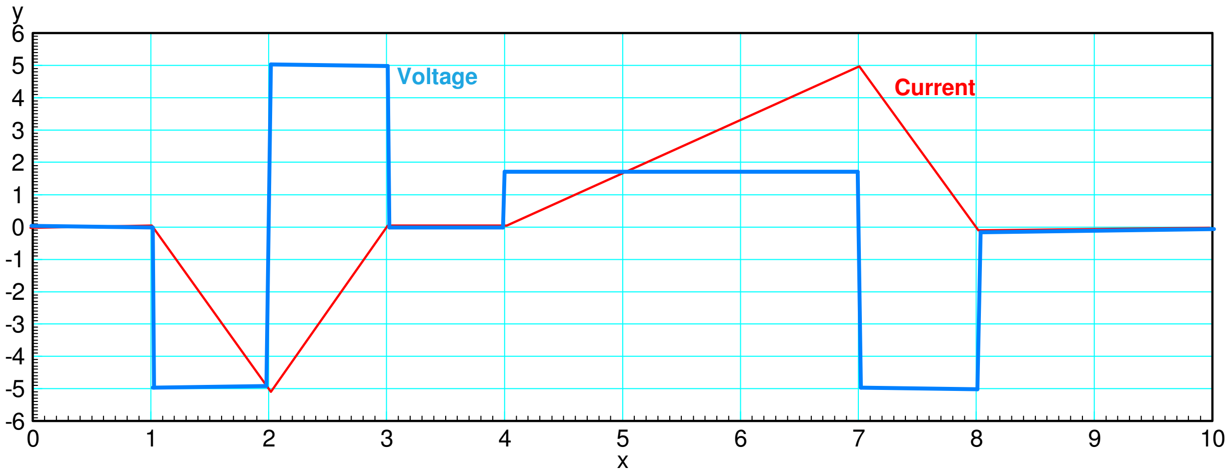
ECE 111 - Homework #8

Week #8: ECE 351 Electromagnetics

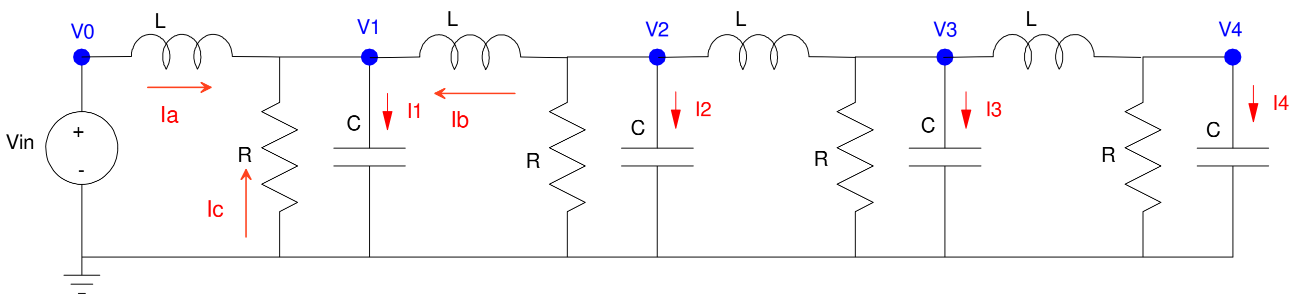
1) Assume the current flowing through a one Henry inductor is shown below. Sketch the voltage.

$$V = L \frac{dI}{dt}$$

The voltage is the derivative of the current (inductors are differentiators)



Problem 2-3) 4-Node RLC Circuit



R = 200Ω, C = 0.05F, L = 0.4H. Repeat for 30 nodes for problems 4-6

2) Write the dynamic equations for the following 4-stage RLC circuit. (i.e. write the node equations)

Write the node equation at V1. The other nodes will be similar

$$I_1 = I_a + I_b + I_c$$

$$I_1 = CV_1' = I_a + I_b + \left(\frac{0-V_1}{R}\right)$$

For inductors

$$V = L\frac{dI}{dt}$$

$$V_0 - V_1 = LI_a'$$

$$V_2 - V_1 = LI_b'$$

Differentiate the second equation (for I1)

$$CV_1'' = I_a' + I_b' - \left(\frac{1}{R}\right)V_1'$$

Substitute for I'

$$CV_1'' = \left(\frac{V_0-V_1}{L}\right) + \left(\frac{V_2-V_1}{L}\right) - \left(\frac{1}{R}\right)V_1'$$

