ECE 111 - Homework #10

ECE 343 Signals & Systems

1) A filter has the following transfer function

$$Y = \left(\frac{2s+50}{s^2+13s+40}\right)X$$

1a) What is the differential equation relating X and Y?

Cross multiply

$$(s^2 + 13s + 40)Y = (2s + 50)X$$

sY means *the derivative of y* or y'

y'' + 13y' + 40y = 2x' + 50x

1b) Find y(t) assuming $x(t) = 4$

At DC, s = 0

$$Y = \left(\frac{2s+50}{s^2+13s+40}\right)_{s=0} \cdot (4)$$

$$Y = 5.00$$

meaning

y(t) = 5.00

1c) Find y(t) assuming $x(t) = 4\cos(6t)$

Using phasor notation

$$s = j6$$

$$X = 4 + j0 \quad \textit{4 cosine + 0 sine}$$

$$Y = \left(\frac{2s+50}{s^2+13s+40}\right)_{s=j6} \cdot (4 + j0)$$

$$Y = 0.7449 - j2.5259$$

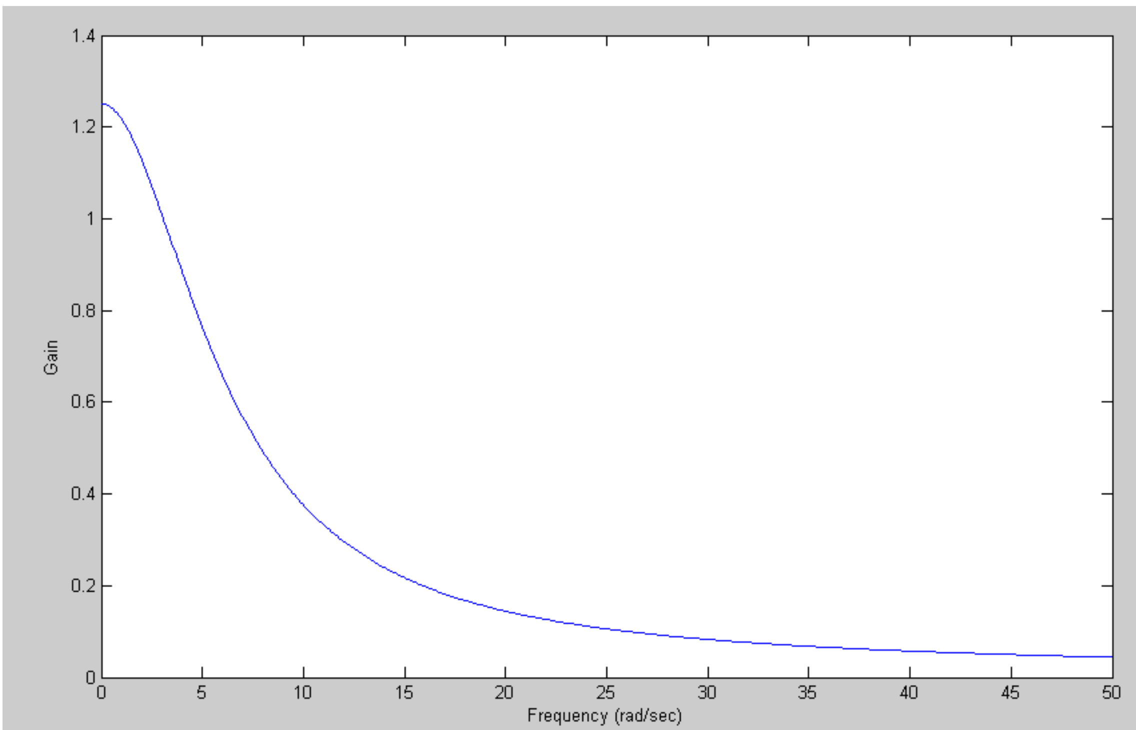
meaning

y(t) = 0.7449 cos(6t) + 2.5259 sin(6t)

2) Plot the gain vs. frequency for this filter from 0 to 50 rad/sec.

