ECE 111 - Homework #9

Week #9: ECE 311 Circuits II

1) Solve for Y

- $Y = \left(\frac{(7+j8)(6+j2)}{(6+j3)} \right)$

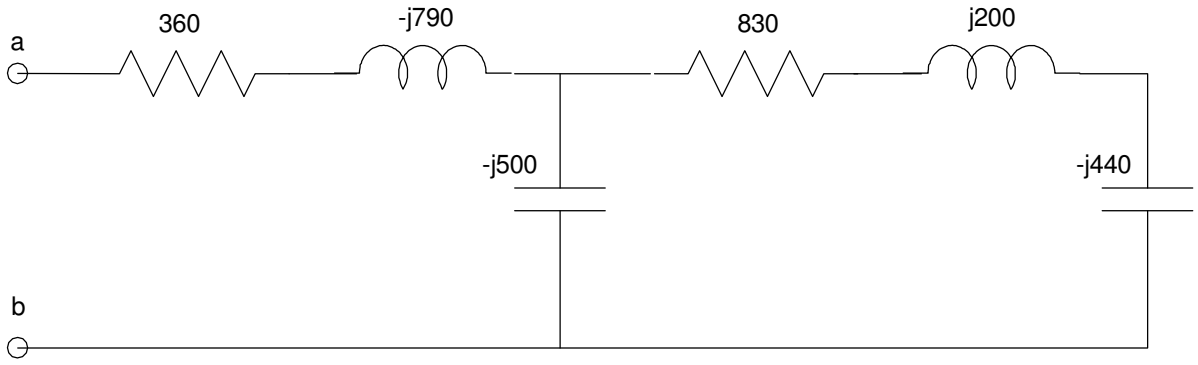
```
>> y = (7+j*8)*(6+j*2) / (6+j*3)

y =   7.6000 + 6.5333i
```

ans:

- $Y = \left(\left(\frac{9+j8}{6-j10} \right) + \left(\frac{2+j9}{5-j5} \right) \right) \left(\frac{6+j7}{9+j10} \right)$

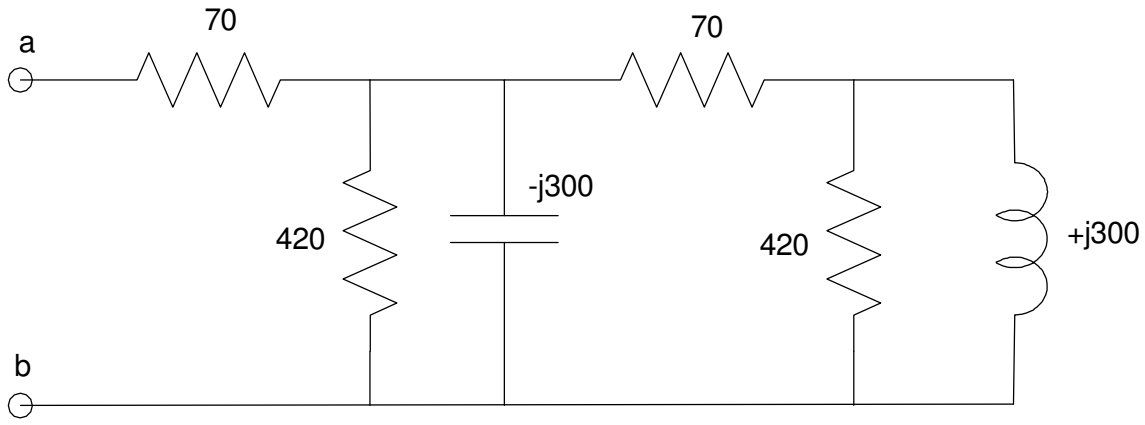
```
>> y = ( (9+j*8)/(6-j*10)  +  (2+j*9)/(5-j*5) ) * ( (6+j*7)/(9+j*10) )

y =    -0.6456 + 1.4340i
```