Group terms

$$V_1'' = \left(\frac{1}{LC}\right)V_0 - \left(\frac{2}{LC}\right)V_1 + \left(\frac{1}{LC}\right)V_2 - \left(\frac{1}{RC}\right)V_1'$$

Ditto for nodes 2 and 3. Node #4 is slightly different

$$V_2'' = \left(\frac{1}{LC}\right)V_1 - \left(\frac{2}{LC}\right)V_2 + \left(\frac{1}{LC}\right)V_3 - \left(\frac{1}{RC}\right)V_2'$$

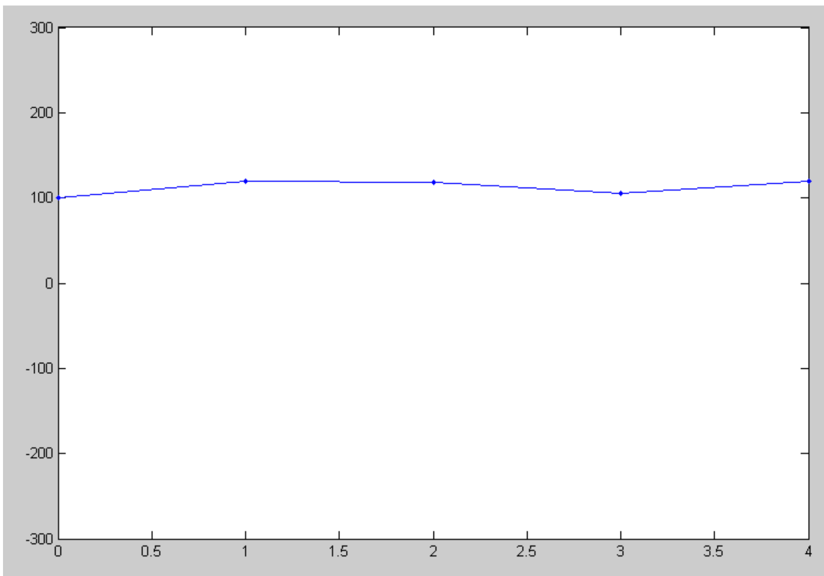
$$V_3'' = \left(\frac{1}{LC}\right)V_2 - \left(\frac{2}{LC}\right)V_3 + \left(\frac{1}{LC}\right)V_4 - \left(\frac{1}{RC}\right)V_3'$$

$$V_4'' = \left(\frac{1}{LC}\right)V_3 - \left(\frac{1}{LC}\right)V_4 - \left(\frac{1}{RC}\right)V_4'$$