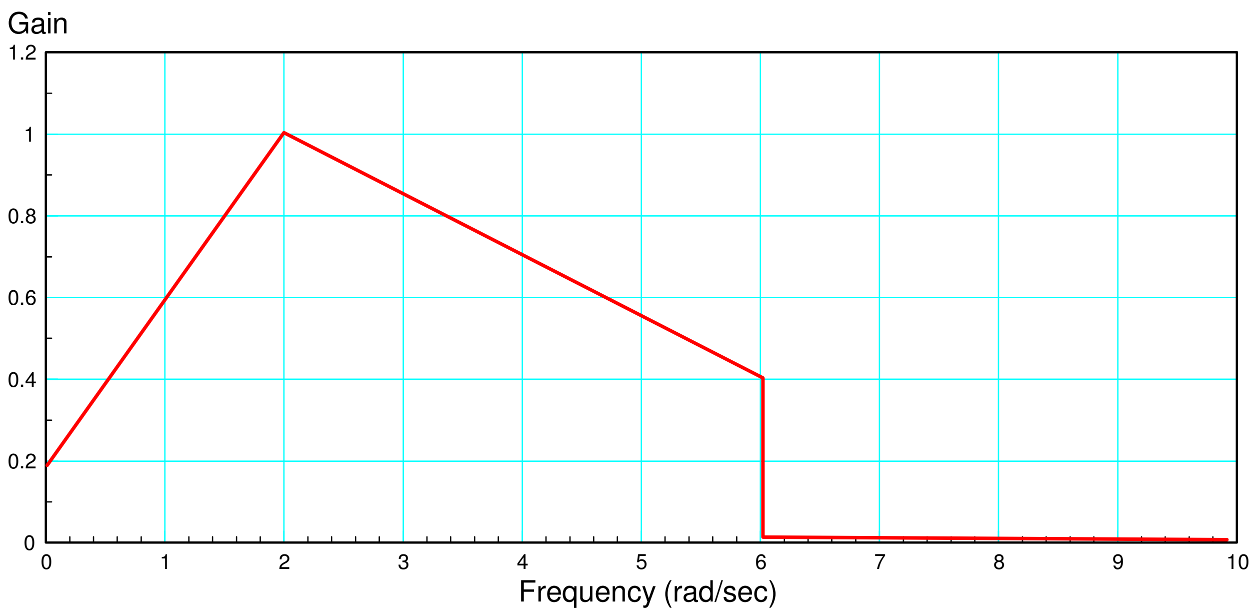
$$Y = \left(\frac{2s+50}{s^2+13s+40}\right)X$$

```
>> w = [0:0.1:50]';
>> s = j*w;
>> Gs = (2*s + 50) ./ (s.^2 + 13*s + 40);
>> plot(w,abs(Gs))
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
>>
```



Problem 3-5) Design a filter of the following form so that the gain matches the graph below:

$$G(s) = \left(\frac{a(s+b)}{\left(s^2+cs+d\right)\left(s^2+es+f\right)} \right)$$