2) Determine the impedance Zab



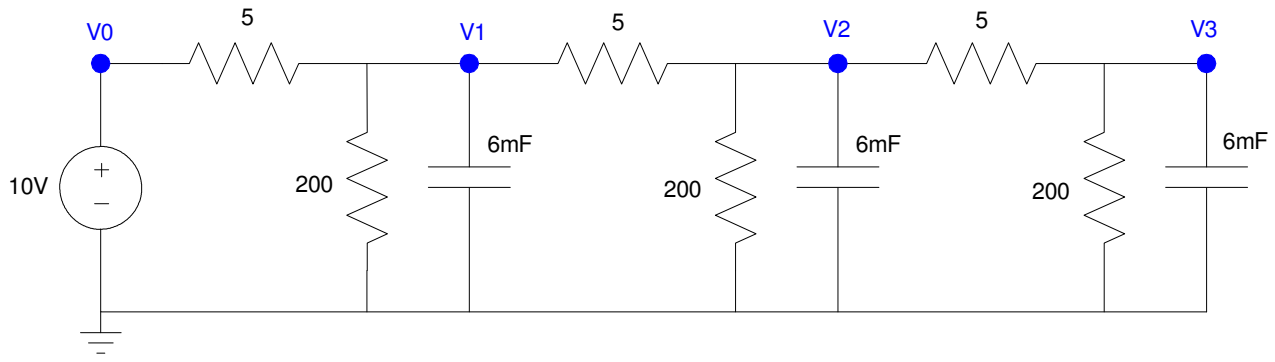

```
>> Z1 = 360 - j*790;
>> Z2 = -j*500;
>> Z3 = 830 + j*200 - j*440;
>> Z23 = 1 / (1/Z2 + 1/Z3)

Z23 =  1.6781e+002 -3.5038e+002i

>> Zab = Z1 + Z23

Zab =  527.81 + 439.616i
```

3) Determine the impeadance Zab




```
>> Z3 = 1 / (1/420 + 1/(j*300))

Z3 =  1.4189e+002 +1.9865e+002i

>> Z4 = 70 + Z3

Z4 =  2.1189e+002 +1.9865e+002i

>> Z5 = 1 / (1/Z4 + 1/420 + 1/(-j*300))

Z5 =  1.9652e+002 -3.9305e+001i

>> Zab = Z5 + 70

Zab =  266.52 -39.305i

>>
```

4) Assume $V_{in} = 10$



a) Determine the impedances of each element at 0 rad/sec

$$R \rightarrow R$$

$$C \rightarrow \infty$$

meaning you can ignore the capacitors at DC (the current will be zero. you can ignore zero)

b) Write the votlage node equations

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{5}\right) + \left(\frac{V_1}{200}\right) + \left(\frac{V_1 - V_2}{5}\right) = 0$$

$$\left(\frac{V_2 - V_1}{5}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{5}\right) = 0$$

$$\left(\frac{V_3 - V_2}{5}\right) + \left(\frac{V_3}{200}\right) = 0$$

c) Solve for V1, V2, and V3.

Group terms

$$V_0 = 10$$

$$-\left(\tfrac{1}{5}\right)V_0 + \left(\tfrac{1}{5}+\tfrac{1}{200}+\tfrac{1}{5}\right)V_1 - \left(\tfrac{1}{5}\right)V_2 = 0$$

$$-\left(\tfrac{1}{5}\right)V_1 + \left(\tfrac{1}{5}+\tfrac{1}{200}+\tfrac{1}{5}\right)V_2 - \left(\tfrac{1}{5}\right)V_3 = 0$$

$$-\left(\tfrac{1}{5}\right)V_2 + \left(\tfrac{1}{5}+\tfrac{1}{200}\right)V_3 = 0$$

