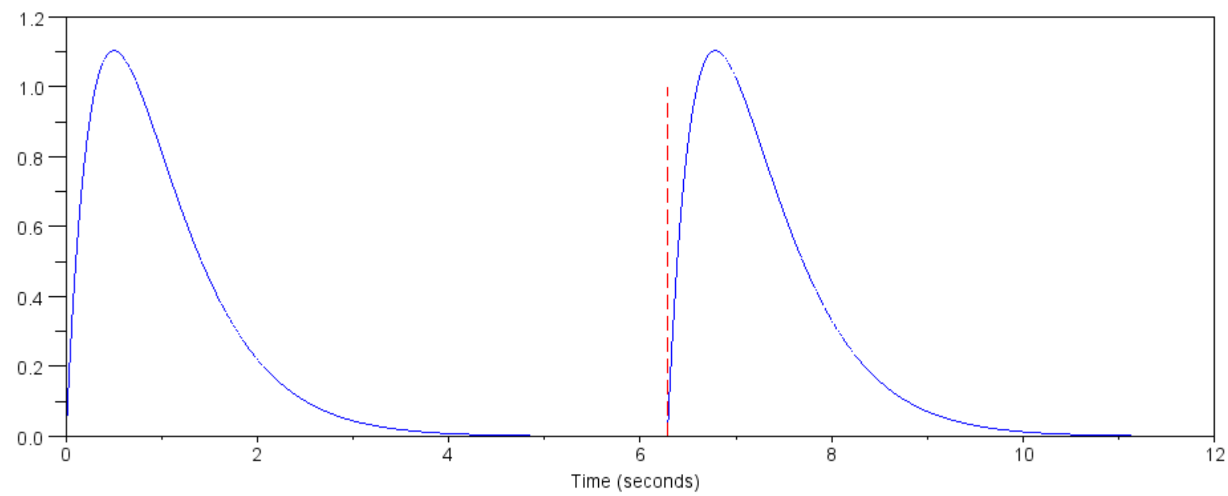
$$x(t) = x(t + 2\pi)$$

Over the interval $(0, 2\pi)$ x(t) is

$$x(t) = 6t \cdot e^{-2t}$$

or in Matlab:

```
t = [0:0.001:2*pi]';
x = 6 * t .* exp(-2*t);
plot(t,x)
```



x(t) Note that x(t) repeats repeats every 2π seconds

Curve Fitting with a power series:

1) Using least squares, approximate x(t) over the interval (0, 2π) as

$$x(t) \approx a + bt + ct^2 + dt^3$$

Plot x(t) along with it's approximation.

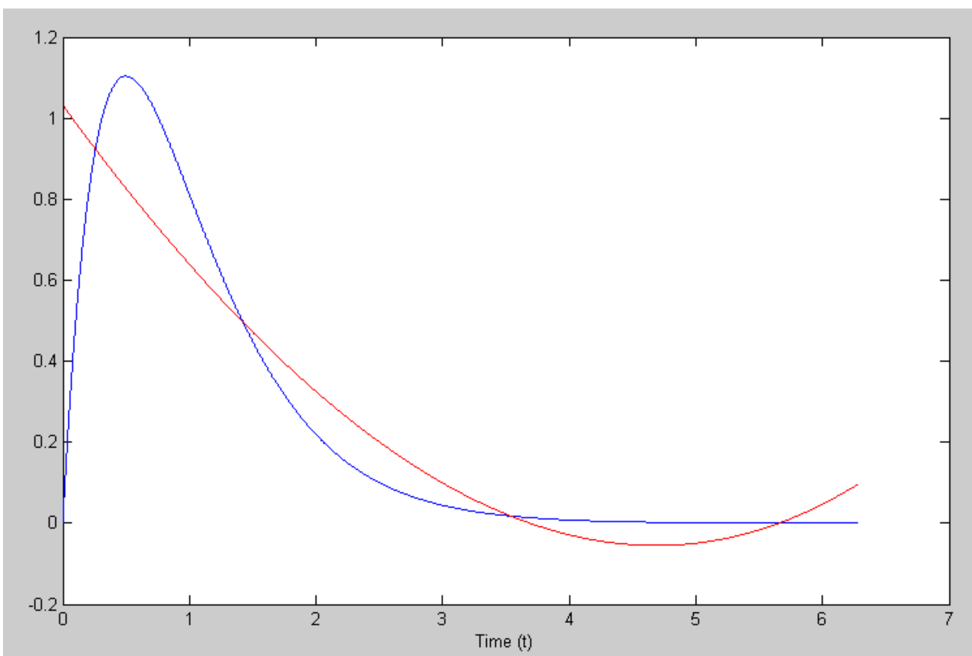
note: You can curve fit anything. This solution doesn't help solve for y(t) in problem #3 since the basis isn't in the form of sinusoids.

```
>> t = [0:0.001:2*pi]';
x = 6 * t .* exp(-2*t);
plot(t,x)
>>
>> B = [t.^0, t, t.^2, t.^3];
>> A = inv(B'*B)*B'*x

A =

    1.0319
   -0.4275
    0.0341
    0.0016

>> plot(t,x,'b',t,B*A,'r');
>> xlabel('Time (t)');
>>
```



x(t) (blue) and its cubic curve fit (red)