3) Write an m-file, cost.m, which

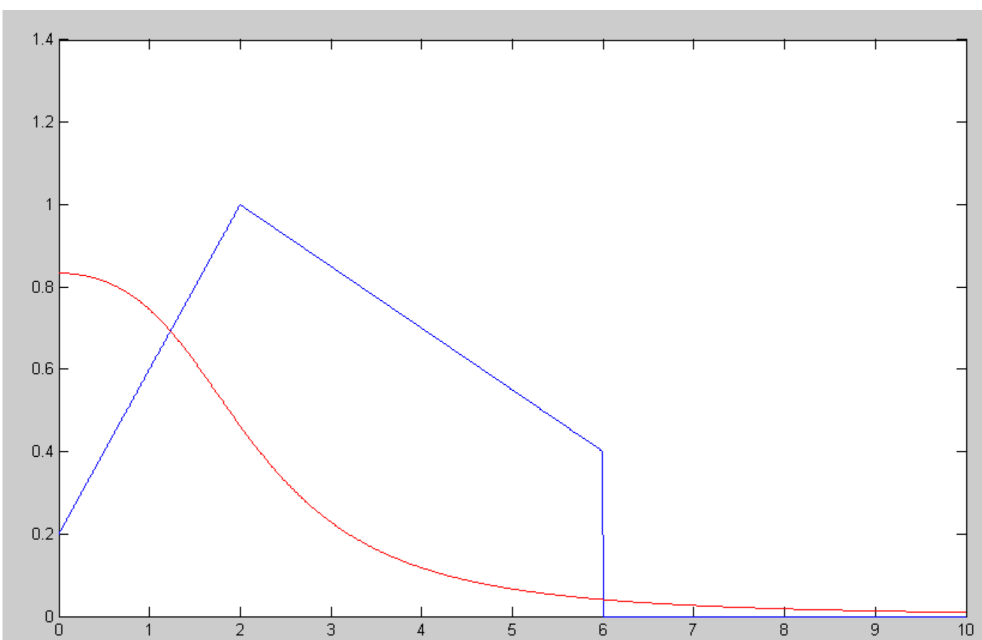
- Is passed a 5-element array, z, with each element representing (a, b, c, d, e)
- Computes the gain, G(s) for this value of (a, b, c, d, e)
- Computes the difference between the gain, G, and the target (above), and
- Returns the sum-squared error in the gain

Code:

```
function [ J ] = costf( z )
 a = z(1);
 b = z(2);
 c = z(3);
 d = z(4);
 e = z(5);
 f = z(6);

 w = [0:0.01:10]';
 s = j*w;

 Gideal = (0.4*w + 0.2).*(w<2) + (1.3 -0.15*w).*(w>=2).*(w<6);

 G = a*(s+b) ./ ( (s.^2 + c*s + d) .* (s.^2 + e*s + f ) );
 G = abs(G);
 E = abs(Gideal) - abs(G);

 J = sum(E .^ 2);

 plot(w,Gideal,w,abs(G),'r');
 ylim([0,1.4]);
 pause(0.01);
end
```

calling:

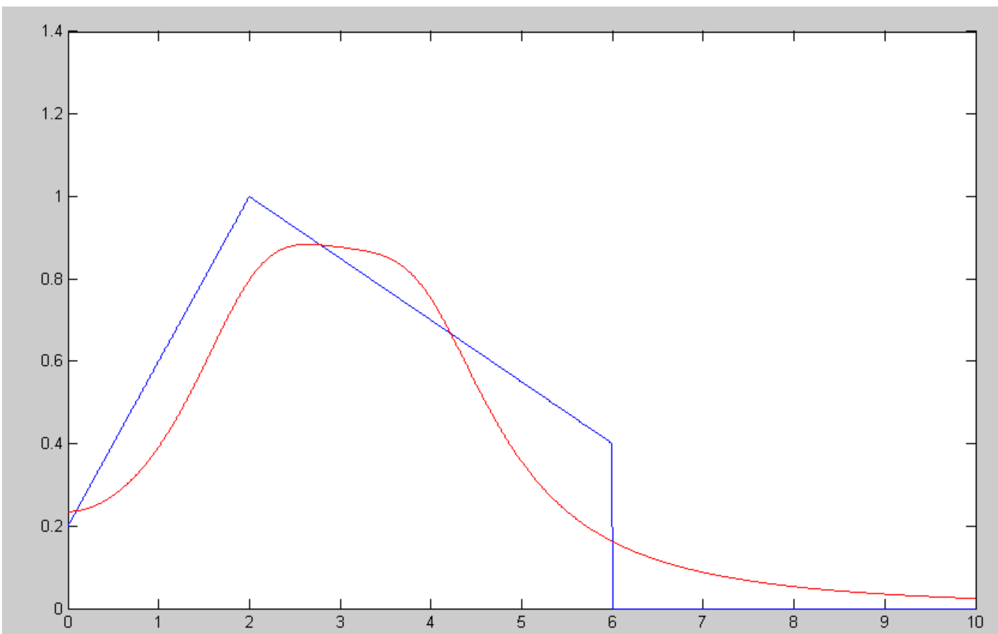
```
>> costf([10,2,3,4,5,6])
   145.2195
```