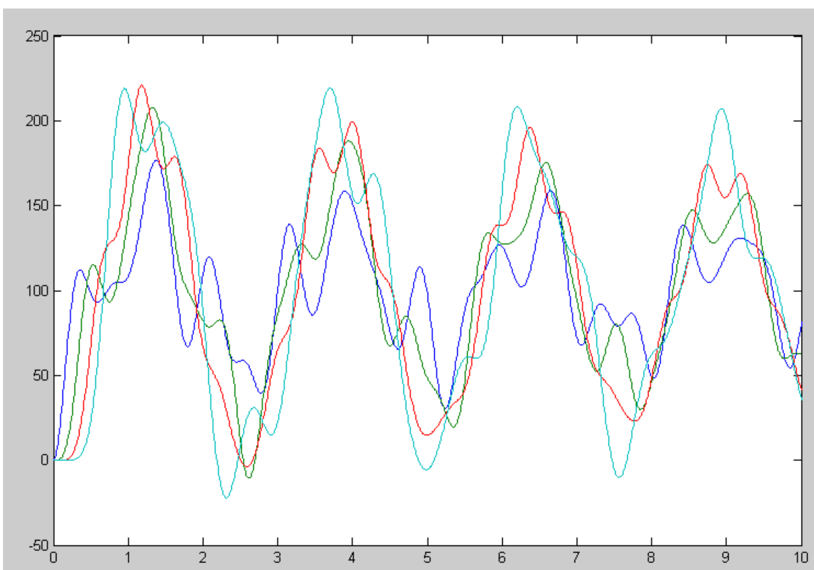
Plugging in numbers

$$V_1'' = 50V_0 - 100V_1 + 50V_2 - 0.1V_1'$$
$$V_2'' = 50V_1 - 100V_2 + 50V_3 - 0.1V_2'$$
$$V_3'' = 50V_2 - 100V_3 + 50V_4 - 0.1V_3'$$
$$V_4'' = 50V_3 - 50V_4 - 0.1V_4'$$

3) Assume Vin = 10V and the initial conditions are zero ($V_1 = V_2 = V_3 = V_4 = 0$). Solve for the voltages at t = 3 seconds. *Hint: Solve numerically using Matlab*