Curve Fitting using a Fourier Series

2) Using least squares, approximate x(t) over the interval (0, 2π) as

$$x(t) = a_0 + a_1\cos(t) + b_1\sin(t) + a_2\cos(2t) + b_2\sin(2t) + a_3\cos(3t) + b_3\sin(3t)$$

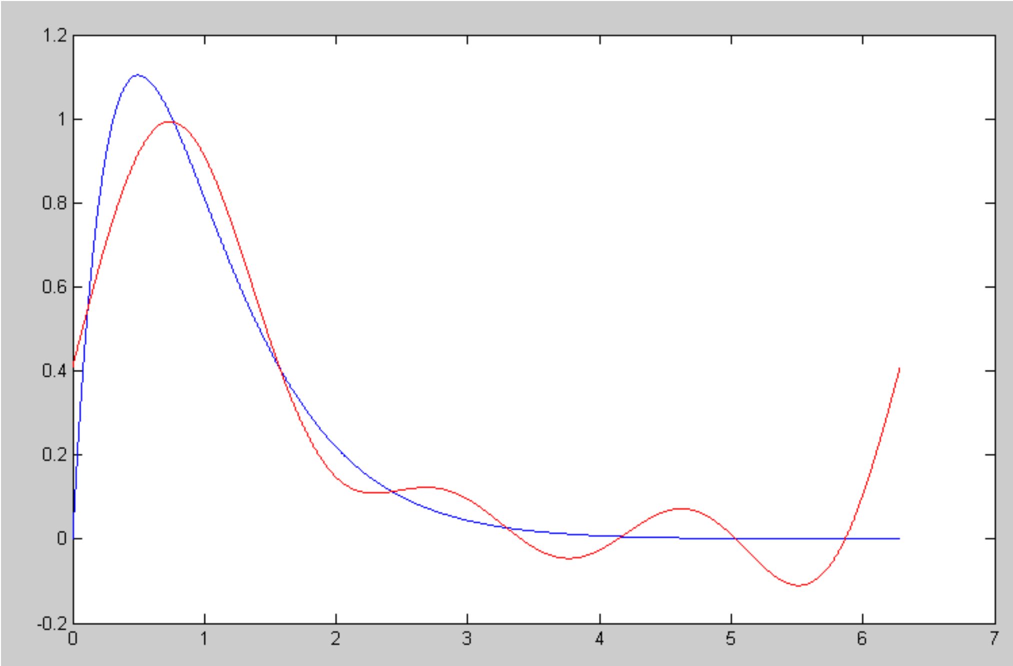
Plot x(t) along with it's approximation.

*note: This solution *does* help solve for y(t). Phasor analysis only works if the input is made up of sinusoids. x(t) is now expressed in terms of sinusoids - meaning you can now proceed with problem #3.*

```
>> B = [t.^0, cos(t), sin(t), cos(2*t), sin(2*t), cos(3*t), sin(3*t)];
>> A = inv(B'*B)*B'*x

a0     0.2387
a1     0.2291
b1     0.3056
a2    -0.0001
b2     0.2387
a3    -0.0566
b3     0.1356

>> plot(t,x,'b',t,B*A,'r');
>>
```



x(t) (blue) and a sine-wave approximation (red)

Superposition

3) Assume X and Y are related by

$$Y = \left(\frac{1}{s^2+1.5s+1}\right)X$$

3a) Determine x(t) in terms of its Fourier Transform out to 3 rad/sec

From problem #2

```
a0    0.2387
a1    0.2291
b1    0.3056
a2   -0.0001
b2    0.2387
a3   -0.0566
b3    0.1356
```

The Fourier coefficients (using the complex Fourier transform) are

```
>> x0 = mean(x)

x0 =    0.2387

>> x1 = 2*mean(x .* exp(-j*t))

x1 =   0.2291 - 0.3055i

>> x2 = 2*mean(x .* exp(-2*j*t))

x2 =  -0.0000 - 0.2387i

>> x3 = 2*mean(x .* exp(-3*j*t))

x3 =  -0.0565 - 0.1356i
```

The Fourier coefficients are identical to what we got with a least-squares curve fit.

Fourier Trransforms is nothing more than a curve fit where you use sine waves for the basis.