4) Use your m-file to determine how 'good' the following filter is:

$$G(s) = \left(\frac{a(s+b)}{\left(s^2+cs+d\right)\left(s^2+es+f\right)} \right) = \left(\frac{20(s+1)}{\left(s^2+2s+5\right)\left(s^2+2s+17\right)} \right)$$

```
>> costf([20,1,2,5,2,17])

ans =

   16.0810
```



5) Use fminsearch() to find the 'best' filter of the form

$$G(s) = \left(\frac{a(s+b)}{\left(s^2+cs+d\right)\left(s^2+es+f\right)} \right)$$

```
>> [Z,e] = fminsearch('costf', [20,1,2,5,2,17])

Z =
 21.9001    1.5453    1.5031    4.2629    1.9397   21.8389

e =
   6.6248
```

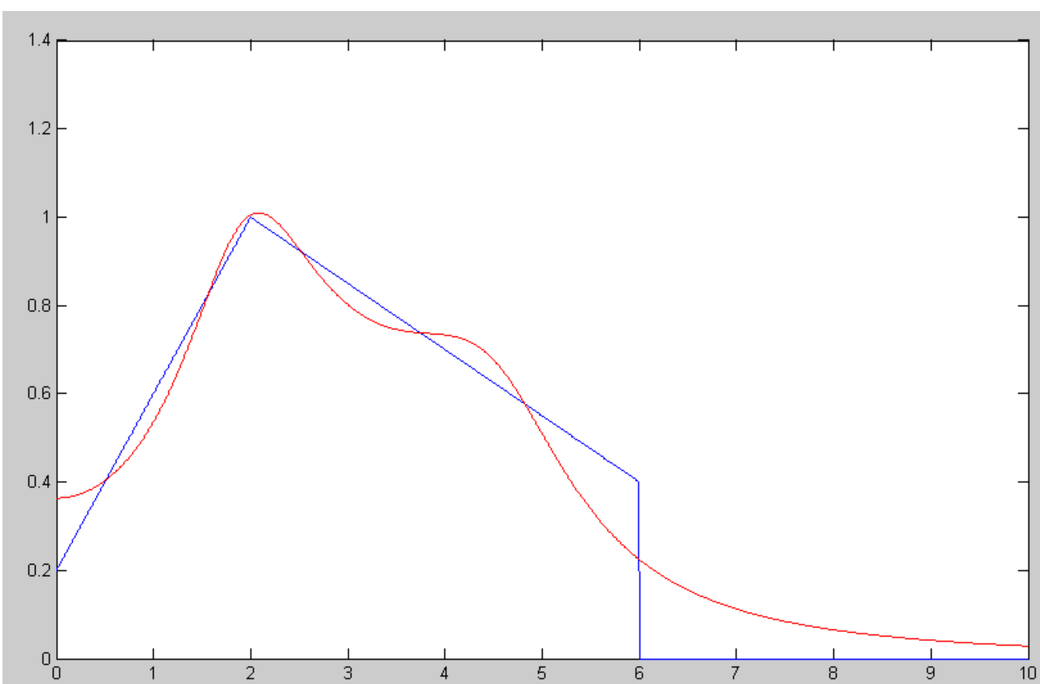
5a) Give the resulting (a, b, c, d, e, f)

```
Z =
    a          b          c          d          e          f
 21.9001    1.5453    1.5031    4.2629    1.9397   21.8389
```

5b) Give the resulting filter, and

$$G(s) = \left(\frac{21.9(s+1.5453)}{\left(s^2+1.6031s+4.2629\right)\left(s^2+1.9397s+21.8389\right)} \right)$$

5c) Plot the 'optimal' filter's gain vs. frequency



'Optimal" filter actual gain vs frequency (red) and desired gain (blue)