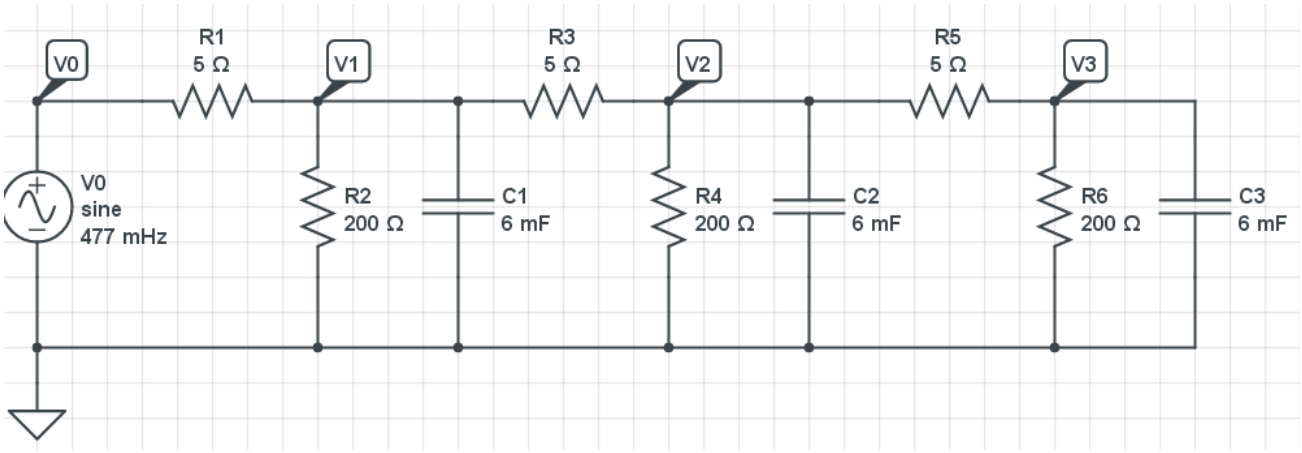
Place in mattrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{5}\right) & \left(\frac{1}{5}+\frac{1}{200}+\frac{1}{5}\right) & \left(\frac{-1}{5}\right) & \\ & \left(\frac{-1}{5}\right) & \left(\frac{1}{5}+\frac{1}{200}+\frac{1}{5}\right) & \left(\frac{-1}{5}\right) \\ & & \left(\frac{-1}{5}\right) & \left(\frac{1}{5}+\frac{1}{200}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

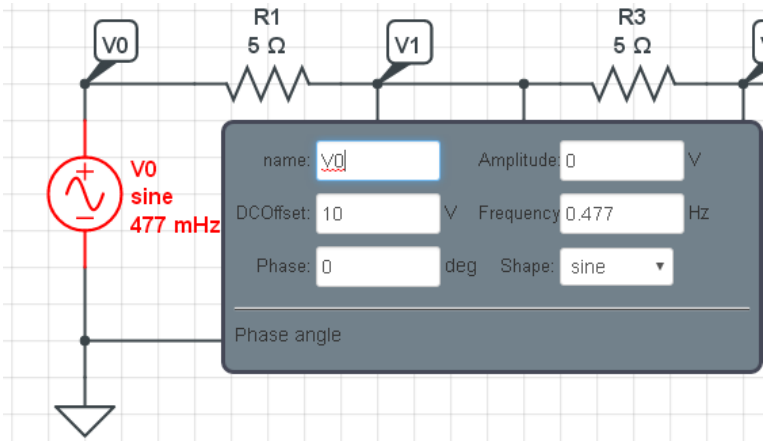
```
>> A = [1,0,0,0 ;
-1/5, 1/5+1/200+1/5, -1/5, 0;
0, -1/5, 1/5+1/200+1/5, -1/5;
0, 0, -1/5, 1/5+1/200]

    1.0000         0         0         0
   -0.2000    0.4050   -0.2000         0
         0   -0.2000    0.4050   -0.2000
         0         0   -0.2000    0.2050

>> B = [10;0;0;0]

    10
     0
     0
     0

>> V = inv(A)*B

V0   10.0000
V1    9.3278
V2    8.8888
V3    8.6720
```