3b) Plot x(t) and its Fourier approximation taken out to 3 rad/sec

Same results as problem #2 so same plot

4) Determine the gain of this filter at each frequency present in problem #2 (i.e. 0, 1, 2, 3 rad/sec)

- *note: You should get a complex number for the gain at each frequency*

DC

```
>> s = 0;
>> G0 = 1 / ( s^2 + 1.5*s + 1)

G0 =     1
```

1 rad/sec

```
>> s = j*1;
>> G1 = 1 / ( s^2 + 1.5*s + 1)

G1 =        0 - 0.6667i
```

2 rad/sec

```
>> s = j*2;
>> G2 = 1 / ( s^2 + 1.5*s + 1)

G2 =  -0.1667 - 0.1667i
```

3 rad/sec

```
>> s = j*3;
>> G3 = 1 / ( s^2 + 1.5*s + 1)

G3 =  -0.0950 - 0.0534i
```

Note: This is a filter. The gain changes with frequency.

5a) Determine the phasor representation for Y(jw) at each frequency.

- *note: You should get a complex number for Y - the phasor representation for y(t) at 0, 1, 2, and 3 rad/sec*

```
>> y0 = G0*x0

y0 =    0.2387

>> y1 = G1*x1

y1 =  -0.2037 - 0.1528i

>> y2 = G2*x2

y2 =  -0.0398 + 0.0398i

>> y3 = G3*x3

y3 =  -0.0019 + 0.0159i
```

5b) From this, determine y(t)

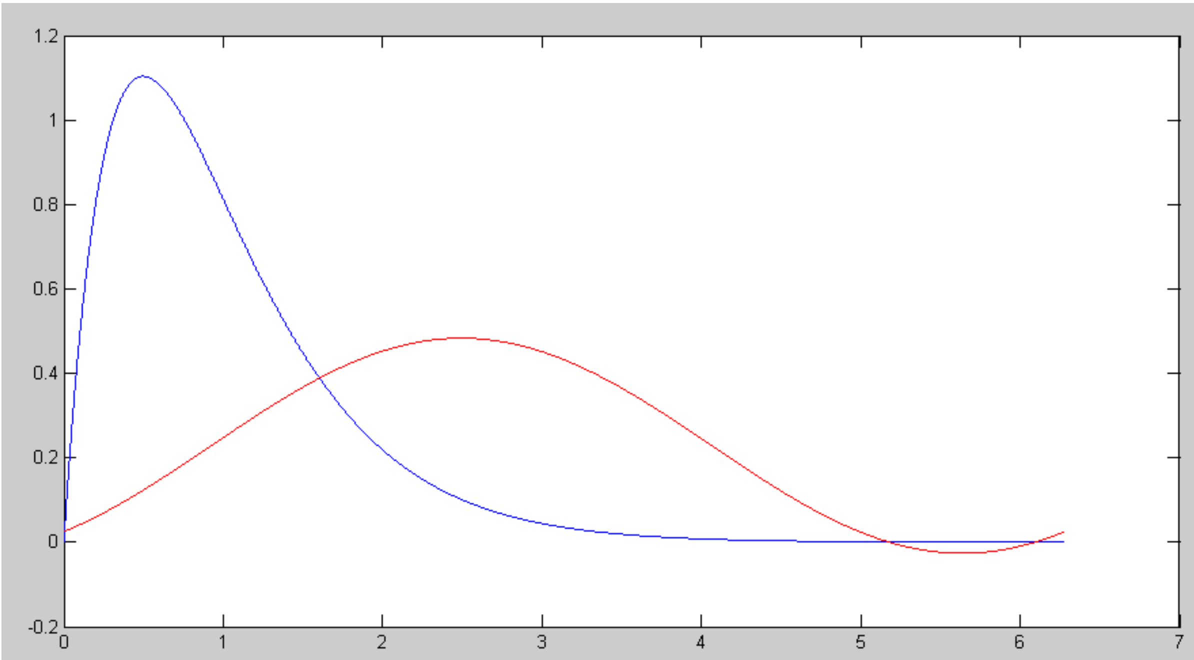
- *convert from phasor form to the time domain*
- real means cosine
- imag means minus sine

```
y(t) = 0.2287
     - 0.2037 cos(t) + 0.1528 sin(t)
     - 0.0398 cos(2t) - 0.0398 sin(2t)
     - 0.0019 cos(3t) - 0.0159 sin(3t)
```

6) Plot x(t) and y(t).

```
y = 0.2287 - 0.2037*cos(t) + 0.1528*sin(t)
     - 0.0398*cos(2*t) - 0.0398*sin(2*t)
     - 0.0019*cos(3*t) - 0.0159*sin(3*t)

plot(t,x,'b',t,y,'r');
```



x(t) (blue) and y(t) (red)

Notes:

- In theory, you need to go out to infinity. In practice, y(t) is almost zero at the 3rd harmonic. Truncating the series at 3 rad/sec is fairly accurate.
- This is actually a really hard problem: find the output of a filter with an arbitrary (i.e. non-sinusoidal) input. Fourier Transforms allow you to solve this type of problem.