Voltages at t = 3 second



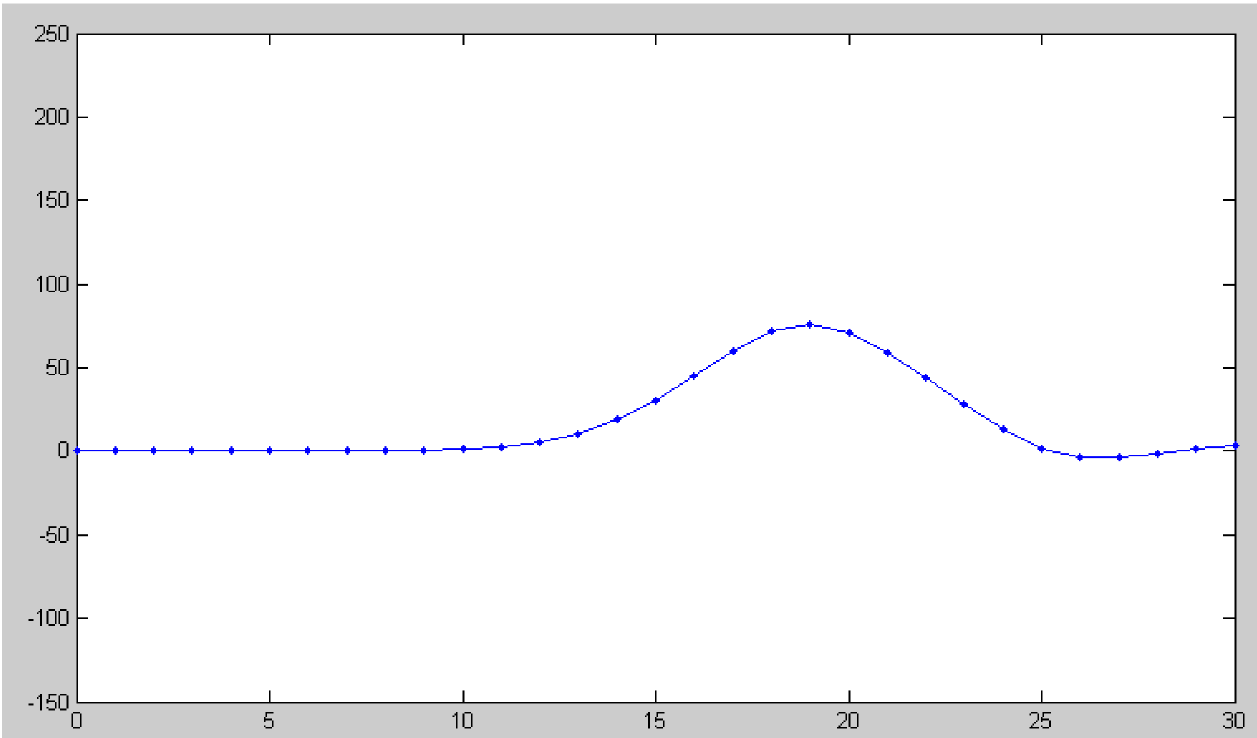
Voltages V1 .. V4 vs time

Code:

```
V0 = 100;
V1 = 0;
V2 = 0;
V3 = 0;
V4 = 0;

dV1 = 0;
dV2 = 0;
dV3 = 0;
dV4 = 0;

V = [];

t = 0;
dt = 0.01;

while(t < 10)

   ddV1 = 50*V0 - 100*V1 + 50*V2 - 0.1*dV1;
   ddV2 = 50*V1 - 100*V2 + 50*V3 - 0.1*dV2;
   ddV3 = 50*V2 - 100*V3 + 50*V4 - 0.1*dV3;
   ddV4 = 50*V3 -  50*V4         - 0.1*dV4;

   dV1 = dV1 + ddV1*dt;
   dV2 = dV2 + ddV2*dt;
   dV3 = dV3 + ddV3*dt;
   dV4 = dV4 + ddV4*dt;

   V1 = V1 + dV1*dt;
   V2 = V2 + dV2*dt;
   V3 = V3 + dV3*dt;
   V4 = V4 + dV4*dt;

   t = t + dt;

   plot([0,1,2,3, 4],[V0,V1,V2,V3, V4],'.-');
   ylim([-300,300]);
   clc
   disp(t)
   pause(0.01);

   V = [V; V1, V2, V3, V4];

   end

t = [1:length(V)]' * dt;
plot(t,V);
```

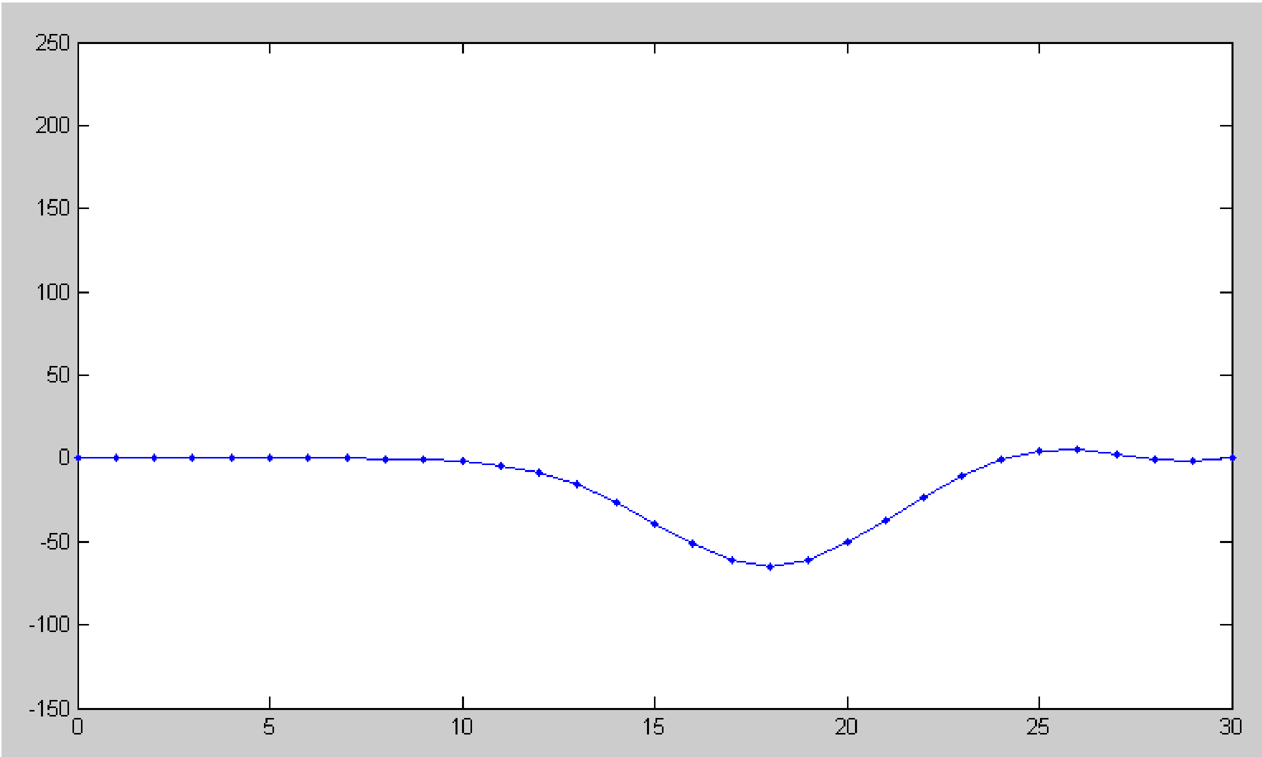
Problem 4-6) 30-Node RLC Circuit (hint: modify the program Wave.m)