5) Check your results in CircuitLab

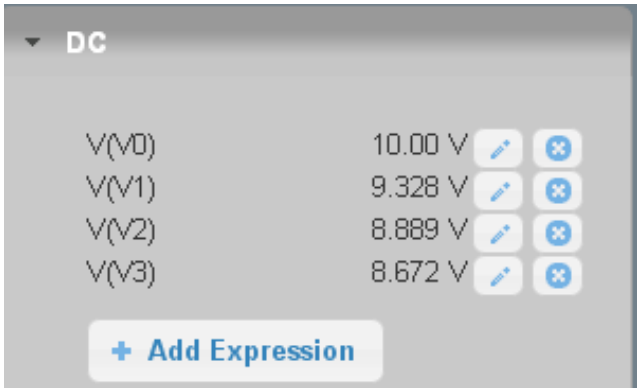


V0 is set up as

- 0V AC
- 10V offset (DC)




```
previous result from
Matlab

V0    10.0000
V1     9.3278
V2     8.8888
V3     8.6720
```



6) Assume Vin is a 10V, 3 rad/sec (0.477Hz)

$$V_{in} = 10\sin(3t)$$

a) Determine the impedances of each element at 3 rad/sec

$$R \rightarrow R$$

$$C \rightarrow \frac{1}{j\omega C} = -j55.56\Omega$$

b) Write the votlage node equations

Same as before but add a term (C)

$$V_0 = -j10$$

$$\left(\frac{V_1 - V_0}{5}\right) + \left(\frac{V_1}{200}\right) + \left(\frac{V_1}{-j55.56}\right) + \left(\frac{V_1 - V_2}{5}\right) = 0$$

$$\left(\frac{V_2 - V_1}{5}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2}{-j55.56}\right) + \left(\frac{V_2 - V_3}{5}\right) = 0$$

$$\left(\frac{V_3 - V_2}{5}\right) + \left(\frac{V_3}{200}\right) + \left(\frac{V_3}{-j55.56}\right) = 0$$

Grouping terms and placing in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{5}\right) & \left(\frac{1}{5}+\frac{1}{200}+\frac{1}{-j55.56}+\frac{1}{5}\right) & \left(\frac{-1}{5}\right) & \\ & \left(\frac{-1}{5}\right) & \left(\frac{1}{5}+\frac{1}{200}+\frac{1}{-j55.56}+\frac{1}{5}\right) & \left(\frac{-1}{5}\right) \\ & & \left(\frac{-1}{5}\right) & \left(\frac{1}{5}+\frac{1}{-j55.56}+\frac{1}{200}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

c) Solve for V1, V2, and V3 as complex numbers

```
A = [1,0,0,0 ;
-1/5, 1/5+1/200+1/(-j*55.56)+1/5, -1/5, 0;
0, -1/5, 1/5+1/200+1/(-j*55.56)+1/5, -1/5;
0, 0, -1/5, 1/5+1/200+1/(-j*55.56)]

B = [-j*10;0;0;0]

V = inv(A)*B
```

```
A =
   1.0000                  0                  0                  0
  -0.2000            0.4050 + 0.0180i  -0.2000                  0
        0           -0.2000             0.4050 + 0.0180i  -0.2000
        0                 0            -0.2000             0.2050 + 0.0180i
B =
        0 -10.0000i
        0
        0
        0
```

```
V =
V0         0 -10.0000i
V1  -1.8978 - 8.6408i
V2  -3.0654 - 7.6684i
V3  -3.6196 - 7.1636i

>> abs(V)

ans =

   10.0000
    8.8467
    8.2584
    8.0261

>>
```

d) Express V1, V2, and V3 in terms of a cosine function:

- real = sine
- imag = -cosine

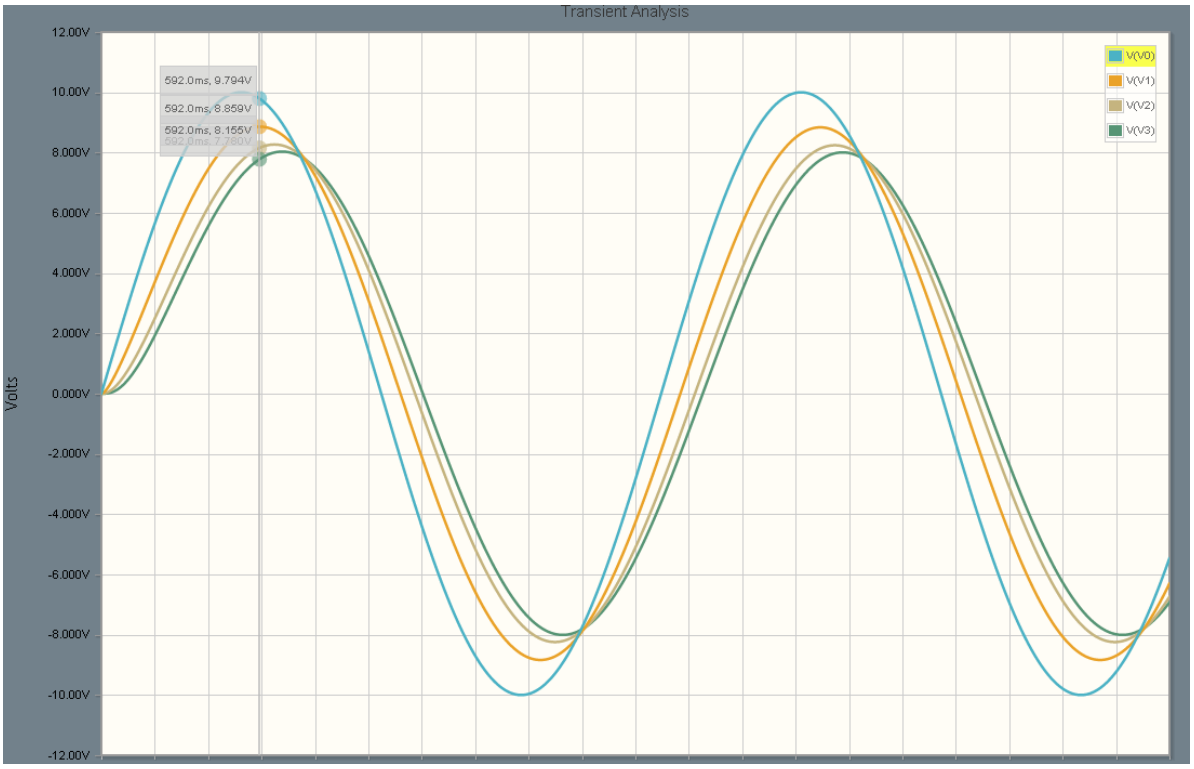
From Matlab

- V0 = 10 sin(3t)
- V1 = -1.8978 cos(3t) + 8.6408 sin(3t)
- V2 = -3.0654 cos(3t) + 7.6684 sin(3t)
- V3 = -3.6196 cos(3t) + 7.1636 sin(3t)

7) Check your results in CircuitLab using a transient simulation for 6 seconds

Change V0 so that

- it's AC amplitude is 10V
- The offset (DC) is 0V
- Run a time-domain simulation



The peak of the waveforms is the amplitude computed in Matlab:

```
>> abs(V)

V0    10.0000          10.00V peak (blue)
V1     8.8467          8.859V peak (orange)
V2     8.2584          8.242V peak (tan)
V3     8.0261          8.006V peak (green)
```

Note: polar form is easier to use for lab data:

- The peak is the magnitude of the voltag in polar form
- The angle is the delay in terms of cycles (-30 degrees is a delay of 30/360 of one cycle)