4) Expand the RLC circuit from problem #2 to 30 nodes. Plot the voltage at t = 8 seconds (just after the reflection) for $1 / R_{30}C = 0.01$



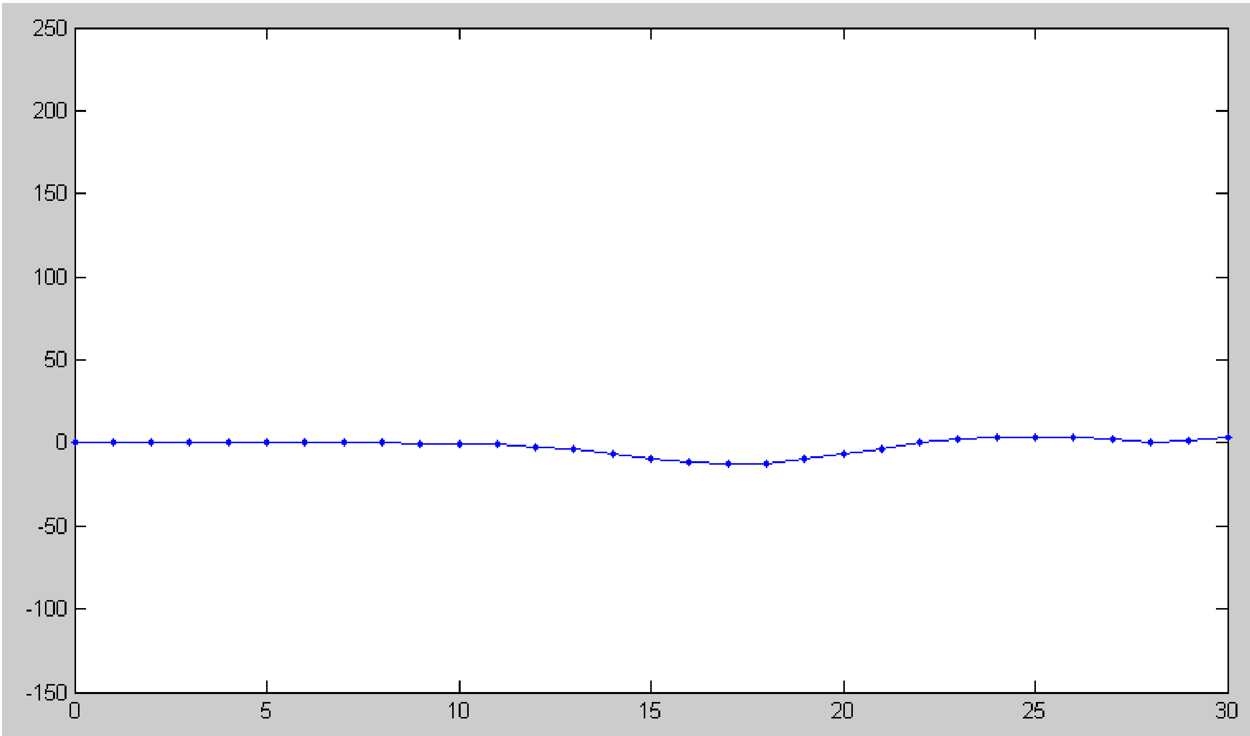
Voltage at t = 7 seconds (just after the reflection) when 1/R30*C = 0.01. (R30 = 2000 Ohms)
A positive reflection results from R being too large

5) Plot the voltage at t = 8 seconds for $1 / R_{30}C = 100$



Voltage at t = 7 seconds (just after the reflection) when 1/R30*C = 100. (R30 = 0.2 Ohms)
A negative reflection results from R being too small

6) Determine experimentally R_{30} so that the reflection is almost zero



Voltage at t = 7 seconds (just after the reflection) when 1/R30*C = 10. (R30 = 2 Ohms)
No reflection results when R is "just right"